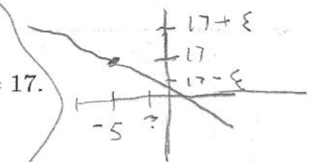
Math 150
Exam 1—Version 1b
Professor Busken

Name: Key

Directions: Make note of which of the 4 versions of the exam you have. Write each final answer on the provided answer line, if appropriate. Absolutely no calculators or cell phones allowed! Your cell phone must not be on your person. Writing in pen will not be accepted either. You will not be allowed to leave to use the restroom. Show all work in a legible manner to receive credit. Make sure you have 5 pages with 18 questions. You should not have to ask me any questions during the exam.

1. (8 points) Prove using the precise definition of a limit that $\lim_{x\to -5}(2-3x) = 17$.



Suppose $\varepsilon > 0$, $\delta = \frac{\varepsilon}{3}$ and $|x-(-5)| < \delta$.

Note that $|f(x) - L| = |2-3x-17| = |-3x-15|$
$= |-3|\cdot|x+5| = 3|x+5|$.

Then, the supposition $\left[|x-(-5)| < \delta\right] \iff$
$\left[|x+5| < \frac{\varepsilon}{3}\right] \iff \left[3|x+5| < \varepsilon\right] \iff \left[|f(x)-L| < \varepsilon\right]$.

$17+\varepsilon = 2-3x$ when $x = -5 - \frac{\varepsilon}{3}$ and $17-\varepsilon = 2-3x$ when $x = -5+\frac{\varepsilon}{3}$. This implies we should take $\delta = \frac{\varepsilon}{3}$ for our proof.

For problem 2, use the definition of the derivative of a function to find $f'(x)$ for the given function.

2. (8 points) $f(x) = \dfrac{5}{1-2x}$

2. $\dfrac{10}{(1-2x)^2}$

$$\lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right] = \lim_{h\to 0}\left[\frac{1}{h}\left[\frac{5}{1-2(x+h)} - \frac{5}{1-2x}\right]\right]$$

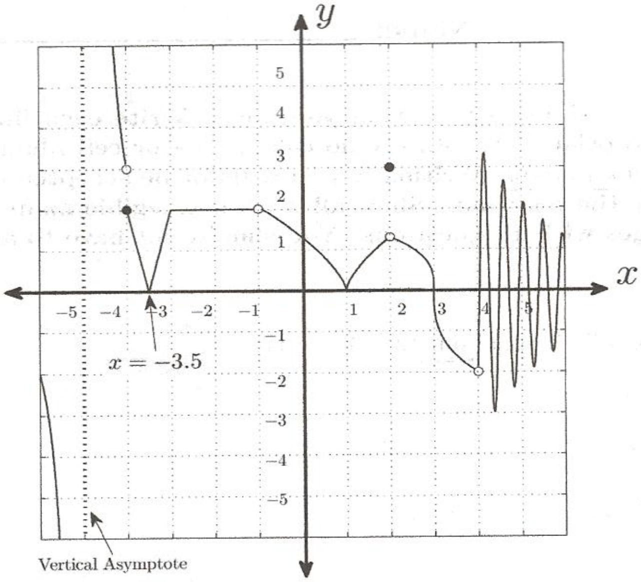
$$= \lim_{h\to 0}\left[\frac{5}{h}\left[\frac{1}{1-2x-2h}\cdot\frac{1-2x}{1-2x} - \frac{1}{1-2x}\cdot\frac{1-2x-2h}{1-2x-2h}\right]\right]$$

$$= \lim_{h\to 0}\left[\frac{5}{h}\left(\frac{1-2x-(1-2x-2h)}{(1-2x-2h)(1-2x)}\right)\right] = 5\lim_{h\to 0}\left[\frac{1-2x-1+2x+2h}{(1-2x-2h)(1-2x)}\right]$$

$$= 5\lim_{h\to 0}\left[\frac{2h}{h(1-2x-2h)(1-2x)}\right] = 10\lim_{h\to 0}\left[\frac{1}{(1-2x-2h)(1-2x)}\right]$$

Key

Use the graph of $f(x)$ below to answer Multiple Choice Questions 3—11.



