Math 150
Exam Part A
Professor Busken **Name:** Key

Directions: **Box your solutions to receive credit.** **Make note of which of the 2 versions of the quiz you have. You are absolutely NOT allowed to use graphing calculators or cell phones or scratch paper! Your cell phone must not be on your person. Writing in pen will not be accepted either. Show all work in a legible manner to receive credit. You should not have to ask me any questions during the exam. You may use a scientific calculator.**

1. (10 points) Let $f(x) = -4x^5 + 20x^3$. Find the zeros of f and classify them as being even or odd zeros. Determine the x intervals for which the graph of the function is completely above or below the x-axis.

$f(x) = -4x^3(x^2-5) = -4x^3(x-\sqrt{5})(x+\sqrt{5})$; also $f(-x) = -f(x)$, so f has origin symmetry.

Zeros	
0	odd
$\pm\sqrt{5}$	odd

$+$ odd $-$ odd $+$ odd $-$; $x, f(x)$; $-\sqrt{5}$, 0, $\sqrt{5}$

$\boxed{\begin{array}{l} f(x) < 0 \quad \forall \; x \in (-\sqrt{5}, 0) \cup (\sqrt{5}, \infty) \\ f(x) \geq 0 \quad \forall \; x \in (-\infty, -\sqrt{5}) \cup (0, \sqrt{5}) \end{array}}$

2. (10 points) Let $f(x) = -4x^5 + 20x^3$. Use limits to determine the end behavior of f's graph.

$\lim_{x\to\infty}[f(x)] = -\infty$ and $\lim_{x\to-\infty}[f(x)] = \infty$

3. (10 points) Let $f(x) = -4x^5 + 20x^3$. Find the x–intervals on which f is increasing ($\uparrow$) and the x–intervals on which f is decreasing ($\downarrow$).

$f'(x) = -20x^4 + 60x^2 = -20x^2(x^2-3) = -20x^2(x-\sqrt{3})(x+\sqrt{3})$

$-$ odd $+$ even $+$ odd $-$; $x, f'(x)$; $-\sqrt{3}$ (local max), 0, $\sqrt{3}$ (local min)

$\boxed{\begin{array}{l} f \downarrow \; \forall \; x \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty) \\ f \uparrow \; \forall \; x \in (-\sqrt{3}, 0) \cup (0, \sqrt{3}) \end{array}}$

Key

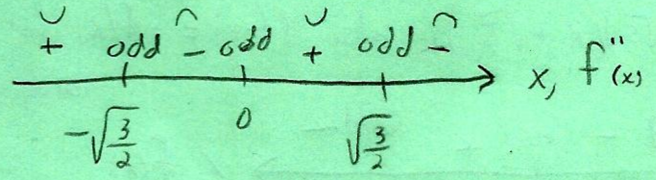
4. (10 points) Let $f(x) = -4x^5 + 20x^3$. Use your sign chart from the previous question to find the extrema of f and state whether they are relative (local) or absolute extrema.

local min $(x,y) = (\sqrt{3}, f(-\sqrt{3})) \approx (-1.73, -41.57)$

local max $(x,y) = (\sqrt{3}, f(\sqrt{3})) \approx (1.73, 41.57)$

5. (10 points) Let $f(x) = -4x^5 + 20x^3$. Find the x-intervals on which f is concave up (CU) or down (CD).

$f''(x) = -80x^3 + 120x = -40x(2x^2-3) = -40x(\sqrt{2}x - \sqrt{3})(\sqrt{2}x + \sqrt{3})$



f is CU $\forall x \in (-\infty, -\sqrt{\frac{3}{2}}) \cup (0, \sqrt{\frac{3}{2}})$

f is CD $\forall x \in (-\sqrt{\frac{3}{2}}, 0) \cup (\sqrt{\frac{3}{2}}, \infty)$

6. (10 points) Let $f(x) = -4x^5 + 20x^3$. Use your sign chart from the previous question to determine if f has any inflection points.

