Green

Math 150
Exam 4
Professor Busken
No Calculators!

Name: Key

1. (3 points) Evaluate $\int_{-3}^{3} \sqrt{9-x^2}\,dx$. Hint: Interpret in terms of area.

$\frac{\pi r^2}{2} = \frac{9\pi}{2}$

1. $\frac{9\pi}{2}$

2. (3 points) Write $\lim_{n\to\infty}\left(\sum_{k=1}^{n} \sqrt{15 + x_k \sec(x_k)}\ \Delta x_k\right)$ as a definite integral on the interval $[-1, 2]$.

2. $\int_{-1}^{2} \sqrt{15 + x\sec(x)}\,dx$

3. (3 points) Evaluate $\int_{1}^{1} \frac{\log(x)}{21 - 3x^7}\,dx$

3. 0

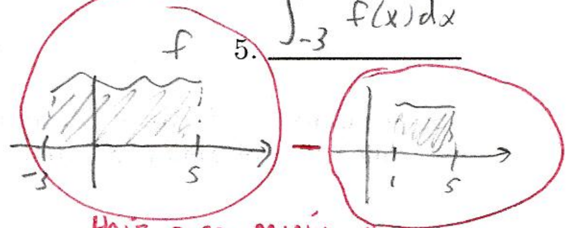
4. (3 points) Evaluate $\int_{-\pi}^{\pi} \tan\left(\frac{x^3 - 2x}{x^4 - 9}\right) dx$

4. 0

5. (3 points) Write as a single integral: $\int_{5}^{1} f(x)\,dx + \int_{-3}^{5} f(x)\,dx$

$= -\int_{1}^{5} f(x)\,dx + \int_{-3}^{5} f(x)\,dx$

$= \int_{-3}^{1} f(x)\,dx$



this area minus this area

5. $\int_{-3}^{1} f(x)\,dx$

6. (8 points) Find the average value of $f(x) = \frac{4x}{\sqrt{x^2+4}}$ on the interval $[0, 3]$.

$\frac{1}{3}\int_{0}^{3} \frac{4x}{\sqrt{x^2+4}}\,dx$

$u = x^2 + 4$

$du = 2x\,dx$

$2\,du = 4x\,dx$

$x: 0 \to 3$

$u: 4 \to 13$

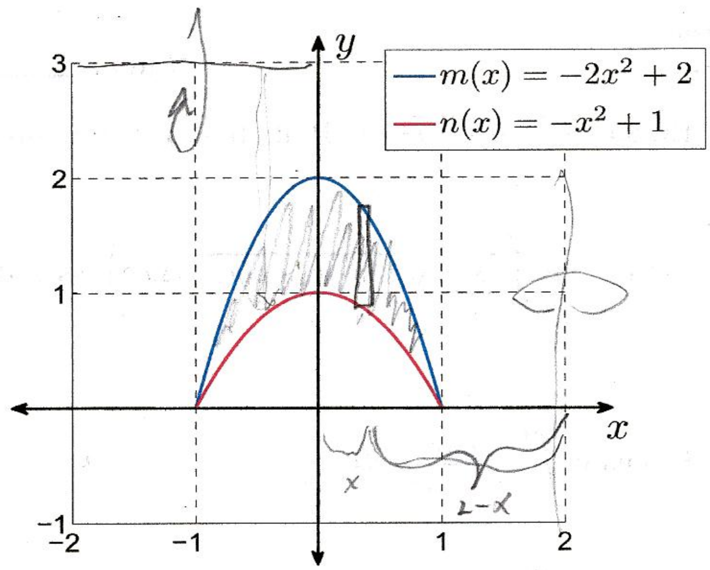
$= \frac{2}{3}\left[\int u^{-1/2}\,du\right]_{x=0}^{x=3} = \frac{2}{3}\cdot\frac{2}{1} u^{1/2}\Big|_{4}^{13} = \frac{4}{3}\left(\sqrt{13} - 2\right)$

or $\boxed{\frac{4\sqrt{13} - 8}{3}}$

6. ________

Green test
colored

For problems 7—10, use the figure given below.



