GROUP PROJECT

CHAPTER 2

One of the best ways to learn how to *solve* a word problem in algebra is to *design* word problems of your own. Creating a word problem makes you very aware of precisely how much information is needed to solve the problem. You must also focus on the best way to present information to a reader and on how much information to give. As you write your problem, you gain skills that will help you solve problems created by others.

The group should design five different word problems that can be solved using an algebraic equation. All of the problems should be on different topics. For example, the group should not have more than one problem on finding a number. The group should turn in both the problems and their algebraic solutions.

Chapter 2 Summary

Definitions and Concepts	Examples
Section 2.1 The Addition Property of Equality	
A linear equation in one variable can be written in the form $ax + b = c$, where a is not zero.	$3x + 7 = 9$ is a linear equation.
Equivalent equations have the same solution.	$2x - 4 = 6$, $2x = 10$, and $x = 5$ are equivalent equations.
The Addition Property of Equality Adding the same number (or algebraic expression) to both sides of an equation or subtracting the same number (or algebraic expression) from both sides of an equation does not change its solution.	• $x - 3 = 8$ $x - 3 + 3 = 8 + 3$ $x = 11$ • $x + 4 = 10$ $x + 4 - 4 = 10 - 4$ $x = 6$
Section 2.2 The Multiplication Property of Equality	
The Multiplication Property of Equality Multiplying both sides of an equation or dividing both sides of an equation by the same nonzero real number (or algebraic expression) does not change the solution.	• $\frac{x}{-5} = 6$ $-5\left(\frac{x}{-5}\right) = -5(6)$ $x = -30$ • $-50 = -5y$ $\frac{-50}{-5} = \frac{-5y}{-5}$ $10 = y$
Equations and Coefficients of -1 If $-x = c$, multiply both sides by -1 to solve for x. The solution is the opposite, or additive inverse, of c.	$-x = -12$ $(-1)(-x) = (-1)(-12)$ $x = 12$

Definitions and Concepts	Examples

Section 2.2 The Multiplication Property of Equality (continued)

Using the Addition and Multiplication Properties

If an equation does not contain fractions,

- Use the addition property to isolate the variable term.
- Use the multiplication property to isolate the variable.

$$\begin{aligned} -2x - 5 &= 11 \\ -2x - 5 + 5 &= 11 + 5 \\ -2x &= 16 \\ \frac{-2x}{-2} &= \frac{16}{-2} \\ x &= -8 \end{aligned}$$

Section 2.3 Solving Linear Equations

Solving a Linear Equation

1. Simplify each side.
2. Collect all the variable terms on one side and all the constant terms on the other side.
3. Isolate the variable and solve. (If the variable is eliminated and a false statement results, the inconsistent equation has no solution. If a true statement results, all real numbers are solutions of the identity.)
4. Check the proposed solution in the original equation.

Solve: $7 - 4(x - 1) = x + 1.$

$$\begin{aligned} 7 - 4x + 4 &= x + 1 \\ -4x + 11 &= x + 1 \\ -4x - x + 11 &= x - x + 1 \\ -5x + 11 &= 1 \\ -5x + 11 - 11 &= 1 - 11 \\ -5x &= -10 \\ \frac{-5x}{-5} &= \frac{-10}{-5} \\ x &= 2 \end{aligned}$$

Check:

$$\begin{aligned} 7 - 4(x - 1) &= x + 1 \\ 7 - 4(2 - 1) &\stackrel{?}{=} 2 + 1 \\ 7 - 4(1) &\stackrel{?}{=} 2 + 1 \\ 7 - 4 &\stackrel{?}{=} 2 + 1 \\ 3 &= 3, \text{ true} \end{aligned}$$

The solution is 2, or the solution set is $\{2\}$.

Equations Containing Fractions

Multiply both sides (all terms) by the least common denominator. This clears the equation of fractions.

Solve: $\frac{x}{5} + \frac{1}{2} = \frac{x}{2} - 1.$

$$\begin{aligned} 10\left(\frac{x}{5} + \frac{1}{2}\right) &= 10\left(\frac{x}{2} - 1\right) \\ 10 \cdot \frac{x}{5} + 10 \cdot \frac{1}{2} &= 10 \cdot \frac{x}{2} - 10 \cdot 1 \\ 2x + 5 &= 5x - 10 \\ -3x &= -15 \\ x &= 5 \end{aligned}$$

The solution is 5, or the solution set is $\{5\}$.