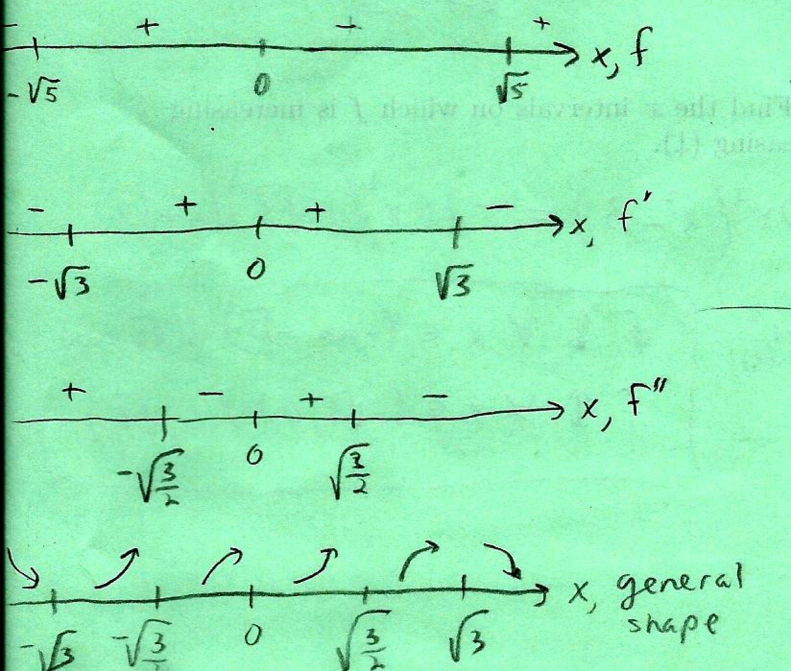
$(0,0)$

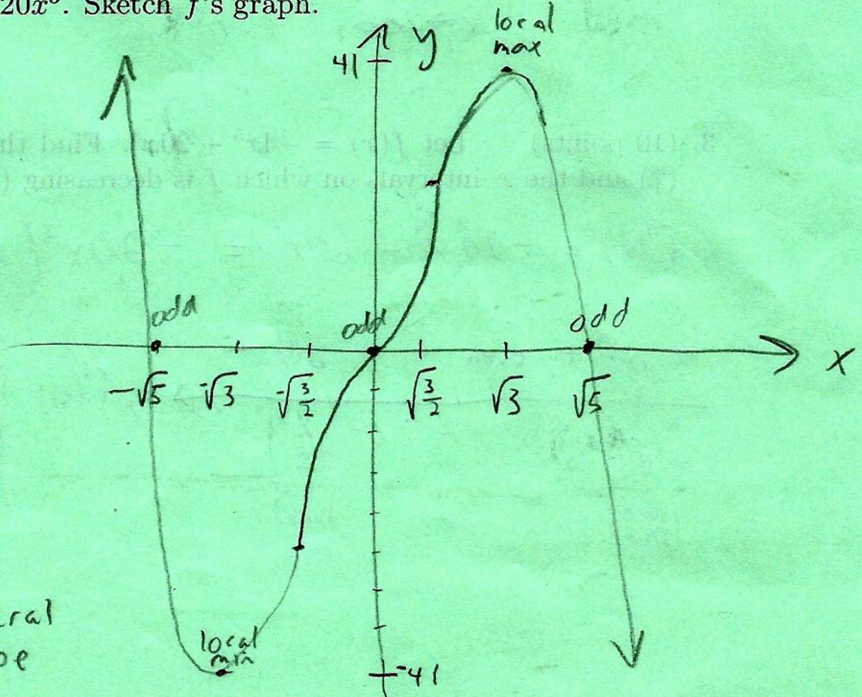
$(-\sqrt{\frac{3}{2}}, f(-\sqrt{\frac{3}{2}})) \approx (-1.2, -25.7)$ | $(\sqrt{\frac{3}{2}}, f(\sqrt{\frac{3}{2}})) \approx (1.2, 25.7)$

7. (10 points) Let $f(x) = -4x^5 + 20x^3$. Determine the range of f.