7. (4 points) Let R be the region bounded by $m(x) = -2x^2+2$, and $n(x) = -x^2+1$, given in the above figure. What integral represents the area of R? ***Do not evaluate the integral.***

use symmetry on all

$$2\int_0^1 \left[(-2x^2+2)-(-x^2+1)\right]dx = 2\int_0^1 (-x^2+1)\,dx$$

8. (4 points) Revolve R around the x-axis. What integral represents the volume of the solid of revolution generated by R? ***Do not evaluate the integral.*** washer

$$V = 2\int_0^1 \pi\left[(-2x^2+2)^2 - (-x^2+1)^2\right] dx$$

symmetry $\quad 2\pi(\text{outer}^2 - \text{Inner}^2)(\text{thickness})$

9. (4 points) Now revolve R around the horizontal line $y=3$. What integral represents the volume of the solid of revolution generated by R? ***Do not evaluate the integral.***

washer

$$V = 2\int_0^1 \pi\left[\left[3-(-x^2+1)\right]^2 - \left[3-(-2x^2+2)\right]^2\right]dx$$

symmetry

10. (4 points) Now revolve R around the vertical line $x=2$. What integral represents the volume of the solid of revolution generated by R? ***Do not evaluate the integral.***

→ use shells and symmetry to set up Volume using a single integral.

note: incorrect $\int_{-1}^{1} 2\pi(2-x)\left[(-2x^2+2)-(-x^2+1)\right]dx$

≈ 16.76

b.c. left of the origin avg radius is $2-(-x) = 2+x$

$$\underset{\text{symmetry}}{2}\cdot 2\pi\int_0^1 \underset{\text{avg radius}}{(2-x)}\underset{\text{ht}}{\left[(-2x^2+2)-(-x^2+1)\right]}\underset{\text{thickness}}{dx} = \boxed{4\pi\int_0^1 (x^3-2x^2-x+2)\,dx \approx 13.61}$$

Green test colored

11. (8 points) Evaluate $\int \left[\frac{2}{\sqrt{1-x^2}} + e^{-5x} + \cos(\pi x) + \sec^2(x) \right] dx$ 11. ____________

$2\sin^{-1}(x) - \frac{1}{5}e^{-5x} + \frac{1}{\pi}\sin(\pi x) + \tan(x) + C$

12. (8 points) Evaluate $\int \tan(x) \cdot \ln\left(\cos(x)\right)\, dx$

$-\frac{1}{2}\left[\ln|\cos(x)|\right]^2 + C$

12. ____________

Let

$u = \cos x$, then

$-du = \sin x\, dx$, and

u-sub nested within a u-sub

$\int \tan(x) \ln(\cos(x))\, dx$

$\overset{u\text{-sub}}{=} -\int \frac{\ln u}{u}\, du \overset{w\text{-sub}}{=} -\int w\, dw = -\frac{w^2}{2} + C$

$w = \ln u.$

$dw = \frac{1}{u}\, du$

$= -\frac{(\ln u)^2}{2} + C$

$= -\frac{(\ln \cos x)^2}{2} + C$

Green
colored test

Extra Credit Problem

13. (8 points) Evaluate $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

13. $\tan^{-1}(e^x)+C$

$u = e^x$

$du = e^x\,dx$

$u^2 = e^{2x}$

$$= \int \frac{1}{1+\underbrace{(e^x)^2}_{u}} \cdot \underbrace{e^x\,dx}_{du} = \int \frac{1}{1+u^2}\,du$$

$$= \tan^{-1}(u) + C$$

$$= \tan^{-1}(e^x) + C$$

Name: Key

Calculator Section**: you may use either the scientific or graphing calculator.**

