Math 150
Exam 2
Professor Busken

Name: Key

Directions: Box off your final answers. There are three different versions of the exam. Absolutely no calculators or cell phones allowed! Your cell phone must not be on your person. Writing in pen will not be accepted either. You will not be allowed to leave to use the restroom. Show all work in a legible manner to receive credit. Make sure you have 5 pages with 18 questions. You should not have to ask me any questions during the exam.

For problems 1–9, find $\frac{dy}{dx}$.

1. (6 points) $y = \sec^3(4x^3) = \left(\sec(4x^3)\right)^3$ 1. ________

$$y' = 3\left(\sec(4x^3)\right)^2 \cdot \frac{d}{dx}\left(\sec(4x^3)\right)$$

$$= 3\sec^2(4x^3) \cdot \sec(4x^3) \cdot \tan(4x^3) \cdot \frac{d}{dx}(4x^3)$$

$$= 3\sec^3(4x^3)\tan(4x^3) \cdot 12x^2$$

$$= \boxed{36x^2\sec^3(4x^3)\tan(4x^3)}$$

10

2. (10 points) $y = (5x-2)^6(1-3x-3x^2)^5$ 2. ________

$$y' = \left[6(5x-2)^5 \cdot \frac{d}{dx}(5x-2)\right](1-3x-3x^2)^5 + (5x-2)^6 \cdot 5(1-3x-3x^2)^4 \cdot \frac{d}{dx}(1-3x-3x^2)$$

$$= 6(5x-2)^5 \cdot 5 \cdot (1-3x-3x^2)^5 + 5(5x-2)^6(1-3x-3x^2)^4(-3-6x)$$

$$= 30(5x-2)^5(1-3x-3x^2)^5 - 15(5x-2)^6(1-3x-3x^2)^4(1+2x)$$

$$= 15(5x-2)^5(1-3x-3x^2)^4\left[2(1-3x-3x^2) - (5x-2)(1+2x)\right]$$

$$= 15(5x-2)^5(1-3x-3x^2)^4\left[2-6x-6x^2-[5x+10x^2-2-4x]\right]$$

$$= 15(5x-2)^5(1-3x-3x^2)^4\left[2-6x-6x^2-x-10x^2+2\right]$$

$$\boxed{= 15(5x-2)^5(1-3x-3x^2)^4(4-7x-16x^2)}$$

3. (6 points) $y = \frac{2}{3}x^3 - \frac{\pi}{x^2} - \frac{1}{\sqrt[5]{x^3}}$.

Key

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2}{3}x^3 - \pi x^{-2} - x^{-3/5}\right)$$

$$= \frac{2}{3}\frac{d}{dx}(x^3) - \pi\frac{d}{dx}(x^{-2}) - \frac{d}{dx}\left(x^{-3/5}\right)$$

$$= \frac{2}{3}\cdot 3x^2 - \pi\cdot(-2)x^{-3} - \left(-\frac{3}{5}\right)x^{-\frac{3}{5}-\frac{5}{5}}$$

$$\boxed{= 2x^2 + 2\pi x^{-3} + \frac{3}{5}x^{-8/5}}$$

4. (6 points) $y = \frac{4e^{3x}}{\cot(x)}$. $= 4e^{3x}\tan(x)$

$$\frac{dy}{dx} = 4\frac{d}{dx}\left(e^{3x}\tan x\right) = 4\left[3e^{3x}\tan x + e^{3x}\sec^2(x)\right]$$

$$\boxed{= 4e^{3x}\left(3\tan(x) + \sec^2(x)\right)}$$

or quotient rule

$$\frac{dy}{dx} = \frac{\cot(x)\cdot 12e^{3x} - 4e^{3x}\cdot(-\csc^2(x))}{\cot^2(x)} = \frac{4e^{3x}\left(3\cot(x) + \csc^2(x)\right)}{\cot^2(x)} = 4e^{3x}\tan^2(x)\left(3\cot(x) + \csc^2(x)\right)$$

$$= \boxed{4e^{3x}\left(3\tan(x) + \sec^2(x)\right)}$$

5. (6 points) $y = \frac{3x-5}{x^2-3x}$.

$$\frac{dy}{dx} = \frac{(x^2-3x)\cdot\frac{d}{dx}(3x-5) - (3x-5)\cdot\frac{d}{dx}(x^2-3x)}{(x^2-3x)^2} = \frac{(x^2-3x)\cdot 3 - (3x-5)(2x-3)}{(x^2-3x)^2}$$

$$= \frac{3x^2-9x-(6x^2-9x-10x+15)}{(x^2-3x)^2} = \boxed{\frac{-3x^2+10x-15}{(x^2-3x)^2}}$$

6. (6 points) $y = \tan^{-1}(x^2)$.