$\mathbb{R}$

8. (10 points) Let $f(x) = -4x^5 + 20x^3$. Sketch f's graph.





10. (10 points) Approximate, to four decimal places, the root of $f(x) = -4x^3 + \ln(x) + 6$ that lies in the x–interval, $[1, 2]$. Use Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{-4x_n^3 + \ln x_n + 6}{-12x_n^2 + \frac{1}{x_n}}$$

$$= x_n + \frac{-4x_n^3 + \ln(x_n) + 6}{\frac{12x_n^3 - 1}{x_n}} = x_n\left(1 + \frac{-4x_n^3 + \ln x_n + 6}{12x_n^3 - 1}\right)$$

$$= x_n\left(\frac{12x_n^3 - 1 - 4x_n^3 + \ln x_n + 6}{12x_n^3 - 1}\right)$$

$$= x_n\left(\frac{8x_n^3 + \ln x_n + 5}{12x_n^3 - 1}\right)$$

Let $x_1 = 1$ (but any x in $[1,2]$ would work)

$x_2 = 1.18$

$x_3 = 1.15445262$

$x_4 = 1.15373705$

$x_5 = 1.1537\ldots$

$$\boxed{1.1537}$$

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1. (10 points) Let $f(x) = -4x^6 + 20x^4$. Find the zeros of f and classify them as being even or odd zeros. Determine the x intervals for which the graph of the function is completely above or below the x-axis.

$f(x) = -4x^6 + 20x^4 = -4x^4(x^2-5) = -4x^4(x-\sqrt{5})(x+\sqrt{5})$

zeros: 0 even, $\pm\sqrt{5}$ odd

$-$ odd $+$ even $+$ odd $-$; $x, f(x)$; $-\sqrt{5}$, 0, $\sqrt{5}$

$f(x) < 0 \;\forall x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$

$f(x) > 0 \;\forall x \in (-\sqrt{5}, 0) \cup (0, \sqrt{5})$

Also, $f(-x) = f(x)$, so f has y-axis symmetry

2. (10 points) Let $f(x) = -4x^6 + 20x^4$. Use limits to determine the end behavior of f's graph.

$\lim_{x\to\infty}[f(x)] = -\infty$ and $\lim_{x\to-\infty}[f(x)] = -\infty$

3. (10 points) Let $f(x) = -4x^6 + 20x^4$. Find the x–intervals on which f is increasing ($\uparrow$) and the x–intervals on which f is decreasing ($\downarrow$).

$f'(x) = -24x^5 + 80x^3 = -8x^3(3x^2-10) = -8x^3(\sqrt{3}x - \sqrt{10})(\sqrt{3}x+\sqrt{10})$

$\nearrow$ $\searrow$ $\nearrow$ $\searrow$

$+$ odd $-$ odd $+$ odd $-$; $x, f'(x)$; $-\sqrt{\frac{10}{3}}$, 0, $\sqrt{\frac{10}{3}}$

max, min, max

$f \uparrow \forall x \in \left(-\infty, -\sqrt{\frac{10}{3}}\right) \cup \left(0, \sqrt{\frac{10}{3}}\right)$

$f \downarrow \forall x \in \left(-\sqrt{\frac{10}{3}}, 0\right) \cup \left(\sqrt{\frac{10}{3}}, \infty\right)$

4. (10 points) Let $f(x) = -4x^6 + 20x^4$. Use your sign chart from the previous question to find the extrema of f and state whether they are relative (local) or absolute extrema.