3. (3 points) ***Multiple Choice Question*** Classify the type of discontinuity $f(x)$ has at $x = 2$.
   (a) jump
   (b) removable
   (c) infinite
   (d) None of the other answers are correct.

   3. B

4. (3 points) ***Multiple Choice Question*** Why is $f(x)$ not differentiable at $x = -3.5$?
   (a) corner
   (b) cusp
   (c) discontinuity
   (d) Vertical tangent line
   (e) None of the other answers are correct.

   4. A

5. (3 points) ***Multiple Choice Question*** Evaluate $\lim_{x \to -4^-} f(x)$
   (a) 2 (b) 3
   (c) ∞ (d) None of the other answers are correct.

   5. B

6. (3 points) ***Multiple Choice Question*** Evaluate $\lim_{x \to -5^+} f(x)$
   (a) $-\infty$ (b) ∞
   (c) 0 (d) None of the other answers are correct.

   6. B

Key

7. (3 points) ***Multiple Choice Question*** Why is $f(x)$ not differentiable at $x = 3$?
   (a) corner (b) cusp
   (c) discontinuity (d) vertical tangent line
   (e) None of the other answers are correct.

   7. D

Key

8. (3 points) ***Multiple Choice Question*** Evaluate $\lim_{x \to -4} f(x)$
   (a) does not exist (b) 2
   (c) $LHL = RHL$ (d) 3
   (e) None of the other answers are correct.

   8. A

9. (3 points) ***Multiple Choice Question*** Classify the type of discontinuity $f(x)$ has at $x = 4$.
   (a) jump (b) infinite
   (c) removable (d) None of the other answers are correct.

   9. C

10. (3 points) ***Multiple Choice Question*** Evaluate $\lim_{x \to -5} f(x)$
   (a) does not exist (b) 3
   (c) $-\infty$ (d) ∞
   (e) None of the other answers are correct.

   10. A

11. (3 points) ***Multiple Choice Question*** Why is $f(x)$ not differentiable at $x = -1$?
   (a) corner (b) cusp
   (c) discontinuity (d) vertical tangent line
   (e) None of the other answers are correct.

   11. C

Key

For problems 12—14, find (or evaluate) the given limits, if they exist. You may use reasoning, tables, algebra, limit theorems and properties covered in class, or what you know from the graph (without your calculator).

12. (4 points) $\lim_{x \to -3} \left(\frac{2}{x+3} \right)$ Let $f(x) = \frac{2}{x+3}$

12. DNE

LHL $= \lim_{x \to -3^-} [f(x)] \to \frac{2}{-0}$

$= -\infty$

RHL $= \lim_{x \to -3^+} (f(x)) \to \frac{2}{0^+}$

$= \infty$

So, LHL $= -\infty$ & RHL $= \infty$.

Since LHL $\neq$ RHL, L DNE

But, we expected that would be the case, since $x = -3$ is an odd asymptote for $f(x)$.

13. (4 points) $\lim_{x \to 2} \left[\frac{-7}{(x-2)^2} \right]$ Let $f(x) = \frac{-7}{(x-2)^2}$

13. $-\infty$

LHL $= \lim_{x \to 2^-} [f(x)] \to \frac{-7}{0^+}$

$= -\infty$

RHL $= \lim_{x \to 2^+} [f(x)] \to \frac{-7}{0^+}$

$= -\infty$

So, LHL $=$ RHL $= -\infty$.

But we expected that result, since $x = 2$ is an even asymptote for $f(x)$

14. (4 points) $\lim_{x \to 5} \left(1 - \frac{3}{5}x - x^2 \right)$

14. -27

We can evaluate the limit directly, since polynomials are continuous.

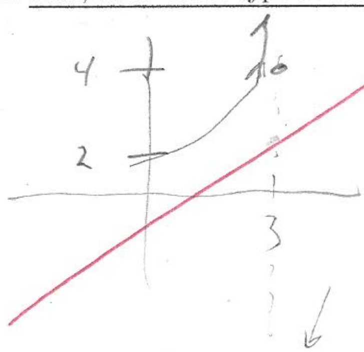
$L = 1 - \frac{3}{5} \cdot 5 - 25 = 1 - 3 - 25 = -27$

For problem 15, use algebra to find the limit.