Key

$$y' = \frac{1}{1+(x^2)^2} \cdot \frac{d}{dx}(x^2) = \boxed{\frac{2x}{1+x^4}}$$

7. (6 points) $y = 7^{\cos(2x)}$.

$$y' = 7^{\cos(2x)} \cdot \ln 7 \cdot \frac{d}{dx}(\cos(2x))$$

$$= 7^{\cos(2x)} \cdot \ln 7 \cdot -2\sin(2x) = (-2\ln 7)(\sin 2x)(7^{\cos 2x})$$

8. (6 points) $y = \Big(\sin(x)\Big)^x$.

$$\ln y = x \ln(\sin x)$$

$$\frac{y'}{y} = \ln(\sin x) + x \cdot \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x)$$

$$y' = y\left(\ln(\sin(x)) + x\cot(x)\right)$$

$$\boxed{\frac{dy}{dx} = (\sin(x))^x \left(\ln(\sin x) + x \cdot \cot x\right)}$$

9. (6 points) $x^2y^2 + x\sin(y) = 1$.

$$\frac{d}{dx}\left(x^2y^2 + x\sin y\right) = \frac{d}{dx} 1$$

$$\frac{d}{dx}(x^2y^2) + \frac{d}{dx}(x\sin y) = 0$$

$$\left[2xy^2 + x^2\, 2y\, y'\right] + \left[\sin(y) + x\cos y \cdot y'\right] = 0$$

$$y'\left(2x^2y + x\cos y\right) = -\left(2xy^2 + \sin(y)\right)$$

$$\boxed{\frac{dy}{dx} = \frac{-2xy^2 - \sin(y)}{2x^2y + x\cos(y)}}$$

10. (6 points) $y = \ln\left[\frac{(5x+1)^2}{(2x-1)^4}\right]$

$y = 2\ln(5x+1) - 4\ln(2x-1)$

$\frac{dy}{dx} = 2\frac{d}{dx}\ln(5x+1) - 4\frac{d}{dx}(\ln(2x-1))$

$= 2\cdot\frac{1}{5x+1}\cdot\frac{d}{dx}(5x+1) - 4\cdot\frac{1}{2x-1}\cdot\frac{d}{dx}(2x-1)$

$= \frac{2}{5x+1}\cdot 5 - \frac{4}{2x-1}\cdot 2 = \boxed{\frac{10}{5x+1} - \frac{8}{2x-1}}$

good enough

$= \frac{10(2x-1)-8(5x+1)}{(5x+1)(2x-1)}$

$= \frac{20x-10-40x-8}{(5x+1)(2x-1)}$

$= \frac{-20x-18}{(5x+1)(2x-1)}$

$= \boxed{\frac{-2(10x+9)}{(5x+1)(2x-1)}}$

A better answer?

11. (6 points) Use a linear approximation (or differentials) to estimate $(4.01)^3$.

Let $f(x) = x^3$, $\Delta x = x_1 - x_0 = 4.01 - 4 = 0.01$.

We know $\Delta y = f(x+\Delta x) - f(x)$ and

$dy = f'(x)\,dx = f'(x)\cdot(\Delta x) = 3x^2\cdot\Delta x$, By the linear approximation thm,

Then, $f(x+\Delta x) = f(x) + \Delta y \approx f(x) + dy = f(x) + f'(x)\,dx = f(x) + 3x^2\cdot\Delta x$,

and $f(4+0.01) = f(4) + \Delta y \approx f(4) + dy = 4^3 + 3\cdot 4^2\cdot(0.01) = 64 + (0.03)\cdot(16)$

$= 64 + 0.48$

$= \boxed{64.48}$

The other way linearize ~~around~~ $f(x) = x^3$ around $(4, 64)$

$f'(x) = 3x^2$, $f'(4) = 3\cdot 16 = 48$

$y - 64 = 48(x-4)$

$y = 48x - 192 + 64 = 48x - 128$

$\boxed{x^3 \approx 48x - 128}$ if $x \approx 4$

$(4.01)^3 \approx 48(4.01) - 128 = 64.48$

(10) <u>The long way</u> – it's too much!!!