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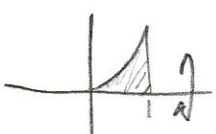
14. (6 points) Let R be the region bounded by $x = 0$, $x = 1$, $y = 0$ and $f(x) = x^4$. Revolve R around the x-axis. What integral represents the surface area of the resulting solid? ***Do not evaluate.***



$$SA = \int_0^1 2\pi x^4 \sqrt{1 + (4x^3)^2}\, dx = \int_0^1 2\pi x^4 \sqrt{1+16x^6}\, dx$$

14. ____________

15. (8 points) ***Calculator Problem.*** Approximate the value of this integral using Simpson's Rule with $\boxed{n = 6}$. Writing a single number as the answer will not receive full credit. It is expected that you will write down the set up of the problem. You may truncate or round to four decimals each functional value in your sum for the sake of time. The majority of credit allocated on this problem is based on how well you organize and show me your work. Numerical answers don't have to exactly match my answer, but they should be close. I recommend you double check your answer.

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}; \quad \frac{\Delta x}{3} = \frac{1}{18}$$

15. 3.4583

$$S_6 = \frac{\Delta x}{3}\left[f(0) + 4f\left(\tfrac{1}{6}\right) + 2f\left(\tfrac{1}{3}\right) + 4f\left(\tfrac{1}{2}\right) + 2f\left(\tfrac{2}{3}\right) + 4f\left(\tfrac{5}{6}\right) + f(1)\right]$$

$$\doteq \frac{1}{18}\left[0 + 4(0.00485) + 2(0.07842) + 4(0.43905) + 2(1.9246) + 4(7.6406) + 25.906\right]$$

$$\approx 3.4583$$

TI-83 check

$\boxed{\text{math}} \rightarrow \text{fnInt}(2\pi x^4\sqrt{1+16x^6}, x, 0, 1) \doteq 3.4365$

<u>note:</u> there should be n+1 function evaluations in your sum.

Name: Key

Calculators Prohibited Section

16. (1 point) Did you write your name on this?

17. (8 points) Evaluate $\int_{-1}^{1} \left(x^3 + 6x \right) dx$

17. 0

$f(x) = x^3 + 6x$

$f(-x) = (-x)^3 + 6(-x)$

$= -x^3 - 6x$

$= -(x^3 + 6)$

$= -f(x)$

since f is odd and the limits of integration are additive inverses, the given integral is equal to zero.

18. (8 points) Evaluate $\int \frac{3}{\sqrt{5x-1}}\, dx$

18. ________

Let $u = 5x - 1$

then $du = 5\,dx$ $\left[\text{or } \frac{du}{5} = dx\right]$

and $3\int \frac{1}{\sqrt{5x-1}}\,dx = \frac{3}{5}\int \frac{1}{\sqrt{u}}\,du = \frac{3}{5}\int u^{-1/2}\,du$

$= \frac{3}{5}\cdot\frac{2}{1}u^{1/2} + C = \frac{6}{5}u^{1/2} + C = \boxed{\frac{6}{5}\sqrt{5x-1} + C}$

or $\frac{2}{75}(5x-1)^{1/2}\left[(5x-1)^3+1\right]+C$

19. (8 points) Evaluate $\displaystyle\int \frac{x}{\sqrt{5x-1}}\,dx$

19. $\frac{2}{75}\sqrt{5x-1}\left[(5x-1)^3+1\right]+C$

Let $u = 5x-1$

then $du = 5\,dx$

or $\boxed{\frac{du}{5} = dx}$

also, $u+1 = 5x$,

or $\boxed{x = \frac{u+1}{5} = \frac{1}{5}(u+1)}$

$$= \int \frac{\frac{1}{5}(u+1)}{\sqrt{u}}\,\frac{du}{5}$$

$$= \frac{1}{25}\int \frac{u+1}{u^{1/2}}\,du = \frac{1}{25}\int\left(\frac{u}{u^{1/2}} + \frac{1}{u^{1/2}}\right)du$$

$$= \frac{1}{25}\int u^{1/2}\,du + \frac{1}{25}\int u^{-1/2}\,du$$

$$= \frac{1}{25}\frac{2}{3}u^{3/2} + \frac{1}{25}\cdot\frac{2}{1}u^{1/2} + C$$

$$= \boxed{\frac{2}{75}(5x-1)^{3/2} + \frac{2}{25}(5x-1)^{1/2} + C}$$

20. (8 points) What integral represents the (arc) length of $f(x) = e^{-2x}\sin(2x)$ from $(x,y) = (0,0)$ to $(x,y) = \left(\frac{\pi}{4}, e^{-\pi/2}\right)$? ***Do not evaluate the integral.***

