Math 150
~~Practice~~ Exam 4
Professor Busken **Name:** ____________________

1. Evaluate $\int_{-5}^{5} \sqrt{25-x^2}\,dx$. Hint: Interpret in terms of area.

2. Write $\lim_{n\to\infty}\left(\sum_{k=1}^{n} x_k \ln(1+x_k^2)\cdot \Delta x_k\right)$ as a definite integral on the interval $[0,e]$.

3. Evaluate $\int_{\pi}^{\pi} x\sin(x)\,dx$

4. Evaluate $\int_{-\pi}^{\pi} \frac{x^2\cos(x^2)}{2x+\sin(x)}\,dx$

5. Suppose $f(-x)=f(x)$ and $\int_{0}^{\pi} f(x)\,dx = 2$. Evaluate $\int_{-\pi}^{\pi} f(x)\,dx$

6. Write as a single integral: $\int_{-2}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx$

7. ***Calculator Problem***. What integral represents the (arc) length of $m(x)=x^2$ from $(x,y)=(0,0)$ to $(x,y)=(3,9)$? Approximate the value of this integral using Simpson's Rule with $n=6$.

8. Evaluate $\int \left[\frac{\sqrt[3]{x^2}-3x^3}{x^2} + e^{-2x} + \frac{1}{x} + \sin(3x) + \frac{1}{1+x^2}\right] dx$

9. Find the average value of $f(x) = -xe^{-x^2}$ on the x interval $\left[0,\frac{1}{2}\right]$.

10. Evaluate $\int_{-5}^{5} \left|x^2-9\right|\,dx$

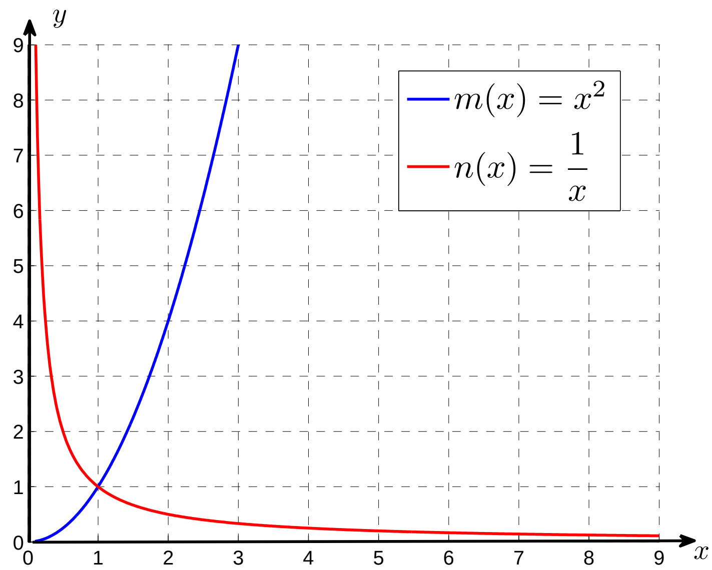
11. Evaluate $\int \frac{1}{2x-1}\,dx$

12. Evaluate $\int \frac{2x}{1-x}\,dx$

13. Find the derivative, $F'(x)$, if $F(x) = \int_{x}^{-3} \sqrt{t+\sin(t)}\,dt$

14. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_{0}^{15} n'(t)\,dt$ represent?

For problems 15—21, use the figure given below.



15. Let R be the region bounded by $m(x) = x^2$, $n(x) = \frac{1}{x}$, and vertical lines $x = \frac{1}{2}$ and $x = e$. Determine the area of R.

16. Let R be the region bounded by $m(x) = x^2$, $n(x) = \frac{1}{x}$, and the vertical line $x = e$ (Recall $e \approx 2.7178$). Revolve R around the x-axis. Use the ***Washer Method*** to find the volume of the resulting solid of revolution? Set up the integral only, do not evaluate.

17. Let R be the region defined in question 16. Revolve R around $y = 9$. Set up the volume integral for the resulting solid. Do not evaluate.

18. Let R be the region bounded by $n(x) = \frac{1}{x}$, $y = 0$ and vertical lines $x = 1$ and $x = 3$. Revolve R around the x-axis. Use the ***Disk Method*** to determine the volume integral associated with R. Set up the integral only, do not evaluate.

19. Let R be the region bounded by $m(x) = x^2$, $y = 0$ and $x = 3$. Revolve R around the y-axis. Use the ***Shell Method*** to determine the volume integral associated with R. Set up the integral only, do not evaluate.

20. Let R be the region defined in question 18. Revolve R around $x = 5$. Set up the volume integral for the resulting solid. Do not evaluate.

21. ***Calculator Problem***. Consider the solid of revolution described in question 19. What integral represents the surface area of the solid? Approximate the value of this integral using the Trapezoid Rule with $n = 6$.

Answers

1. $\frac{25\pi}{2}$; the integrand function is a half circle with radius 5.

2. $\int_0^e x\ln(1+x^2)\ dx$

3. 0; the limits of integration are the same.

4. 0; the integrand is an odd function.

5. 4

6. $\int_{-1}^5 f(x)\ dx$

7. $L = \int_0^3 \sqrt{1+4x^2}\ dx \approx 9.75$

8. $-3x^{-1/3} - \frac{3x^2}{2} + \frac{1}{2}e^{-2x} + \ln|x| - \frac{1}{3}\cos(3x) + \tan^{-1}(x) + C$

9. $e^{1/4} - 1$

10. $\frac{196}{3}$

11. $\frac{1}{2}\ln|2x-1| + C$ or $\ln\sqrt{2x-1} + C$

12. $-2\ln|1-x| + 2(1-x) + C = \ln(1-x)^{-2} + 2 - 2x + C$
$= \ln(1-x)^{-2} - 2x + C$
We say the 2 is "absorbed by C."

13. $F'(x) = -\sqrt{x+\sin(x)}$

14. The number of bees in the population after 15 weeks. (In other words, the given expression is the initial number of bees plus the net change in the bee population over 15 weeks.)

15. $\int_{\frac{1}{2}}^1 \left(\frac{1}{x} - x^2\right)dx + \int_1^e \left(x^2 - \frac{1}{x}\right)dx = \frac{e^3}{3} - \ln\left(\frac{1}{2}\right) - \frac{39}{24} = \frac{e^3}{3} + \ln(2) - \frac{39}{24}$

16. $V = \pi\int_1^e \left(x^4 - \frac{1}{x^2}\right)dx$

17. $V = \pi\int_1^e \left[\left(9 - \frac{1}{x}\right)^2 - \left(9 - x^2\right)^2\right]dx$

18. $V = \pi\int_1^3 \left(\frac{1}{x}\right)^2 dx$

19. $V = 2\pi\int_0^3 x^3 dx$

20. $V = 2\pi\int_1^3 (5-x)\cdot\frac{1}{x}\ dx$

21. $SA = \int_0^3 2\pi x^2\sqrt{1+4x^2}\ dx \approx 268.4$