Equations Containing Decimals

An equation may be cleared of decimals by multiplying every term on both sides by a power of 10. The exponent on 10 will equal the greatest number of decimal places in the equation.

Solve: $1.4(x - 5) = 3x - 3.8$

$$\begin{aligned} 1.4x - 7 &= 3x - 3.8 \\ 10(1.4x) - 10(7) &= 10(3x) - 10(3.8) \\ 14x - 70 &= 30x - 38 \\ -16x &= 32 \\ x &= -2 \end{aligned}$$

The solution is -2, or the solution set is $\{-2\}$.

Definitions and Concepts	Examples

Section 2.3 Solving Linear Equations (continued)

Inconsistent equations with no solution (solution set: $\varnothing$) result in false statements in the solution process.
Identities, true for every real number (solution set: $\{x \mid x \text{ is a real number}\}$) result in true statements in the solution process.

Solve:
$$3x + 2 = 3(x + 5).$$
$$3x + 2 = 3x + 15$$
$$3x + 2 - 3x = 3x + 15 - 3x$$
$$2 = 15 \quad \text{(false)}$$

No solution: $\varnothing$

Solve:
$$2(x + 4) = x + x + 8.$$
$$2x + 8 = 2x + 8$$
$$2x + 8 - 2x = 2x + 8 - 2x$$
$$8 = 8 \quad \text{(true)}$$

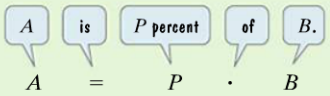
$\{x \mid x \text{ is a real number}\}$

Section 2.4 Formulas and Percents

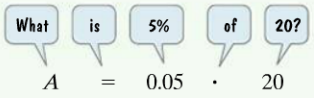
To solve a formula for one of its variables, use the steps for solving a linear equation and isolate the specified variable on one side of the equation.

Solve for l:
$$w = \frac{P - 2l}{2}.$$
$$2w = 2\left(\frac{P - 2l}{2}\right)$$
$$2w = P - 2l$$
$$2w - P = P - P - 2l$$
$$2w - P = -2l$$
$$\frac{2w - P}{-2} = \frac{-2l}{-2}$$
$$\frac{2w - P}{-2} = l$$

A Formula Involving Percent



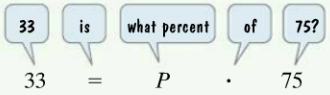
In the formula $A = PB$, P is expressed as a decimal.

- 

$$A = 1$$
Thus, 1 is 5% of 20.

- 

$$\frac{6}{0.3} = B$$
$$20 = B$$
Thus, 6 is 30% of 20.

- 

$$\frac{33}{75} = P$$
$$P = 0.44 = 44\%$$
Thus, 33 is 44% of 75.

Definitions and Concepts	Examples

Section 2.5 An Introduction to Problem Solving

Strategy for Solving Word Problems

Step 1. Let x represent one of the quantities.

Step 2. Represent other unknown quantities in terms of x.

Step 3. Write an equation that models the conditions.

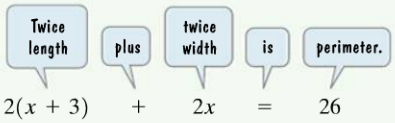
Step 4. Solve the equation and answer the question.

Step 5. Check the proposed solution in the original wording of the problem.

The length of a rectangle exceeds the width by 3 inches. The perimeter is 26 inches. What are the rectangle's dimensions?

Let x = the width.

$x + 3$ = the length



$$2(x + 3) + 2x = 26$$

$$2x + 6 + 2x = 26$$

$$4x + 6 = 26$$

$$4x = 20$$

$$x = 5$$

The width (x) is 5 inches and the length ($x + 3$) is $5 + 3$, or 8 inches.

$$\text{Perimeter} = 2(5 \text{ in.}) + 2(8 \text{ in.})$$

$$= 10 \text{ in.} + 16 \text{ in.} = 26 \text{ in.}$$

This checks with the given perimeter.