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1+(f'(x))^2}\,dx$$

$$\boxed{= \int_0^{\frac{\pi}{4}} \sqrt{1+\left[2e^{-2x}\left(\cos(2x)-\sin(2x)\right)\right]^2}\,dx}$$

20. ____________

$$f'(x) = \frac{d}{dx}\left(e^{-2x}\sin(2x)\right)$$

$$= -2e^{-2x}\sin(2x) + e^{-2x}\cdot 2\cos(2x)$$

$$= 2e^{-2x}\left(\cos(2x)-\sin(2x)\right)$$

21. (8 points) Evaluate $\int \frac{3x^4}{2x^5 - 1}\,dx$ 21. ____________

Let $u = 2x^5 - 1$

then $du = 10x^4\,dx$

or $\frac{du}{10} = x^4\,dx$

$$= 3\int \frac{x^4\,dx}{2x^5-1} = \frac{3}{10}\int \frac{1}{u}\,du$$

$$= \frac{3}{10}\ln|u| + C$$

$$= \boxed{\frac{3}{10}\ln|2x^5-1| + C}$$

22. (8 points) Evaluate $\int \frac{4}{7x - 1}\,dx$ 22. ____________

Let $u = 7x-1$

then $du = 7\,dx$

and $\frac{du}{7} = dx$

$$= 4\int \frac{1}{7x-1}\,dx$$

$$= \frac{4}{7}\int \frac{1}{u}\,du$$

$$= \frac{4}{7}\ln|u| + C$$

$$= \frac{4}{7}\ln|7x-1| + C$$

Extra Credit Problem

23. (6 points) Evaluate $\int \frac{2^x}{2^x + 4}\, dx$ 23. ____________

Let $u = 2^x + 4$

then $du = 2^x \ln 2\, dx$

and $\frac{du}{\ln 2} = 2^x\, dx$

$$= \int \frac{1}{2^x + 4} \cdot 2^x\, dx$$

$$= \frac{1}{\ln 2} \int \frac{1}{u}\, du$$

$$= \frac{1}{\ln 2} \ln |2^x + 4| + C$$

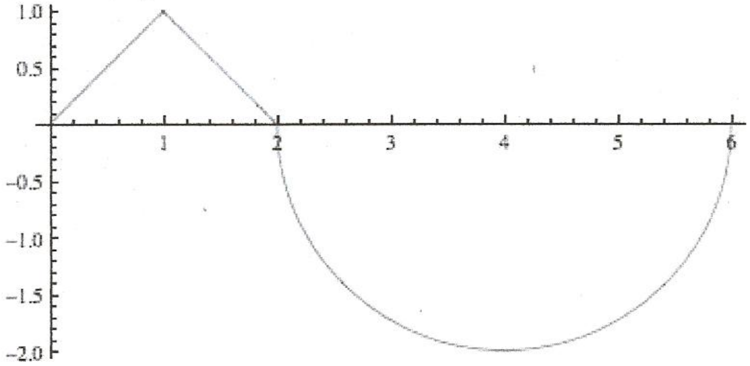
$$= \boxed{\frac{1}{\ln 2} \ln (2^x + 4) + C}$$

Name: ____________________

Signature: ____________________

This is an in class quiz. **Calculators are allowed**. You must complete this quiz on your own, use methods learned in this class, and show all work in a legible manner to receive credit. By signing your quiz you acknowledge that you have neither given or received help from other students. You have 10 minutes to complete this quiz.