$$\frac{dy}{dx} = \frac{\cancel{(2x-1)}^{4}}{(5x+1)^2} \cdot \frac{(2x-1)^4 \cdot \frac{d}{dx}(5x+1)^2 - (5x+1)^2 \cdot \frac{d}{dx}(2x-1)^4}{(2x-1)^{\cancel{8}}}$$

$$= \frac{1}{(5x+1)^2} \cdot \frac{(2x-1)^4 \cdot 2 \cdot (5x+1) \cdot 5 - (5x+1)^2 \cdot 4(2x-1)^3 \cdot 2}{(2x-1)^4}$$

$$= \frac{2\cancel{(2x-1)}^{3}\cancel{(5x+1)}\left[(2x-1)\cdot 5 - 4(5x+1)\right]}{\cancel{(5x+1)^2}(2x-1)^{\cancel{4}}}$$

$$= \frac{2}{(5x+1)(2x-1)}\left[10x - 5 - 20x - 4\right] = \frac{2(-10x-9)}{(5x+1)(2x-1)}$$

$$= \frac{-20x-18}{(5x+1)(2x-1)} = \frac{-2(10x+9)}{(5x+1)(2x-1)}$$

12. (6 points) The radius of a sphere is increasing at a rate of 4 mm/sec. How fast is the volume increasing when the diameter is 80 mm?

given $\frac{dr}{dt} = 4\,\frac{mm}{s}$, find $\frac{dV}{dt}$ when $r = 40$.

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} = 4\pi \cdot (40)^2 \cdot 4 = 16\pi \cdot 1600 = \boxed{25{,}600\pi\ \frac{mm^3}{sec}}$$

13. (6 points) Find $\frac{d^2y}{dx^2}$ for $y = \frac{3x+7}{x^2}$

Key

$$y = (3x+7)x^{-2}$$

$$\frac{dy}{dx} = 3x^{-2} - 2x^{-3}(3x+7) = x^{-3}\left(3x - 2(3x+7)\right) = x^{-3}(3x - 6x - 14)$$

$$= x^{-3}(-3x - 14) = -x^{-3}(3x+14)$$

$$\frac{d^2y}{dx^2} = 3x^{-4}(3x+14) - 3x^{-3}$$

$$= 3x^{-4}\left[3x + 14 - x\right] = 3x^{-4}(2x+14)$$

$$\boxed{= 6x^{-4}(x+7) \quad \text{or} \quad \frac{6(x+7)}{x^4}}$$

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For problems 1–10, find $\frac{dy}{dx}$.

1. (6 points) $y = \csc^3(3x^4) = \left(\csc(3x^4)\right)^3$ 1. ________

$$\frac{dy}{dx} = 3\left(\csc(3x^4)\right)^2 \cdot \frac{d}{dx}\left(\csc(3x^4)\right)$$

$$= 3\csc^2(3x^4)\left(-\csc(3x^4)\cot(3x^4)\right) \cdot \frac{d}{dx}(3x^4)$$

$$\boxed{= -36x^3\csc^3(3x^4)\cot(3x^4)}$$

2. (10 points) $y = (12x+7)^5(1-3x-3x^2)^6$ 2. ________

$$\frac{dy}{dx} = 5(12x+7)^4 \cdot \left[\frac{d}{dx}(12x+7)\right](1-3x-3x^2)^6 + (12x+7)^5 \cdot 6(1-3x-3x^2)^5 \cdot \frac{d}{dx}(1-3x-3x^2)$$

$$= 5(12x+7)^4 \cdot 12 \cdot (1-3x-3x^2)^6 + 6(12x+7)^5(1-3x-3x^2)^5(-3-6x)$$

$$= 60(12x+7)^4(1-3x-3x^2)^6 - 18(12x+7)^5(1-3x-3x^2)^5(1-2x)$$

$$= 6(12x+7)^4(1-3x-3x^2)^5\left[10(1-3x-3x^2) - 3(12x+7)(1-2x)\right]$$

$$= 6(12x+7)^4(1-3x-3x^2)^5\left[10-30x-30x^2-3\left[12x-24x^2+7-14x\right]\right]$$

$$= 6(12x+7)^4(1-3x-3x^2)^5\left[10-30x-30x^2+6x+72x^2-21\right]$$

$$\boxed{= 6(12x+7)^4(1-3x-3x^2)^5\left[42x^2-24x-11\right]}$$

3. (6 points) $y = \frac{3}{2}x^2 + \frac{\pi}{x^3} - \frac{1}{\sqrt[3]{x^2}}$.

