3.2 Conditional Probability and the Multiplication Rule

Definition

A **conditional probability** is the probability of an event occurring, given that another event has already occurred. The conditional probability of event B occurring, given that event A has occurred, is denoted by P(B|A) and is read as "the probability of B, given A."

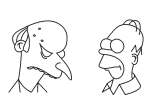
Results from Experiments with Polygraph Instruments

	No (Did Not Lie)	Yes (Lied)
Positive Test Result (The polygraph test indicated that the subject *lied*.)	15 (false positive)	42 (true positive)
Negative Test Result (The polygraph test indicated that the subject did not *lie*.)	32 (true negative)	9 (false negative)

☞ **Example**: If one of the 98 subjects is randomly selected, find the probability that the subject had a positive test result, given that the subject actually lied. That is find P(positive test result|subject lied).

☞ **Example**: If one of the 98 subjects is randomly selected, find the probability that the subject actually lied, given that he or she had a positive test result.

	Nonsmoker	Light Smoker	Heavy Smoker	Total
Men	306	74	66	446
Women	345	68	81	494
Total	651	142	147	940



Consider the following events:

Event N: The person selected is a nonsmoker
Event L: The person selected is a light smoker
Event H: The person selected is a heavy smoker
Event M: The person selected is a male
Event F: The person selected is a female

Example: Suppose one of the 940 subjects is chosen at random. Compute the following probabilities:

a. $P(N|F)$

b. $P(F|N)$

c. $P(H|M)$

d. $P(\text{the person selected is a smoker})$

Try This! The human resources division at the Krusty-O cereal factory reports a breakdown of employees by job type and sex, summarized in the table below.

	Sex		
Job Type	Male	Female	total
Management	7	6	13
Supervision	8	12	20
Production	45	72	117
total	60	90	150

One of these workers is randomly selected.

(a) Find the probability that the worker is a female.

(b) Find the probability that the worker is a male supervisor.

(c) Find the probability that the worker is female, given that the person works in works in production.

Try This! Voter Support for political term limits is strong in many parts of the U.S. A poll conducted by the Field Institute in California showed support for term limits by a 2–1 margin. The results of this poll of $n = 347$ registered voters are given in the table.

	For (F)	**Against (A)**	**No Opinion (N)**	**Total**
Republican (R)	0.28	0.10	0.02	0.40
Democrat (D)	0.31	0.16	0.03	0.50
Other (O)	0.06	0.04	0.00	0.10
Total	0.65	0.30	0.05	1.00

If one individual in drawn at random from this group of 347 people, calculate the following probabilities:

(a) $P(\overline{N})$

(b) $P(R|N)$

(c) $P(A|D)$

(d) $P(D|A)$

The Multiplication Rule

Notation

$P(B|A)$ represents the probability of event B occurring after it is assumed that event A has already occurred (read $B|A$ as "B given A").

Definition

Two events A and B are **independent** if the occurrence of one does not affect the probability of the occurrence of the other. If A and B are not independent, they are said to be **dependent** .

Definition

Two events A and B are said to be **independent** if and only if either

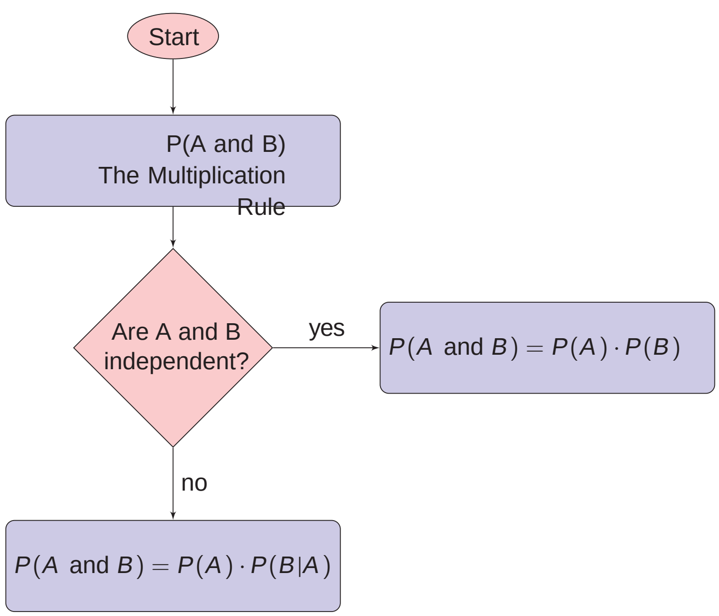
$$P(B|A) = P(B) \quad \text{or} \quad P(A|B) = P(A)$$

Theorem (The Multiplication Rule)

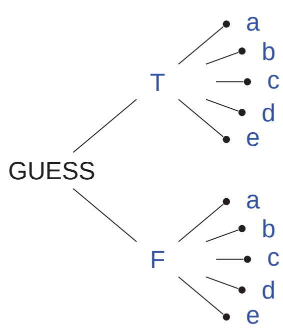
$P(A \text{ and } B) = P(A) \cdot P(B)$ *(if A and B are independent)*

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ *(if A and B are dependent)*

Applying the Multiplication Rule



Example: Suppose you are given a two-question quiz, where the first question is a true/false question and the second question is a multiple choice question with 5 possible answers. Suppose you guess on both questions. What is the probability that you correctly answered both questions?