1. The graph of $f(x)$ below consists of straight lines and semi-circles. Use it to evaluate the following definite integrals.



   (a) (3 pts) $\int_0^2 f(x)\,dx$

   $\int_0^2 f(x)\,dx = \frac{1}{2}(2)(1) = 1$

   +1 for knowing the area is positive

   +1 for knowing the formula for the area of a triangle

   +1 for simplifying to the correct answer of "1"

   (b) (4 pts) $\int_0^6 f(x)\,dx$

   $\int_0^6 f(x)\,dx = 1 - \frac{1}{2}\pi(2)^2 = 1 - 2\pi$

   +1 for the "1" from part (a)

   +1 for knowing the area of the semi-circle counts as negative

   +1 for knowing the area of a semi-circle is $\frac{1}{2}\pi(2)^2$

   +1 for getting the correct answer of $1 - 2\pi$

2. (3 pts) If $g(x) = \int_x^0 \cos(t^2)\,dt$, find $g'(x)$

   $g'(x) = -\cos(x^2)$

   +1 for the (-) sign.

   +1 for $\cos(x^2)$, if they have $\cos(t^2)$ then +0.5

   +1 for not having anything "extra" i.e. for not multiplying by the derivative of $\cos(x^2)$ or something silly like that.

Math 150
Exam 4
Professor Busken
No Calculators! **Name:** Key

1. (3 points) Evaluate $\int_{-2}^{2} \sqrt{4-x^2}\, dx$. Hint: Interpret in terms of area.

$\frac{\pi r^2}{2} = \pi \frac{2^2}{2} = 2\pi$

1. 2π

2. (3 points) Write $\lim_{n\to\infty} \left(\sum_{k=1}^{n} x_k^4 \tan(x_k)\ \Delta x_k \right)$ as a definite integral on the interval $[-1, 2]$.

2. $\int_{-1}^{2} x^4 \tan(x)\,dx$

3. (3 points) Evaluate $\int_{1}^{1} \frac{\tanh^{-1}(x)}{52-317x^7}\, dx$

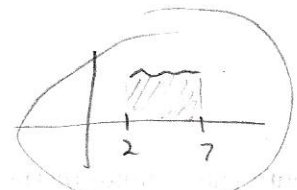
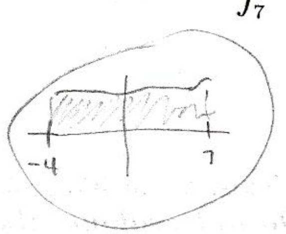
3. 0

4. (3 points) Evaluate $\int_{-\pi}^{\pi} \tan\left(\frac{x^3-2x}{x^4+4}\right) dx$

4. 0

5. (3 points) Write as a single integral: $\int_{7}^{2} f(x)\, dx + \int_{-4}^{7} f(x)\, dx$

5. $\int_{-4}^{2} f(x)\,dx$

$-$  $+$ 

$= -\int_{2}^{7} f(x)\,dx + \int_{-4}^{7} f(x)\,dx = \int_{-4}^{2} f(x)\,dx$

6. (8 points) Find the average value of $f(x) = \frac{6x}{\sqrt{x^2+9}}$ on the interval $[0, 2]$.