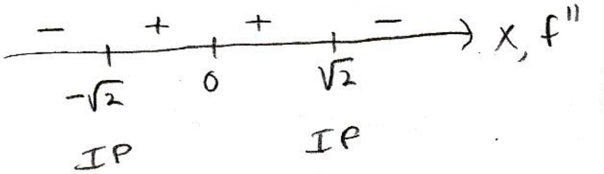
local min $(0,0)$

absolute max's $\left(\pm\sqrt{\frac{10}{3}}, f\left(\pm\sqrt{\frac{10}{3}}\right)\right) \approx (\pm 0.54, 74)$

5. (10 points) Let $f(x) = -4x^6 + 20x^4$. Find the x–intervals on which f is concave up (CU) or down (CD).

$$f''(x) = -8\frac{d}{dx}(3x^5 - 10x^3) = -8(15x^4 - 30x^2) = -120x^2(x^2-2)$$

$$= -120x^2(x-\sqrt{2})(x+\sqrt{2})$$



$\boxed{\begin{array}{l} f \text{ is CD } \forall x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty) \\ f \text{ is CU } \forall x \in (-\sqrt{2}, 0) \cup (0, \sqrt{2}) \end{array}}$

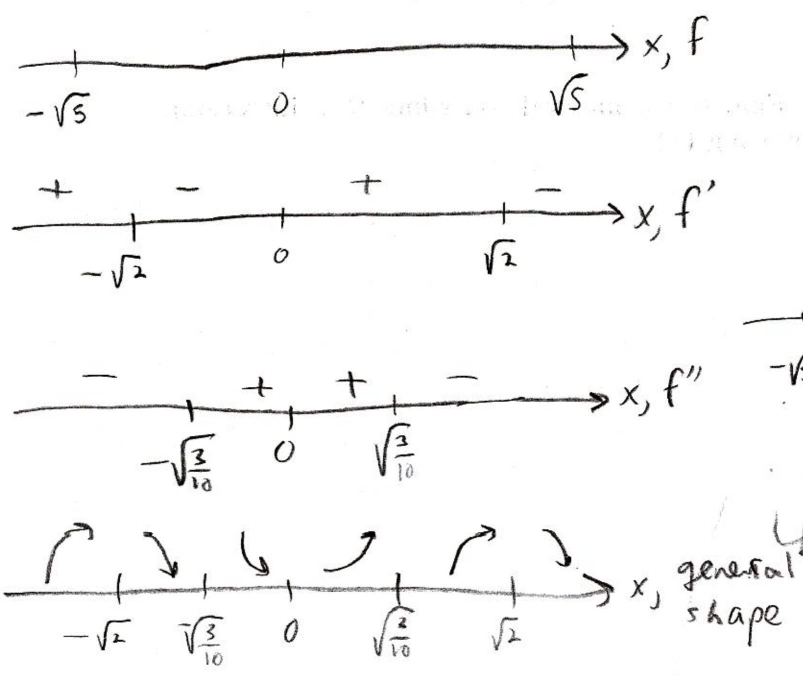
6. (10 points) Let $f(x) = -4x^6 + 20x^4$. Use your sign chart from the previous question to determine if f has any inflection points.

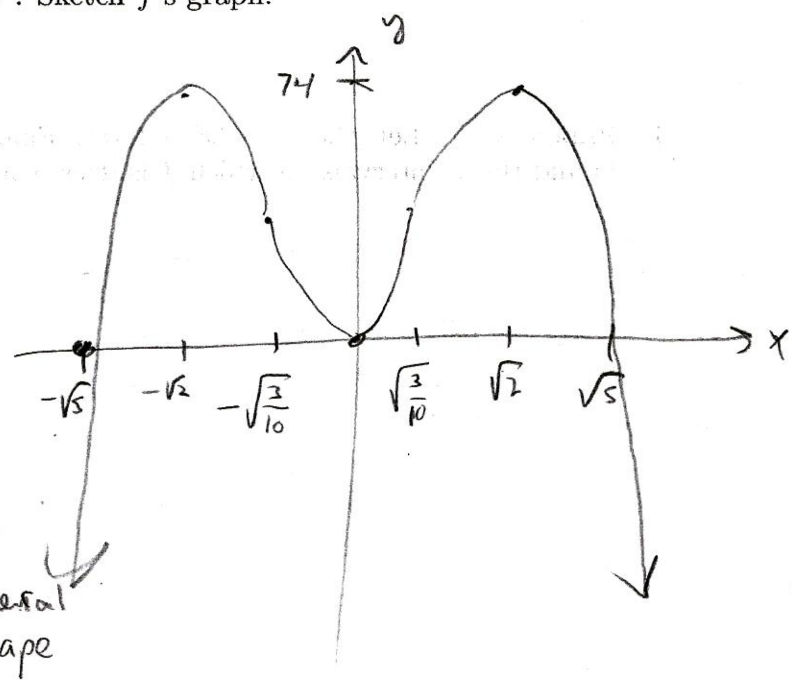
$(x,y) = (\pm\sqrt{2}, f(\pm\sqrt{2})) \approx (\pm 1.4, 48)$