Section 2.6 Problem Solving in Geometry

Solving geometry problems often requires using basic geometric formulas. Formulas for perimeter, area, circumference, and volume are given in **Table 2.3** (page 169), **Table 2.4** (page 170), and **Table 2.5** (page 171) in Section 2.6.

A sailboat's triangular sail has an area of 24 ft^2 and a base of 8 ft. Find its height.

$$A = \frac{1}{2}bh$$

$$24 = \frac{1}{2}(8)h$$

$$24 = 4h$$

$$6 = h$$

The sail's height is 6 ft.

Definitions and Concepts	Examples

Section 2.6 Problem Solving in Geometry (continued)

The sum of the measures of the three angles of any triangle is 180°.

In a triangle, the first angle measures 3 times the second and the third measures 40° less than the second. Find each angle's measure.

$$\text{Second angle} = x$$
$$\text{First angle} = 3x$$
$$\text{Third angle} = x - 40$$

Sum of measures is 180°.

$$x + 3x + (x - 40) = 180$$
$$5x - 40 = 180$$
$$5x = 220$$
$$x = 44$$

The angles measure $x = 44$, $3x = 3 \cdot 44 = 132$, and $x - 40 = 44 - 40 = 4$.

The angles measure 44°, 132°, and 4°.

Two complementary angles have measures whose sum is 90°. Two supplementary angles have measures whose sum is 180°. If an angle measures x, its complement measures $90 - x$, and its supplement measures $180 - x$.

An angle measures five times its complement. Find the angle's measure.

$$x = \text{angle's measure}$$
$$90 - x = \text{measure of complement}$$
$$x = 5(90 - x)$$
$$x = 450 - 5x$$
$$6x = 450$$
$$x = 75$$

The angle measures 75°.

Section 2.7 Solving Linear Inequalities

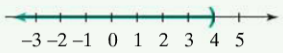
A linear inequality in one variable can be written in one of these forms:

$$ax + b < c \qquad ax + b \le c.$$
$$ax + b > c \qquad ax + b \ge c.$$

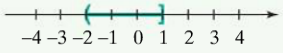
$3x + 6 > 12$ is a linear inequality.

Graphs of solutions to linear inequalities are shown on a number line by shading all points representing numbers that are solutions. Square brackets, [ ], indicate endpoints that are solutions. Parentheses, ( ), indicate endpoints that are not solutions.

- Graph the solutions of $x < 4$.



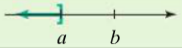
- Graph the solutions of $-2 < x \le 1$.



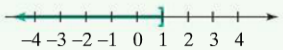
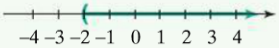
Definitions and Concepts	Examples

Section 2.7 Solving Linear Inequalities (continued)

Solutions of inequalities can be expressed in interval notation or set-builder notation.

Inequality	Interval Notation	Set-Builder Notation	Graph
$x > b$	(b, ∞)	$\{x \mid x > b\}$	
$x \leq a$	$(-\infty, a]$	$\{x \mid x \leq a\}$	

Express the solution set in interval notation and graph the interval:

- $x \leq 1$
  $(-\infty, 1]$ 
- $x > -2$
  $(-2, \infty)$ 

The Addition Property of Inequality

Adding the same number to both sides of an inequality or subtracting the same number from both sides of an inequality does not change the solutions.

$$x + 3 < 8$$
$$x + 3 - 3 < 8 - 3$$
$$x < 5$$

The Positive Multiplication Property of Inequality

Multiplying or dividing both sides of an inequality by the same positive number does not change the solutions.

$$\frac{x}{6} \geq 5$$
$$6 \cdot \frac{x}{6} \geq 6 \cdot 5$$
$$x \geq 30$$

The Negative Multiplication Property of Inequality

Multiplying or dividing both sides of an inequality by the same negative number and reversing the direction of the inequality sign does not change the solutions.

$$-3x \leq 12$$
$$\frac{-3x}{-3} \geq \frac{12}{-3}$$
$$x \geq -4$$

Solving Linear Inequalities

Use the procedure for solving linear equations. When multiplying or dividing by a negative number, reverse the direction of the inequality symbol. Express the solution set in interval or set-builder notation, and graph the set on a number line. If the variable is eliminated and a false statement results, the inequality has no solution. The solution set is $\varnothing$, the empty set. If a true statement results, the solution is the set of all real numbers: $(-\infty, \infty)$ or $\{x \mid x \text{ is a real number}\}$.