15. (4 points) $\lim_{x \to -\infty} \left(\dfrac{10x^2 + 4}{2x^3 + 3x - 7} \right)$ 15. 0

16. (8 points) Show, using the definition of continuity, that $f(x) = \begin{cases} \dfrac{x^2 - 5x - 6}{x - 3} & x \neq 3 \\ 4 & x = 3 \end{cases}$ is not continuous at $x = 3$. Also, state which type of discontinuity exists for f at $x = 3$.

9-15-6



typo

17. (8 points) Suppose $f(x) = \frac{1}{3}x^2e^x$. We will show next week that $f'(x) = \frac{1}{3}x(x+2)e^x$. What is the equation of the line tangent to the graph of $f(x)$ at $x = 1$?

17. ____________

use $y - y_1 = m(x - x_1)$ with $(x_1, y_1) = (1, \frac{e}{3})$

and $m = f'(1) = e$. Then

$$y - \frac{e}{3} = e(x-1)$$, or

$$y = ex - e + \frac{e}{3}$$, or

$$y = ex + \left(\frac{-3e}{3} + \frac{e}{3}\right)$$, or $$\boxed{y = ex - \frac{2e}{3}}$$

For problem 18, find (or evaluate) the limit, using algebra.

18. (8 points) $\lim_{x \to -\infty} \left(\frac{\sqrt{16x^2 + 8}}{x + 5} \right)$

18. -4

$$= \lim_{x \to -\infty} \left[\frac{\sqrt{x^2\left(16 + \frac{8}{x^2}\right)}}{x + 5} \right]$$

$$= \lim_{x \to -\infty} \left[\frac{|x|\sqrt{16 + \frac{8}{x^2}}}{x + 5} \right] = \lim_{x \to -\infty} \left[\frac{-x\sqrt{16 + \frac{8}{x^2}}}{x + 5} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right]$$

$$= \lim_{x \to -\infty} \left[\frac{-\sqrt{16 + \frac{8}{x^2}}}{1 + \frac{5}{x}} \right] = \frac{-\sqrt{\lim_{x \to -\infty}(16) + 8\lim_{x \to -\infty}\left(\frac{1}{x^2}\right)}}{\lim_{x \to -\infty}(1) + 5\lim_{x \to -\infty}\left(\frac{1}{x}\right)}$$

$$= \frac{-\sqrt{16 + 8 \cdot 0}}{1 + 5 \cdot 0} = -\sqrt{16} = -4$$

Math 150
Exam 1—Version 1d
Professor Busken

Name: Key

Directions: Make note of which of the 4 versions of the exam you have. Write each final answer on the provided answer line, if appropriate. Absolutely no calculators or cell phones allowed! Your cell phone must not be on your person. Writing in pen will not be accepted either. You will not be allowed to leave to use the restroom. Show all work in a legible manner to receive credit. Make sure you have 5 pages with 18 questions. You should not have to ask me any questions during the exam.

1. (8 points) Prove using the precise definition of a limit that $\lim_{x\to 2}(3-4x) = -5$.

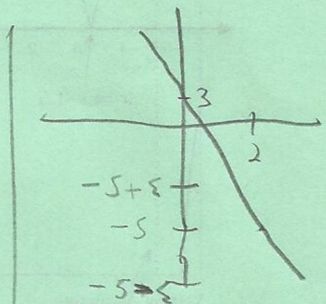
Suppose $\varepsilon > 0$, $\delta = \frac{\varepsilon}{4}$ and $|x-2| < \delta$.

Note that $|f(x)-L| = |3-4x-(-5)| = |-4x+8|$

$= |-4|\cdot|x-2| = 4|x-2|$.

Then, $\left[\,|x-2| < \delta\,\right] \Leftrightarrow \left[\,4|x-2| < 4\delta\,\right] \Leftrightarrow$

$\left[\,|f(x)-L| < 4\delta\,\right] \Leftrightarrow \left[\,|f(x)-L| < \varepsilon\,\right]$.