$$\frac{dy}{dx} = \frac{3}{2}\frac{d}{dx}(x^2) + \pi\frac{d}{dx}x^{-3} - \frac{d}{dx}x^{-2/3}$$

$$= \boxed{3x - 3\pi x^{-4} + \frac{2}{3}x^{-5/3}}$$

4. (6 points) $y = \frac{2e^{5x}}{\sec(x)} = 2e^{5x}\cos(x)$

$$\frac{dy}{dx} = 2\frac{d}{dx}\left(e^{5x}\cos(x)\right)$$

$$= 2[5e^{5x}\cos x - e^{5x}\sin x)$$

$$= \boxed{2e^{5x}\left(5\cos(x) - \sin(x)\right)}$$

or quotient rule

$$\frac{dy}{dx} = \frac{10e^{5x}\sec(x) - 2e^{5x}\sec(x)\tan(x)}{\sec^2(x)}$$

$$= \frac{2e^{5x}\cancel{\sec(x)}\left(5 - \tan(x)\right)}{\cancel{\sec(x)}\cdot\sec(x)}$$

$$= \frac{2e^{5x}(5 - \tan(x))}{\sec(x)}$$

$$= 2e^{5x}\cos(x)\left(5 - \tan(x)\right)$$

$$= \boxed{2e^{5x}\left(5\cos(x) - \sin(x)\right)}$$

5. (6 points) $y = \frac{2x-7}{1-x-2x^2}$.

$$\frac{dy}{dx} = \frac{(1-x-2x^2)\cdot\frac{d}{dx}(2x-7) - (2x-7)\cdot\frac{d}{dx}(1-x-2x^2)}{(1-x-2x^2)^2}$$

$$= \frac{(1-x-2x^2)\cdot 2 - (2x-7)(-1-4x)}{(1-x-2x^2)^2} = \frac{2 - \cancel{2x} - 4x^2 + \cancel{2x} + 8x^2 - 7 - 28x}{(1-x-2x^2)^2}$$

$$= \boxed{\frac{4x^2 - 28x - 5}{(1-x-2x^2)^2}}$$

Key

6. (6 points) $\quad y = \sin^{-1}(\sqrt{x})$.

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{d}{dx} x^{1/2} = \frac{1/2}{\sqrt{1-x}} \cdot x^{-1/2} = \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

7. (6 points) $\quad y = 5^{\sin(3x)}$.

$$\frac{dy}{dx} = 5^{\sin(3x)} \cdot \ln 5 \cdot \frac{d}{dx}(\sin(3x)) = \boxed{5^{\sin(3x)} \ln 5 \cdot 3\cos(3x)}$$

8. (6 points) $\quad y = x^{x^3}$.

$$\ln y = x^3 \ln x$$

$$\frac{y'}{y} = 3x^2 \ln x + x^3 \frac{1}{x}$$

$$y' = y\left(3x^2 \ln x + x^2\right)$$

$$= x^{x^3} \cdot x^2 \left(3\ln x + 1\right) = \boxed{x^{x^3+2}\left(\ln x^3 + 1\right)}$$

9. (8 points) $\quad x^3y^3 + x\cos(y) = 34$.

$$\frac{d}{dx}(x^3y^3) + \frac{d}{dx}(x\cos(y)) = \frac{d}{dx}(34)$$

$$3x^2y^3 + 3x^3y^2y' + \cos y - xy'\sin(y) = 0$$

$$y'\left(3x^3y^2 - x\sin(y)\right) = -\left(3x^2y^3 + \cos y\right)$$

$$\boxed{y' = \frac{-3x^2y^3 - \cos y}{3x^3y^2 - x\sin y}}$$

10. (6 points) $y = \ln\left[\dfrac{(3x-8)^4}{(5x+4)^3}\right] = 4\ln(3x-8) - 3\ln(5x+4)$

$$\frac{dy}{dx} = 4\frac{d}{dx}\left(\ln(3x-8)\right) - 3\frac{d}{dx}\ln(5x+4)$$

$$= 4\cdot\frac{1}{3x-8}\cdot\frac{d}{dx}(3x-8) - 3\cdot\frac{1}{5x+4}\cdot\frac{d}{dx}(5x+4)$$

$$= \boxed{\frac{12}{3x-8} - \frac{15}{5x+4}}$$

good enough

Key

$15 \cdot 8 = 120$, $168 / 3 = 56$

better??

$$= \frac{12(5x+4)-15(3x-8)}{(3x-8)(5x+4)} = \frac{60x+48-45x+120}{(3x-8)(5x+4)} = \frac{15x+168}{(3x-8)(5x+4)} = \boxed{\frac{3(5x+56)}{(3x-8)(5x+4)}}$$

11. (6 points) Use a linear approximation (or differentials) to estimate $(2.01)^3$.