$f_{av} = \frac{1}{2-0}\int_0^2 \frac{6x}{\sqrt{x^2+9}}\,dx$

$u = x^2+9$

$du = 2x\,dx$

$3\,du = 6x\,dx$

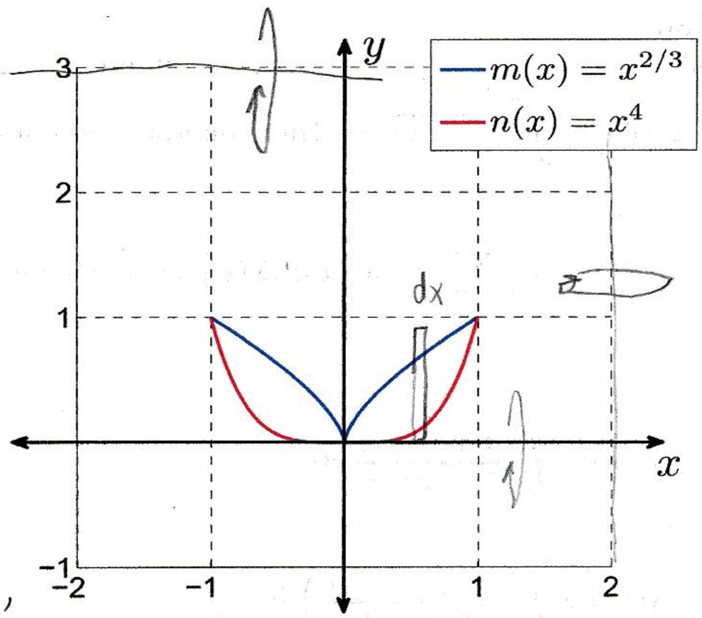
$0 \xrightarrow{} 2$ as X, $9 \xrightarrow{} 13$ U

$= \frac{3}{2}\left[\int \frac{1}{\sqrt{u}}\,du\right]_{x=0}^{x=2}$

6. $3(\sqrt{13}-3)$

$= \frac{3}{2}\left[\int u^{-1/2}\,du\right]_{u=9}^{u=13} = \frac{3}{2}\frac{2}{1}u^{1/2}\Big|_9^{13} = 3(\sqrt{13}-3)$ or $3\sqrt{13}-9$

For problems 7—10, use the figure given below.



Plz use symmetry for all 4 questions, although its only required on #10.

7. (4 points) Let R be the region bounded by $m(x) = x^{2/3}$, and $n(x) = x^4$, given in the above figure. What integral represents the area of R? ***Do not evaluate the integral.***

$$A = 2\int_0^1 \left(x^{2/3} - x^4\right) dx$$

8. (4 points) Revolve R around the x-axis. What integral represents the volume of the solid of revolution generated by R? ***Do not evaluate the integral.*** washers & symmetry

$$V = 2\pi\int_0^1 \left[\left(x^{2/3}\right)^2 - \left(x^4\right)^2\right] dx = 2\pi\int_0^1 \left(x^{4/3} - x^8\right) dx$$

9. (4 points) Now revolve R around the horizontal line $y = 3$. What integral represents the volume of the solid of revolution generated by R? ***Do not evaluate the integral.***

washers

$$V = 2\pi\int_0^1 \left[\left(3 - x^4\right)^2 - \left(3 - x^{2/3}\right)^2\right] dx$$

or

shells

$$2\cdot 2\pi\int_0^1 (3-y)\left(y^{1/4} - y^{3/2}\right) dy$$

10. (4 points) Now revolve R around the vertical line $x = 2$. What integral represents the volume of the solid of revolution generated by R? ***Do not evaluate the integral.***

Shells & symmetry

$$V = 2\cdot 2\pi\int_0^1 (2-x)\left[\left(x^{2/3}\right)^2 - \left(x^4\right)^2\right] dx$$

symmetry

$$= 4\pi\int_0^1 \left[x^9 - 2x^8 - x^{4/3} + 2x^{1/3}\right] dx \approx 0.95$$

note incorrect

$$\int_{-1}^1 2\pi(2-x)\left[\left(x^{2/3}\right)^2 - \left(x^4\right)^2\right] dx$$

≈ -1.3 b.c. left of the origin avg radius is $2-(-x)$ $= 2+x$

~~Green~~ Key

11. (8 points) Evaluate $\int \left[e^{-3x} + \sec^2(x) + \cos(\pi x) + \frac{2}{\sqrt{1-x^2}}\right] dx$ 11. ____________

$$\boxed{-\frac{1}{3}e^{-3x} + \tan(x) + \frac{1}{\pi}\sin(\pi x) + 2\sin^{-1}(x) + C}$$