Solve: $x + 4 \geq 6x - 16$

$$x + 4 - 6x \geq 6x - 16 - 6x$$
$$-5x + 4 \geq -16$$
$$-5x + 4 - 4 \geq -16 - 4$$
$$-5x \geq -20$$
$$\frac{-5x}{-5} \leq \frac{-20}{-5}$$
$$x \leq 4$$

Solution set: $(-\infty, 4]$ or $\{x \mid x \leq 4\}$

CHAPTER 2 REVIEW EXERCISES

2.1 *Solve each equation in Exercises 1–5 using the addition property of equality. Be sure to check proposed solutions.*

1. $x - 10 = 22$

2. $-14 = y + 8$

3. $7z - 3 = 6z + 9$

4. $4(x + 3) = 3x - 10$

5. $6x - 3x - 9 + 1 = -5x + 7x - 3$

2.2 *Solve each equation in Exercises 6–13 using the multiplication property of equality. Be sure to check proposed solutions.*

6. $\frac{x}{8} = 10$

7. $\frac{y}{-8} = 7$

8. $7z = 77$

9. $-36 = -9y$

10. $\frac{3}{5}x = -9$

11. $30 = -\frac{5}{2}y$

12. $-x = 25$

13. $\frac{-x}{10} = -1$

Solve each equation in Exercises 14–18 using both the addition and multiplication properties of equality. Check proposed solutions.

14. $4x + 9 = 33$

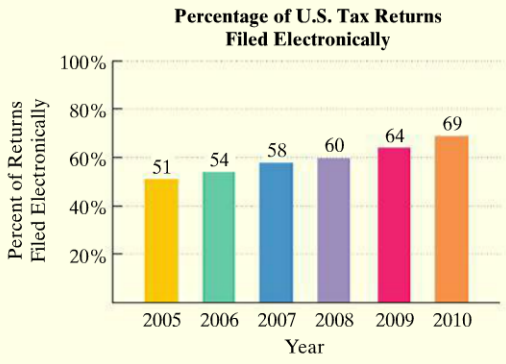
15. $-3y - 2 = 13$

16. $5z + 20 = 3z$

17. $5x - 3 = x + 5$

18. $3 - 2x = 9 - 8x$

19. The percentage of tax returns filed electronically in the United States exceeded 50% for the first time in 2005. The bar graph shows the percentage of electronically filed tax returns from 2005 through 2010.



Source: IRS

The mathematical model

$$p = 3.5n + 51$$

describes the percentage of U.S. tax returns filed electronically, p, n years after 2005.

a. Does the formula underestimate or overestimate the percentage of tax returns filed electronically in 2009? By how much?

b. If trends shown by the formula continue, when will 93% of tax returns be filed electronically?

2.3 *Solve and check each equation in Exercises 20–30.*

20. $5x + 9 - 7x + 6 = x + 18$

21. $3(x + 4) = 5x - 12$

22. $1 - 2(6 - y) = 3y + 2$

23. $2(x - 4) + 3(x + 5) = 2x - 2$

24. $-2(y - 4) - (3y - 2) = -2 - (6y - 2)$

25. $\frac{2x}{3} = \frac{x}{6} + 1$

26. $\frac{x}{2} - \frac{1}{10} = \frac{x}{5} + \frac{1}{2}$

27. $0.5x + 8.75 = 13.25$

28. $0.1(x - 3) = 1.1 - 0.25x$

29. $3(8x - 1) = 6(5 + 4x)$

30. $4(2x - 3) + 4 = 8x - 8$

31. The formula $H = 0.7(220 - a)$ can be used to determine target heart rate during exercise, H, in beats per minute, by a person of age a. If the target heart rate is 133 beats per minute, how old is that person?

2.4 *In Exercises 32–36, solve each formula for the specified variable.*

32. $I = Pr$ for r

33. $V = \frac{1}{3}Bh$ for h

34. $P = 2l + 2w$ for w

35. $A = \frac{B + C}{2}$ for B

36. $T = D + pm$ for m

37. What is 8% of 120?

38. 90 is 45% of what?

39. 36 is what percent of 75?