$3-4x = -5+\varepsilon$ when $x = 2-\frac{\varepsilon}{4}$

and $3-4x = -5-\varepsilon$ when $x = 2+\frac{\varepsilon}{4}$.

This implies we should take $\frac{\varepsilon}{4}$ in our proof

For problem 2, use the definition of the derivative of a function to find $f'(x)$ for the given function.

2. (8 points) $f(x) = 2\sqrt{3-5x}$

2. $-5/\sqrt{3-5x}$

$$\lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right] = \lim_{h\to 0}\left[\frac{2\sqrt{3-5(x+h)} - 2\sqrt{3-5x}}{h}\right]$$

$$= 2\lim_{h\to 0}\left[\frac{1}{h}\cdot\left(\frac{\sqrt{3-5x-5h}-\sqrt{3-5x}}{1}\right)\cdot\frac{\sqrt{3-5x-5h}+\sqrt{3-5x}}{\sqrt{3-5x-5h}+\sqrt{3-5x}}\right]$$

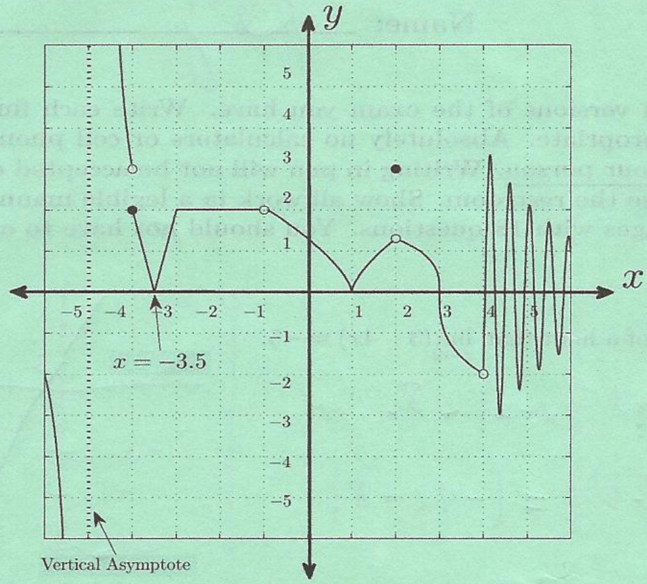
$$= 2\lim_{h\to 0}\left[\frac{(3-5x-5h)-(3-5x)}{h(\sqrt{3-5x-5h}+\sqrt{3-5x})}\right] = 2\lim_{h\to 0}\left[\frac{-5h}{h(\sqrt{3-5x-5h}+\sqrt{3-5x})}\right]$$

$$= -10\lim_{h\to 0}\left[\frac{1}{\sqrt{3-5x-5h}+\sqrt{3-5x}}\right] = -10\cdot\frac{1}{\sqrt{3-5x-5\cdot 0}+\sqrt{3-5x}}$$

$$= -10\,\frac{1}{2\sqrt{3-5x}} = \frac{-5}{\sqrt{3-5x}}$$

Key

Use the graph of $f(x)$ below to answer Multiple Choice Questions 3—11.



3. (3 points) ***Multiple Choice Question*** Classify the type of discontinuity $f(x)$ has at $x = 4$.
   (a) jump
   (b) removable
   (c) infinite
   (d) None of the other answers are correct.

   3. B

4. (3 points) ***Multiple Choice Question*** Why is $f(x)$ not differentiable at $x = 1$?
   (a) discontinuity
   (b) cusp
   (c) vertical tangent line
   (d) None of the other answers are correct.

   4. B

5. (3 points) ***Multiple Choice Question*** Evaluate $\lim_{x\to\infty} f(x)$
   (a) $-\infty$ (b) ∞
   (c) 0 (d) None of the other answers are correct.

   5. C

Key

6. (3 points) ***Multiple Choice Question*** Evaluate $\lim_{x\to -5^-} f(x)$
   (a) $-\infty$ (b) ∞
   (c) 0 (d) None of the other answers are correct.