Notice that the notation $P(\text{both correct})$ is equivalent to $P(\text{the first answer is correct AND the second answer is correct})$. The sample space,

$$S = \{Ta, Tb, Tc, Td, Te, Fa, Fb, Fc, Fd, Fe\},$$

has 10 simple events. Only one of these is a correct outcome, so

$$P(\text{both correct}) = \frac{1}{10} = 0.1$$

Suppose the correct answers are T and c. We can also obtain the correct probability by multiplying the individual probabilities:

$$\begin{aligned} P(\text{both correct}) &= P(T \text{ and } c) \\ &= P(T) \cdot P(c) = \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10} = 0.1 \end{aligned}$$

Try This: If 28% of U.S. medical degrees are conferred to women, find the probability that 2 randomly selected medical school graduates are **women**. Would you consider this event to be unusual?

Find the probability that 3 randomly selected medical school graduates are **women**. Would you consider this event to be unusual?

Try This: Find the probability that 3 randomly selected medical school graduates are **men**. Would you consider this event to be unusual?

Try This: A candy dish contains four red candies, seven yellow candies and fourteen blue candies. You close your eyes, choose two candies one at a time (without replacement) from the dish, and record their colors.

(a) Find the probability that both candies are red.
(b) Find the probability that the first candy is red and the second candy is blue.

Try This Pick two cards **without replacement** at random from a shuffled deck of playing cards. Find the probability the first card is an ace and the second card is an ten.

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Try This: Use the data in the following table, which summarizes blood type and Rh types for 100 subjects.

		Blood Type			
		O	A	B	AB
Rh Type	Rh^+	39	35	8	4
	Rh^-	5	6	2	1

If 2 out of the 100 subjects are randomly selected, find the probability that they are both blood group O and Rh type Rh^+.

1. Assume that the selections are made with replacement.
2. Assume that the selections are made without replacement.

	Nonsmoker	Light Smoker	Heavy Smoker	Total
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Consider the following events:

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(a) Now suppose that two people are selected from the group, ***without replacement***. Let A be the event "the first person selected is a nonsmoker," and let B be the event "the second person is a light smoker." What is $P(A \cap B)$?

(b) Two people are selected from the group, ***with replacement***. What is the probability that both people are nonsmokers?

The Probability of "at least one"

Example: It is reported that 16% of households regularly eat Krusty-O cereal. Choose 4 households at random. Find the probability that

(a) none regularly eat Krusty-O cereal
(b) all of them regularly eat Krusty-O cereal
(c) at least one regularly eats Krusty-O cereal

Let A be the event "a randomly selected household regularly eats Krusty-O cereal." Then $P(A) = 0.16$ and the complement of A (the event "a randomly selected household <u>does not</u> regularly eat Krusty-O cereal"), $P(\overline{A}) = 1 - P(A) = 1 - 0.16 = 0.84$.

(a) P(none regularly eat Krusty-O cereal)
= P(1st does not AND 2nd does not AND 3rd does not AND 4th does not)
$= (0.84) \cdot (0.84) \cdot (0.84) \cdot (0.84) = (0.84)^4 = \boxed{0.4979}$

(b) P(all 4 of them regularly eat Krusty-O cereal)
= P(1st does AND 2nd does AND 3rd does AND 4th does)
$= (0.16) \cdot (0.16) \cdot (0.16) \cdot (0.16) = (0.16)^4 = \boxed{0.000655}$

(c) P(at least one regularly eats Krusty-O cereal)
$= 1 -$ P(none regularly eat Krusty-O cereal) $= 1 - 0.4979 = \boxed{0.5021}$.

Try This! 24% of teens go online “almost constantly,” facilitated by the widespread availability of smartphones . (source: pew research 2013). Choose 3 teens at random. Find the probability that

(a) none go online “almost constantly,”
(b) all of them go online “almost constantly,”
(c) at least one goes online “almost constantly,”

Try This! Four in ten adults in the U.S. are caring for an adult or child with significant health issues, up from 30% in 2010. Caring for a loved one is an activity that cuts across most demographic groups, but is especially prevalent among adults ages 30 to 64, a group traditionally still in the workforce. (source: pew research 2013).

Randomly select 4 U.S. Adults. Find the probability that
(a) none are caregivers
(b) all of them are caregivers
(c) at least one is a caregiver