$f(x) = x^3$

$\Delta y = f(x+\Delta x) - f(x)$

$dy = f'(x)\,dx = 3x^2\,dx = 3x^2(\Delta x)$

$\Delta x = x_1 - x_0 = 2.01 - 2.00 = 0.01$

$f(x+\Delta x) = f(x) + \Delta y \approx f(x) + dy = f(x) + f'(x)\,\Delta x$

$f(2.01) = f(2) + \Delta y \approx f(2) + dy = f(2) + f'(2)\cdot 0.01$

$= 8 + 3\cdot 2^2\cdot(0.01)$

$= 8 + (0.03)\cdot 4$

$= 8 + 0.12 = \boxed{8.12}$

The other way. $f(x) = x^3$, $f'(x) = 3x^2$, $f'(2) = 3\cdot 4 = 12$. Use $(x_1, y_1) = (2, 8)$ and $y - y_1 = 12(x - x_1)$

$y - 8 = 12(x-2)$

$\boxed{y = 12x - 16}$

$x^3 \approx 12x - 16$

$(2.01)^3 \approx 12(2.01) - 16 = 8.12$

Problem 10: the long way its toomuch!!!

$$y' = \frac{(5x+4)^3}{(3x-8)^4} \cdot \frac{d}{dx}\left(\frac{(3x-8)^4}{(5x+4)^3}\right)$$

$$= \frac{\cancel{(5x+4)^3}}{(3x-8)^4} \cdot \frac{(5x+4)^3 \cdot \frac{d}{dx}(3x-8)^4 - (3x-8)^4 \cdot \frac{d}{dx}(5x+4)^3}{(5x+4)^{\cancel{6}\,3}}$$

$$= \frac{1}{(3x-8)^4} \cdot \frac{(5x+4)^3 \cdot 4(3x-8)^3 \cdot 3 - (3x-8)^4 \cdot 3 \cdot (5x+4)^2 \cdot 5}{(5x+4)^3}$$

$$= \frac{1}{(3x-8)^{\cancel{4}}} \cdot \frac{3\cancel{(5x+4)^2}\cancel{(3x-8)^3}\left[4(5x+4) - 5(3x-8)\right]}{(5x+4)^{\cancel{3}}}$$

$$= \frac{3(20x+16-15x+40)}{(3x-8)(5x+4)} = \frac{3(5x+56)}{(3x-8)(5x+4)}$$

unfactored $\dfrac{15x+168}{(3x-8)(5x+4)}$

Key

12. (6 points) An inflatable cube is being expanded at a rate of 1080 $\frac{cm^3}{sec}$. Find the rate of growth of each edge when the edges are 6cm ($V = s^3$).

given $\frac{dV}{dt} = 1080 \frac{cm^3}{sec}$, find $\frac{ds}{dt}$ if $s = 6$.

$12 \cdot 9 = 108$

$$\frac{d}{dt}(V) = \frac{d}{dt}(s^3)$$

$$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$$

$$1080 = 3(6)^2 \cdot \frac{ds}{dt}$$

$$\frac{ds}{dt} = \frac{1080}{3 \cdot 36} = \frac{12 \cdot 9 \cdot 10}{3 \cdot 12 \cdot 3} = 10$$

$$\boxed{\frac{ds}{dt} = 10 \text{ cm/sec}}$$

13. (10 points) Find $\frac{d^2y}{dx^2}$ for $y = \frac{4x+7}{x^3} = (4x+7)x^{-3}$

$$\frac{dy}{dx} = -3x^{-4}(4x+7) + 4x^{-3}$$

$$= x^{-4}\left[4x - 3(4x+7)\right] = x^{-4}(4x - 12x - 21)$$

$$= x^{-4}(-8x-21) = -x^{-4}(8x+21)$$

$$\frac{d^2y}{dx^2} = 4x^{-5}(8x+21) - 8x^{-4}$$

$$= 4x^{-5}(8x+21-2x)$$

$$= 4x^{-5}(6x+21) = \boxed{12x^{-5}(2x+7) \text{ or } \frac{12(2x+7)}{x^5}}$$