   6. A

Key

7. (3 points) ***Multiple Choice Question*** Evaluate $\lim_{x\to -4^+} f(x)$
   (a) 2 (b) 3
   (c) ∞ (d) None of the other answers are correct.

   7. A

8. (3 points) ***Multiple Choice Question*** Why is $f(x)$ not differentiable at $x = -3$?
   (a) corner (b) cusp
   (c) discontinuity (d) vertical tangent line
   (e) None of the other answers are correct.

   8. A

9. (3 points) ***Multiple Choice Question*** Evaluate $\lim_{x\to 2} f(x)$
   (a) does not exist (b) 3
   (c) $LHL \neq RHL$ (d) 1
   (e) None of the other answers are correct.

   9. E

10. (3 points) ***Multiple Choice Question*** Classify the type of discontinuity $f(x)$ has at $x = -1$.
    (a) jump (b) removable
    (c) infinite (d) None of the other answers are correct.

    10. B

11. (3 points) ***Multiple Choice Question*** Why is $f(x)$ not differentiable at $x = 3$?
    (a) corner (b) cusp
    (c) discontinuity (d) vertical tangent line
    (e) None of the other answers are correct.

    11. D or d

Key

For problems 12—14, find (or evaluate) the given limits, if they exist. You may use reasoning, tables, algebra, limit theorems and properties covered in class, or what you know from the graph (without your calculator).

12. (4 points) $\lim_{x \to 2} \frac{-4}{(x-2)^4}$ 12. $-\infty$

See other exam key

13. (4 points) $\lim_{x \to -3} \left(\frac{-2}{x+3} \right)$ 13. DNE

-12

14. (4 points) $\lim_{x \to -\infty} \left(\frac{10x^3+4}{2x^3+3x-7} \right)$ 14. -5

For problem 15, use algebra to find the limit.

15. (4 points) $\lim_{x \to 3} \left(1 - \frac{7}{3}x - x^2\right)$ $= 1 - \frac{7}{3}(3) - 3^2$ 15. -15

$= 1 - 7 - 9 = -15$

16. (8 points) Suppose $f(x) = \frac{1}{3}x^2 e^x$. We will show next week that $f'(x) = \frac{1}{3}x(x+2)e^x$. What is the equation of the line tangent to the graph of $f(x)$ at $x = 1$?

16. ______________

See problem 17

on other exam

17. (8 points) Show, using the definition of continuity, that $f(x) = \left\{ \begin{array}{ll} \dfrac{x^2 - 7x + 12}{x - 3} & x \neq 3 \\ 4 & x = 3 \end{array} \right\}$ is not continuous at $x = 3$. Also, state which type of discontinuity exists for f at $x = 3$.

For problem 18, find (or evaluate) the limit, using algebra.

18. (8 points) $\lim_{x \to -\infty} \left(\dfrac{2x^2 - 3}{\sqrt{x^4 + 2x}} \right) \rightarrow \dfrac{-\infty}{-\infty}$

18. 2

$$= \lim_{x \to -\infty} \left[\frac{2x^2 - 3}{\sqrt{x^4 + 2x}} \cdot \frac{\frac{1}{(-x)^2}}{\frac{1}{(-x)^2}} \right] = \lim_{x \to -\infty} \left[\frac{2 - \frac{3}{x^2}}{\sqrt{\frac{x^4 + 2x}{x^4}}} \right]$$

$$= \lim_{x \to -\infty} \left[\frac{2 - \underbrace{\frac{3}{x^2}}_{\to 0}}{\sqrt{1 + \underbrace{\frac{2}{x^3}}_{\to 0}}} \right] = \frac{\lim_{x \to -\infty}(2) - 3 \cancel{\lim_{x \to -\infty}\left(\frac{1}{x^2}\right)}^{\,0}}{\sqrt{\lim_{x \to -\infty}(1) + 2 \lim_{x \to -\infty}\left(\frac{1}{x^3}\right)}}$$

$$= \frac{2}{\sqrt{1}} = 2$$