40. If 6 is increased to 12, the increase is what percent of the original number?

41. If 5 is decreased to 3, the decrease is what percent of the original number?

42. A college that had 40 students for each lecture course increased the number to 45 students. What is the percent increase in the number of students in a lecture course?

43. Consider the following statement:

> My portfolio fell 10% last year, but then it rose 10% this year, so at least I recouped my losses.

Is this statement true? In particular, suppose you invested $10,000 in the stock market last year. How much money would be left in your portfolio with a 10% fall and then a 10% rise? If there is a loss, what is the percent decrease in your portfolio?

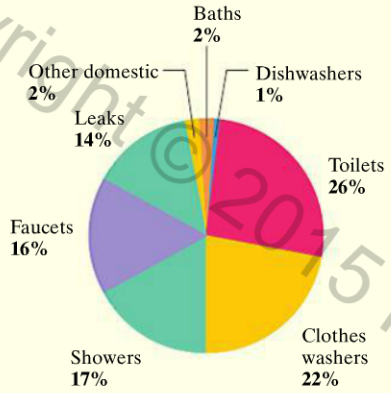
44. The radius is one of two bones that connect the elbow and the wrist. The formula $r = \frac{h}{7}$ models the length of a woman's radius, r, in inches, and her height, h, in inches.

a. Solve the formula for h.

b. Use the formula in part (a) to find a woman's height if her radius is 9 inches long.

45. Every day, the average U.S. household uses 91 gallons of water flushing toilets. The circle graph shows that this represents 26% of the total number of gallons of water used per day. How many gallons of water does the average U.S. household use per day?

Where United States Households Use Water



Source: American Water Works Association

2.5 *In Exercises 46–53, use the five-step strategy to solve each problem.*

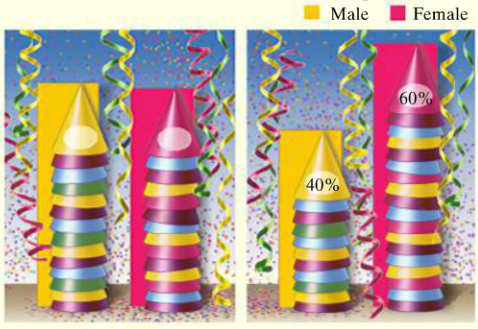
46. Six times a number, decreased by 20, is four times the number. Find the number.

47. In 2010, the wealthiest Americans were Bill Gates and Warren Buffett. At that time, Gates's net worth exceeded that of Buffett's by $9 billion. Combined, the two men were worth $99 billion. Determine each man's net worth in 2010. (*Source: Forbes*)

48. Two pages that face each other in a book have 93 as the sum of their page numbers. What are the page numbers?

49. The graph shows the gender breakdown of the U.S. population at each end of the age spectrum.

Gender Breakdown of the American Population

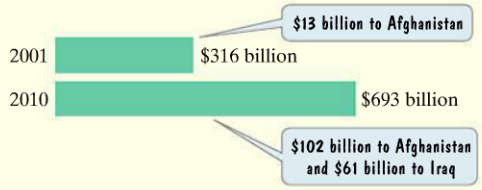


Source: U.S. Census Bureau

For Americans under 20, the percentage of males is greater than the percentage of females and these percents are consecutive odd integers. What percentage of Americans younger than 20 are females and what percentage are males?

50. In 2001, the U.S. defense budget was $316 billion, increasing by approximately $42 billion per year for the period shown by the bar graph. If this trend continues, in how many years after 2001 will the defense budget be $904 billion? In which year will that be?

U.S. Defense Budget



Source: Office of Management and Budget, www.whitehouse.gov <http://www.whitehouse.gov>

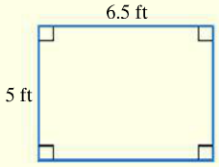
51. A bank's total monthly charge for a checking account is $6 plus $0.05 per check. If your total monthly charge is $6.90, how many checks did you write during that month?

52. A rectangular field is three times as long as it is wide. If the perimeter of the field is 400 yards, what are the field's dimensions?

53. After a 25% reduction, you purchase a table for $180. What was the table's price before the reduction?

2.6 *Use a formula for area to find the area of each figure in Exercises 54–57.*

54.