7. (10 points) Let $f(x) = -4x^6 + 20x^4$. Determine the range of f.

$\left(-\infty, f\left(\pm\sqrt{\frac{3}{10}}\right)\right] \approx (-\infty, 74]$

8. (10 points) Let $f(x) = -4x^6 + 20x^4$. Sketch f's graph.





9. (10 points) Factor completely: $f(x) = x^5 - 10x^4 + 29x^3 - 32x^2 + 12x$

$$= x(x^4 - 10x^3 + 29x^2 - 32x + 12)$$

RZT $\Rightarrow$ the possible rational zeros of f are in the set

$$\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12\}$$

$$\begin{array}{c|ccccc} 1 & 1 & -10 & 29 & -32 & 12 \\ & \downarrow & 1 & -9 & 20 & -12 \\ \hline & 1 & -9 & 20 & -12 & 0 \end{array}$$

$$\Rightarrow f(x) = x(x-1)(x^3 - 9x^2 + 20x - 12)$$

$$\begin{array}{c|cccc} 1 & 1 & -9 & 20 & -12 \\ & \downarrow & 1 & -8 & 12 \\ \hline & 1 & -8 & 12 & 0 \end{array}$$

$$\Rightarrow f = x(x-1)^2(x^2 - 8x + 12)$$

$$\boxed{f(x) = x(x-1)^2(x-2)(x-6)}$$

Key

10. (10 points) Approximate, to four decimal places, the root of $f(x) = 8 - 2x^5 + \ln(x)$ that lies in the x–interval, $[1, 2]$. Use Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$X_{n+1} = X_n - \frac{8 - 2x_n^5 + \ln(x_n)}{-10x^4 + \frac{1}{x_n}}$$

$$= X_n + \frac{8 - 2x_n^5 + \ln(x_n)}{\frac{10x^5 - 1}{x_n}}$$

$$= X_n \left(1 + \frac{8 - 2x_n^5 + \ln(x_n)}{10x_n^5 - 1} \right)$$

$$= X_n \left(\frac{10x_n^5 - 1 + 8 - 2x_n^5 + \ln(x_n)}{10x_n^5 - 1} \right)$$

$$= X_n \left(\frac{8x_n^5 + \ln(x_n) + 7}{10x_n^5 - 1} \right)$$

Let $x_1 = 1$, but any x in $[1,2]$ would work!

$X_2 = 1.\overline{6}$

$X_3 = 1.44188573$

$X_4 = 1.34551171$

$X_5 = 1.32917816$

$X_6 = 1.32875417$

$X_7 = 1.3287\ldots$

$$\boxed{X \doteq 1.3287}$$

Math 150
Exam 3
Professor Busken

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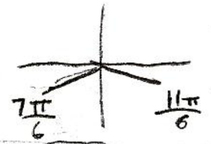
1. (17 points) Use the Extreme Value Theorem to find the local and absolute extrema of $f(x) = 2\sin(x) - \cos(2x)$ on the x–interval, $[0, 2\pi]$.

$$f'(x) = 2\cos(x) + 2\sin(2x)$$
$$= 2\cos(x) + 4\sin(x)\cos(x)$$
$$= 2\cos(x)\left(1 + 2\sin(x)\right)$$

and $f'(x) = 0$ when $2\cos(x) = 0$

or when $\sin(x) = -\frac{1}{2}$.