12. (8 points) Evaluate $\int \frac{\sqrt[4]{1-(1/x)}}{x^2}\, dx$

12. ____________

Let $u = 1 - \frac{1}{x} = 1 - x^{-1}$

then $du = x^{-2}\,dx = \frac{1}{x^2}\,dx$

and $\int (1 - 1/x)^{1/4} \cdot \frac{1}{x^2}\,dx = \int u^{1/4}\,du$

$= \frac{4}{5}u^{5/4} + C$

$= \boxed{\frac{4}{5}\left(1 - \frac{1}{x}\right)^{5/4} + C}$

Extra Credit Problem

13. (8 points) Evaluate $\int \frac{x^2 - x - 6}{x + 7}\, dx$

13. ____________

$$\frac{x^2 - x - 6}{x+7} = x+7 \overline{\big) x^2 - x - 6}$$

$$x - 8 + \frac{50}{x+7}$$

$$\frac{\begin{array}{r} x^2 - x - 6 \\ -x^2 + 7x \end{array}}{\begin{array}{r} -8x - 6 \\ \pm 8x \pm 56 \end{array}}$$

$$50$$

$$\int \frac{x^2 - x - 6}{x+7}\, dx = \int \left(x - 8 + \frac{50}{x+7}\right) dx$$

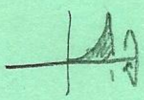
$$= \int x\, dx - \int 8\, dx + 50 \int \frac{1}{x+7}\, dx$$

$$\boxed{= \frac{1}{2}x^2 - 8x + 50 \ln|x+7| + C}$$

Name: Key

***Calculator Section*:** **you may use either the scientific or graphing calculator.**

14. (6 points) Let R be the region bounded by $x = 0$, $x = 1$, $y = 0$ and $f(x) = x^3$. Revolve R around the x-axis. What integral represents the surface area of the resulting solid? ***Do not evaluate.***



$$SA = \int_0^1 2\pi x^3 \sqrt{1 + (3x^2)^2}\, dx = \boxed{\int_0^1 2\pi x^3 \sqrt{1 + 9x^4}\, dx}$$

14. ____________

15. (8 points) ***Calculator Problem.*** Approximate the value of this integral using the Trapezoid Rule with $\boxed{n = 6}$. Writing a single number as the answer will not receive full credit. It is expected that you will write down the set up of the problem. You may truncate or round to four decimals each functional value in your sum for the sake of time. The majority of credit allocated on this problem is based on how well you organize and show me your work. Numerical answers need only be a ballpark estimation, but I recommend you double check your answer.

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6} \; ; \; \frac{\Delta x}{2} = \frac{1}{12}$$

15. ____________

$$T_6 = \frac{\Delta x}{2}\left[f(0) + 2f(\tfrac{1}{6}) + 2f(\tfrac{1}{3}) + 2f(\tfrac{1}{2}) + 2f(\tfrac{2}{3}) + 2f(\tfrac{5}{6}) + f(1)\right]$$

$$\doteq \frac{1}{12}\left[0 + 2(0.0291) + 2(0.2453) + 2(0.98175) + 2(3.1028) + 2(8.4027) + 19.869\right]$$

$$\approx 3.7826$$

TI-83 check

$\boxed{\text{Math}} \rightarrow \text{fnInt}(2\pi x^3\sqrt{1+9x^4}, x, 0, 1) \doteq 3.56$

Name: Key

Calculators Prohibited Section

16. (1 point) Did you write your name on this?

17. (6 points) Evaluate $\int_{-1}^{1} \left(x^5 + 3x^3 + x \right) dx$ 17. 0

$f(x) = x^5 + 3x^3 + x$ is an odd function

since $f(-x) = (-x)^5 + 3(-x)^3 + (-x) = -x^5 - 3x^3 - x = -(x^5 + 3x^3 + x)$

$= -f(x)$.

Also, the limits of integration are additive inverses, thus the given integral is equal to zero.