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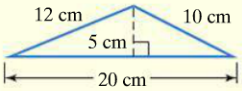

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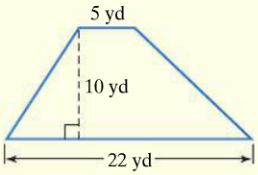

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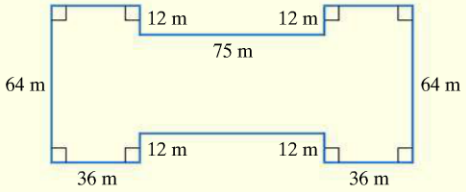


58. Find the circumference and the area of a circle with a diameter of 20 meters. Round answers to the nearest whole number.

59. A sailboat has a triangular sail with an area of 42 square feet and a base that measures 14 feet. Find the height of the sail.

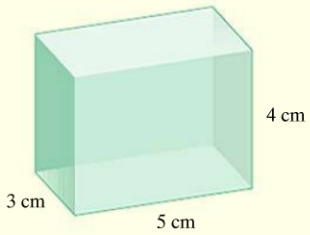
60. A rectangular kitchen floor measures 12 feet by 15 feet. A stove on the floor has a rectangular base measuring 3 feet by 4 feet, and a refrigerator covers a rectangular area of the floor measuring 3 feet by 4 feet. How many square feet of tile will be needed to cover the kitchen floor not counting the area used by the stove and the refrigerator?

61. A yard that is to be covered with mats of grass is shaped like a trapezoid. The bases are 80 feet and 100 feet, and the height is 60 feet. What is the cost of putting the grass mats on the yard if the landscaper charges $0.35 per square foot?

62. Which one of the following is a better buy: a medium pizza with a 14-inch diameter for $6.00 or two small pizzas, each with an 8-inch diameter, for $6.00?

Use a formula for volume to find the volume of each figure in Exercises 63–65. Where applicable, express answers in terms of π. Then round to the nearest whole number.

63.



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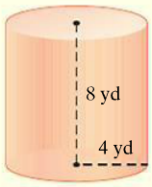

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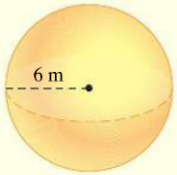
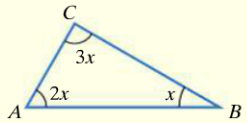


66. A train is being loaded with freight containers. Each box is 8 meters long, 4 meters wide, and 3 meters high. If there are 50 freight containers, how much space is needed?

67. A cylindrical fish tank has a diameter of 6 feet and a height of 3 feet. How many tropical fish can be put in the tank if each fish needs 5 cubic feet of water?

68. Find the measure of each angle of the triangle shown in the figure.



69. In a triangle, the measure of the first angle is 15° more than twice the measure of the second angle. The measure of the third angle exceeds that of the second angle by 25°. What is the measure of each angle?

70. Find the measure of the complement of a 57° angle.

71. Find the measure of the supplement of a 75° angle.

72. How many degrees are there in an angle that measures 25° more than the measure of its complement?

73. The measure of the supplement of an angle is 45° less than four times the measure of the angle. Find the measure of the angle and its supplement.

2.7

In Exercises 74–75, graph the solution of each inequality on a number line.

74. $x < -1$

75. $-2 < x \le 4$

In Exercises 76–77, express the solution set of each inequality in interval notation and graph the interval.

76. $x \ge \frac{3}{2}$

77. $x < 0$

In Exercises 78–85, solve each inequality and graph the solution set on a number line. It is not necessary to provide graphs if the inequality has no solution or is true for all real numbers.

78. $2x - 5 < 3$

79. $\frac{x}{2} > -4$

80. $3 - 5x \le 18$

81. $4x + 6 < 5x$

82. $6x - 10 \ge 2(x + 3)$

83. $4x + 3(2x - 7) \le x - 3$

84. $2(2x + 4) > 4(x + 2) - 6$

85. $-2(x - 4) \le 3x + 1 - 5x$

86. To pass a course, a student must have an average on three examinations of at least 60. If a student scores 42 and 74 on the first two tests, what must be earned on the third test to pass the course?