$\cos(x) = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin(x) =$ when $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

$\nearrow$ 1.57 $\searrow$ 3.66 $\nearrow$ 4.71 $\searrow$ 5.75 $\nearrow$

$+ \quad | \quad - \quad | \quad + \quad | \quad - \quad | \quad +$ $\longrightarrow x, f'$

$\frac{\pi}{2} \quad \frac{7\pi}{6} \quad \frac{3\pi}{2} \quad \frac{11\pi}{6}$

Test Pts

$f'(1) \approx 2.89$ | $f'(2) \approx -2.3$ | $f'(4) \approx 0.67$
$f'(4.8) \approx -0.17$ | $f'(6) \approx 0.84$

$f(0) = f(\pi) = -1$, $f\left(\frac{\pi}{2}\right) = 2 + 1 = 3$, $f\left(\frac{7\pi}{6}\right) = 2\cdot\frac{-1}{2} - \frac{1}{2} = -3/2$

$f\left(\frac{3\pi}{2}\right) = -2 + 1 = -1$, $f\left(\frac{11\pi}{6}\right) = -1 - \frac{1}{2} = -3/2$

$$\boxed{\begin{array}{l} \text{abs max } \left(\frac{\pi}{2}, 3\right) \\ \text{local max } \left(\frac{3\pi}{2}, -1\right) \\ \text{abs min } \left(\frac{7\pi}{6}, -3/2\right) \\ \text{abs min } \left(\frac{11\pi}{6}, -3/2\right) \end{array}}$$

2. (17 points) Find the critical numbers of $f(x) = \sqrt{2x^2 - 1} \cdot (x + 2)^3$,

$$f'(x) = \frac{d}{dx}\left((2x^2-1)^{1/2}(x+2)^3\right) = \frac{1}{2}(2x^2-1)^{-1/2} \cdot 4x \cdot (x+2)^3 + (2x^2-1)^{1/2}\, 3(x+2)^2$$
$$= 2x(2x^2-1)^{-1/2}(x+2)^3 + 3(2x^2-1)^{1/2}(x+2)^2$$
$$= (2x^2-1)^{-1/2}(x+2)^2\left[2x(x+2) + 3(2x^2-1)\right]$$
$$= (2x^2-1)^{-1/2}(x+2)^2\left[2x^2 + 4x + 6x^2 - 3\right] = \frac{(x+2)^2(8x^2+4x-3)}{\sqrt{2x^2-1}}$$

and $\mathrm{dom}(f) \equiv \left(-\infty, \frac{-\sqrt{2}}{2}\right] \cup \left[\frac{\sqrt{2}}{2}, \infty\right)$.

CN's of f satisfy $(x+2)^2 = 0$, $8x^2+4x+3 = 0$ or $\sqrt{2x^2-1} = 0$, so long as they are in dom(f).

$$\left[8x^2+4x+3=0\right] \Leftrightarrow \left[x = \frac{-4 \pm \sqrt{16 - 4(8)(-3)}}{2\cdot 8} = \frac{-4 \pm \sqrt{112}}{16} = \frac{-4 \pm 4\sqrt{7}}{16} = \frac{-4}{16} \pm \frac{4\sqrt{7}}{16} = \frac{-1}{4}(1 \pm \sqrt{7}) \approx \{-0.9, 0.4\}\right]$$

$$\boxed{\text{CN's of } f: \left\{-2, \pm\frac{\sqrt{2}}{2}, -\frac{1}{4}(1+\sqrt{7})\right\}}$$

3. (17 points) Find $f(x)$ if $f''(x) = 2\cos(x) - 5\sin(x)$ and $f(\pi) = 2 + 6\pi$ and $f'(\pi) = 3$.