18. (8 points) Evaluate $\int \frac{4}{\sqrt[3]{3x-1}}\, dx$ 18. ______

Let $u = 3x - 1$

then $du = 3\,dx$

or $\frac{du}{3} = dx$

$= \frac{4}{3} \int \frac{1}{u^{1/3}}\, du$

$= \frac{4}{3} \int u^{-1/3}\, du$

$= \frac{4}{3} \cdot \frac{3}{2} u^{2/3} + C$

$= \boxed{2(3x-1)^{2/3} + C}$

19. (8 points) What integral represents the (arc) length of $f(x) = e^{-2x}\cos(2x)$ from $(x, y) = (0, 1)$ to $(x, y) = \left(\frac{\pi}{4}, 0\right)$? ***Do not evaluate the integral.***

$$f'(x) = \frac{d}{dx}\left[e^{-2x}\cos(2x)\right]$$

19. ______________

$$= -2e^{-2x}\cos(2x) - 2e^{-2x}\sin(2x)$$

$$\boxed{L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \left[-2e^{-2x}\left(\cos(2x) + \sin(2x)\right)\right]^2}\, dx}$$

20. (8 points) Evaluate $\displaystyle\int \frac{x}{\sqrt{3x-1}}\,dx$

20. ______________

Let $u = 3x-1$

then $du = 3\,dx$

and $\frac{du}{3} = dx$

Also

$$\frac{u+1}{3} = x$$

$$= \int \frac{\frac{u+1}{3}}{\sqrt{u}}\,\frac{du}{3} = \frac{1}{3}\int \frac{\frac{1}{3}(u+1)}{u^{1/2}}\,du$$

$$= \frac{1}{9}\int \frac{u+1}{u^{1/2}}\,du = \frac{1}{9}\int \left(\frac{u}{u^{1/2}} + \frac{1}{u^{1/2}}\right) du$$

$$= \frac{1}{9}\int \left(u^{1/2} + u^{-1/2}\right) du$$

$$= \frac{1}{9}\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\right) + C$$

$$= \boxed{\frac{2}{27}(3x-1)^{3/2} + \frac{2}{9}(3x-1)^{1/2} + C}$$

21. (8 points) Evaluate $\int \frac{2}{3x-1}\,dx$ 21. ____________

Let $u = 3x-1$

then $du = 3\,dx$

and $\frac{du}{3} = dx$

$= 2\int \frac{1}{3x-1}\,dx$

$= 2\int \frac{1}{u}\frac{du}{3}$

$= \frac{2}{3}\int \frac{1}{u}\,du = \frac{2}{3}\ln|u| + c$

$= \boxed{\frac{2}{3}\ln|3x-1| + C}$

22. (8 points) Evaluate $\int \frac{2x^3}{3x^4-1}\,dx$ 22. ____________

Let $u = 3x^4-1$

then $du = 12x^3\,dx$

and $\frac{du}{6} = 2x^3\,dx$

$= \int \frac{1}{3x^4-1}\cdot 2x^3\,dx$

$= \int \frac{1}{u}\frac{du}{6} = \frac{1}{6}\int \frac{1}{u}\,du$

$= \frac{1}{6}\ln|u| + C$

$= \boxed{\frac{1}{6}\ln|3x^4-1| + C}$

Extra Credit Problem

23. (6 points) Evaluate $\displaystyle\int \frac{3^x}{1+9^x}\,dx$

23. ____________

Let $u = 3^x$

then $du = 3^x \ln 3 \, dx$,

$$\frac{du}{\ln 3} = 3^x \, dx$$

and

$$u^2 = (3^x)^2 = (3^2)^x$$

$$= 9^x$$

$$= \int \frac{1}{1+u^2} \frac{du}{\ln 3}$$

$$= \frac{1}{\ln 3} \int \frac{1}{1+u^2} \, du$$

$$= \frac{1}{\ln 3} \tan^{-1}(u) + C$$

$$\boxed{= \frac{1}{\ln 3} \tan^{-1}(3^x) + C}$$