87. You can spend at most $2000 for a catered party. The caterer charges a setup fee of $350 and $55 per person. How many people can you invite while staying within your budget?

CHAPTER 2 TEST

CHAPTER Test Prep VIDEOS

Step-by-step test solutions are found on the Chapter Test Prep Videos available in MyMathLab® or on YouTube (search "BlitzerCombinedAlg" and click on "Channels").

In Exercises 1–7, solve each equation.

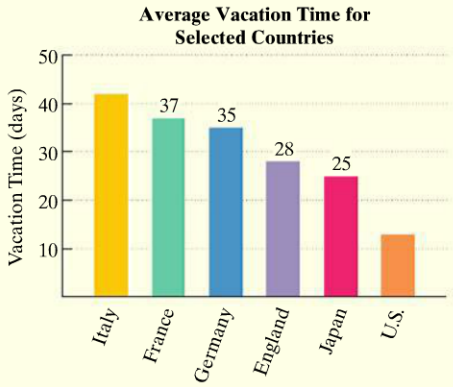
1. $4x - 5 = 13$
2. $12x + 4 = 7x - 21$
3. $8 - 5(x - 2) = x + 26$
4. $3(2y - 4) = 9 - 3(y + 1)$
5. $\frac{3}{4}x = -15$
6. $\frac{x}{10} + \frac{1}{3} = \frac{x}{5} + \frac{1}{2}$
7. $9.2x - 80.1 = 21.3x - 19.6$
8. The formula $P = 2.4x + 180$ models U.S. population, P, in millions x years after 1960. How many years after 1960 is the U.S. population expected to reach 324 million? In which year is this expected to occur?

In Exercises 9–10, solve each formula for the specified variable.

9. $V = \pi r^2 h$ for h
10. $l = \frac{P - 2w}{2}$ for w
11. What is 6% of 140?
12. 120 is 80% of what?
13. 12 is what percent of 240?

In Exercises 14–18, solve each problem.

14. The product of 5 and a number, decreased by 9, is 306. What is the number?
15. Compared with other major countries, American employees have less vacation time. The bar graph shows the average number of vacation days per person for selected countries.



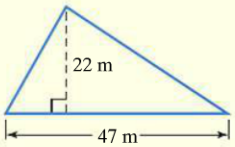
Source: World Development Indicators

The average time Italians spend on vacation exceeds the average American vacation time by 29 days. The combined average vacation time for Americans and Italians is 55 days. On average, how many days do Americans spend on vacation and how many days do Italians spend on vacation?

16. A phone plan has a monthly fee of \$15.00 and a rate of \$0.05 per minute. How many calling minutes can you use in a month for a total cost, including the \$15.00, of \$45.00?
17. A rectangular field is twice as long as it is wide. If the perimeter of the field is 450 yards, what are the field's dimensions?
18. After a 20% reduction, you purchase a new Stephen King novel for \$28. What was the book's price before the reduction?

In Exercises 19–21, find the area of each figure.

19.



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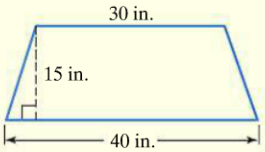

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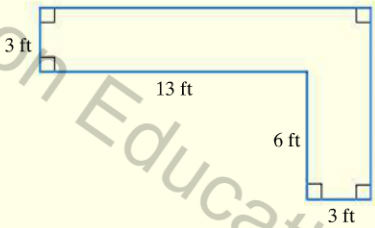
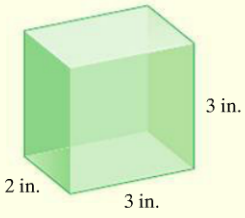
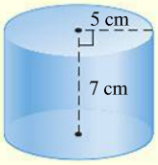


In Exercises 22–23, find the volume of each figure. Where applicable, express answers in terms of π. Then round to the nearest whole number.

22.



23.



24. What will it cost to cover a rectangular floor measuring 40 feet by 50 feet with square tiles that measure 2 feet on each side if a package of 10 tiles costs \$13 per package?
25. A sailboat has a triangular sail with an area of 56 square feet and a base that measures 8 feet. Find the height of the sail.
26. In a triangle, the measure of the first angle is three times that of the second angle. The measure of the third angle is 30° less than the measure of the second angle. What is the measure of each angle?