$$\left[f'(x) = 2\sin(x) + 5\cos(x) + C\right] \text{ and } \left[f'(\pi) = 3\right] \Leftrightarrow \left[2\cancel{\sin(\pi)}^{0} + 5\cancel{\cos(\pi)} + C = 3\right]$$

$$\Leftrightarrow \left[-5 + C = 3\right] \Leftrightarrow \left[C = 8\right]. \therefore, \left[f'(x) = 2\sin(x) + 5\cos(x) + 8\right]. \text{ Moreover,}$$

$$\left[f(x) = -2\cos(x) + 5\sin(x) + 8x + C\right] \text{ and } \left[f(\pi) = 2 + 6\pi\right] \Leftrightarrow$$

$$\left[-2\cancel{\cos(\pi)}^{-1} + 5\cancel{\sin(\pi)}^{0} + 8\pi + C = 2 + 6\pi\right] \Leftrightarrow \left[2 + 8\pi + C = 2 + 6\pi\right]$$

$$\Leftrightarrow \left[C = -2\pi\right]. \therefore, \boxed{f(x) = -2\cos(x) + 5\sin(x) + 8x - 2\pi}$$

4. (16 points) Find $f(x)$ if $f'(x) = \dfrac{6 - 7x^2}{\sqrt[5]{x}}$

$$f'(x) = x^{-1/5}(6 - 7x^2) = 6x^{-1/5} - 7x^{2-\frac{1}{5}} = 6x^{-1/5} - 7x^{9/5}$$

Then
$$f(x) = 6\left(\frac{x^{-1/5+1}}{-\frac{1}{5}+1}\right) - 7\left(\frac{x^{9/5+1}}{\frac{9}{5}+1}\right) + C$$

$$= 6\left(\frac{x^{4/5}}{4/5}\right) - 7\left(\frac{x^{14/5}}{14/5}\right) + C$$

$$= 6 \cdot \frac{5}{4} x^{4/5} - 7 \cdot \frac{5}{14} x^{14/5} + C = \frac{15}{2} x^{4/5} - \frac{5}{2} x^{14/5} + C$$

$$\boxed{f(x) = \frac{5}{2} x^{4/5}(3 - x^2) + C}$$

5. (17 points) Sketch the graph of a function that has the following properties:

$f(0) = 1, \;\; f(2) = 3; \;\; f'(0) = f'(2) = 0,$

$f'(x) < 0 \;\; \forall\, x \in (-\infty, 0) \cup (2, \infty)$

$f'(x) > 0 \;\; \forall\, x \in (0, 2) \cup (6, 9)$

$f''(x) > 0 \;\; \forall\, x \in (-\infty, 1)$

$f''(x) < 0 \;\; \forall\, x \in (1, \infty)$

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1. (17 points) Use the Extreme Value Theorem to find the local and absolute extrema of $f(x) = 2\sin(x) - \cos(2x)$ on the x–interval, $[0, 2\pi]$.

See other key

2. (17 points) Find the critical numbers of $f(x) = \sqrt{3x^2-1}\cdot(x+2)^3$,

$$f'(x) = \frac{d}{dx}\left((3x^2-1)^{1/2}(x+2)^3\right) = \frac{1}{2}(3x^2-1)^{-1/2}\cdot 6x\cdot(x+2)^3 + (3x^2-1)^{1/2}\cdot 3(x+2)^2$$

$$= 3x(3x^2-1)^{-1/2}(x+2)^3 + 3(3x^2-1)^{1/2}(x+2)^2$$

$$= 3(3x^2-1)^{-1/2}(x+2)^2\left[x(x+2) + (3x^2-1)\right] = 3(3x^2-1)^{-1/2}(x+2)^2\left[x^2+2x+3x^2-1\right]$$

$$= \frac{3(x+2)^2(4x^2+2x-1)}{\sqrt{3x^2-1}} \text{ and } \mathrm{dom}(f) \equiv \left(-\infty, -\frac{\sqrt{3}}{3}\right] \cup \left[\frac{\sqrt{3}}{3}, \infty\right).$$

CN's of f satisfy $3(x+2)^2 = 0$ or $4x^2+2x-1=0$ or $\sqrt{3x^2-1}=0$, so long as they are values in dom f. $\boxed{\text{CN's of } f: \left\{\pm\frac{\sqrt{3}}{3}, -\frac{1}{4}(1+\sqrt{5})\right\}}$

$$\left[x = \frac{-2\pm\sqrt{4-4\cdot4(-1)}}{2\cdot4} = \frac{-2\pm\sqrt{20}}{8} = \frac{-2\pm2\sqrt{5}}{8} = -\frac{1}{4}(1\pm\sqrt{5}) \approx \{-0.8, 0.3\}\right]$$

3. (17 points) Find $f(x)$ if $f''(x) = 2\cos(x) - 5\sin(x)$ and $f(\pi) = 2 + 6\pi$ and $f'(\pi) = 3$.

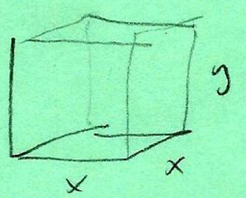
See other key

4. (16 points) Find $f(x)$ if $f'(x) = \dfrac{6 - 7x^2}{\sqrt[5]{x}} = x^{-1/5}(6-7x^2) = 6x^{-1/5} - 7x^{2-1/5}$

$f'(x)$

See other key

6. (16 points) An open box has a square base and volume 1728 cm^3. Find the dimensions of the box that minimize the surface area. (Round to 1 decimal)



$$\boxed{\begin{array}{c}\text{constraint eqn}\\ V = x^2 y = 1728\end{array}}$$

$$SA = 4xy + x^2$$

$$SA(x) = 4x\left(\frac{1728}{x^2}\right) + x^2$$

$$= 6912x^{-1} + x^2$$

$$\frac{d(SA)}{dx} = -6912x^{-2} + 2x$$

$$= \frac{-6912}{x^2} + \frac{2x^3}{x^2}$$

CN's of $SA(x)$, $x = (6912/2)^{1/3} \approx 15.1$

$-$ 15.1 $+$ $\to$ x, SA'

$(3456)^{1/3}$

Moreover, $SA''(x) = 13824x^{-3} + 2$

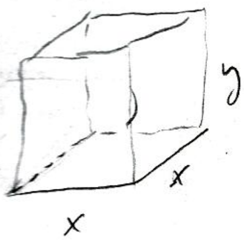
and $SA''\left((3456)^{1/3}\right) > 0$, $\therefore$

the 2nd derivative test $\Rightarrow$

$$\boxed{\begin{array}{c}\text{min dimensions}\\ (x, y) \approx (15.1\text{ cm}, 7.6\text{cm})\end{array}}$$

Check $1728 \approx (15.1)^2 \cdot (\ldots)$

6. (16 points) An open box has a square base and volume 726 cm^3. Find the dimensions of the box that minimize the surface area. (Round to 1 decimal)



constraint eqn

$$\boxed{V = x^2 y = 726}$$

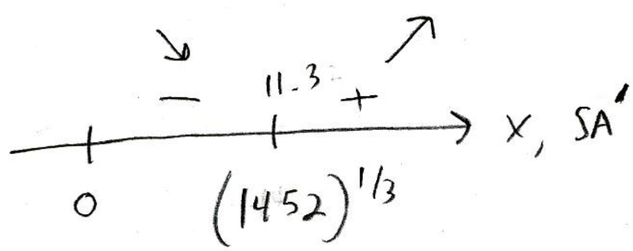
minimize surface area

$$SA = 4xy + x^2, \quad \text{or}$$

$$SA(x) = 4x\left(\frac{726}{x^2}\right) + x^2 = 2904x^{-1} + x^2$$

$$\frac{d(SA)}{dx} = -2904x^{-2} + 2x = -2x^{-2}(1452 - x^3) = \frac{2(x^3 - 1452)}{x^2}$$

CN's of $SA(x)$ are $x = 0, (1452)^{1/3}$ (But $x=0$ not in domain)



$SA''(x) = 5808x^{-3} + 2$ and $SA''\left((1452)^{1/3}\right) > 0$, $\therefore$ the 2nd Derivative test $\Longrightarrow$

$$\boxed{(x, y) \approx (11.3\text{ cm}, 5.7\text{ cm}) \quad \text{min dimensions}}$$