Test 1 Review

Disclaimer: I do not claim that the following questions are representative of what will appear on the exam. There are many topics we have covered that do not appear on this review. Doing these problems is a good start to studying for the exam, it is not sufficient on its own. You will never be allowed to use a calculator on an exam. There will always be more than one version of the exam.

1. Using the definition of the derivative of a function find $f'(x)$ for the following:
   (a) $f(x) = 3x^2 + 4x - 5$
   (b) $f(x) = \dfrac{3x}{x+2}$
   (c) $f(x) = \sqrt{x+3}$
2. Find the following limits:
   (a) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{3x^2+2x}}{x+2}$
   (b) $\displaystyle\lim_{x\to-\infty} \frac{x^2-5}{\sqrt{x^4+3x}}$
   (c) $\displaystyle\lim_{x\to\infty} x - \sqrt{x^2+3x-5}$
   (d) $\displaystyle\lim_{x\to-\infty} \frac{x^3+5x-4}{4x^3+2x^2+x-6}$
   (e) $\displaystyle\lim_{x\to\infty} \frac{x^2+4}{x^3+3x^2-7}$
3. Evaluate the following limits, use algebra to find them:
   (a) $\displaystyle\lim_{x\to0} \frac{\frac{1}{x+3}-\frac{1}{3}}{x}$
   (b) $\displaystyle\lim_{h\to0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$
   (c) $\displaystyle\lim_{t\to0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right)$
4. Prove using the precise definition of a limit that $\displaystyle\lim_{x\to3}(3x-2) = 7$.
5. Find the average rate of change of the function, $f(x) = x^3 - 2x$, on the interval $[2,3]$.
6. Find the instantaneous rate of change of the function $f(x) = x^2 - x$ at $x = 3$.
7. Show, using the definition of continuity, that

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & x \neq 3 \\ 4 & x = 3 \end{cases}$$

   is not continuous at $x = 3$.
8. Prove that the equation $f(x) = x^4 - 3x^2 + 1$ has a root using the Intermediate Value Theorem.
9. Plot $f(x) = x^3 - x$ on a set of axes. On the same set of axes plot $f'(x)$. **Hint:** Use where $f(x)$ has slope zero to help you get started.
10. Find the equation of the tangent line to $y = 3x^2 + 2$ at $x = -1$.
11. Find the equation of the tangent line to $y = \dfrac{1}{x}$ at $x = 3$.

1a) $6x+4$ 1b) $\dfrac{6}{(x+2)^2}$ 1c) $\dfrac{1}{2\sqrt{x+3}}$

2a) $\sqrt{3}$ 2b) 1 2c) $-\frac{3}{2}$ 2d) $\frac{1}{4}$ 2e) 0

3a) $-\frac{1}{9}$ 3b) $\frac{1}{2\sqrt{2}}$ 3c) 1

4) $\delta = \frac{\epsilon}{3}$

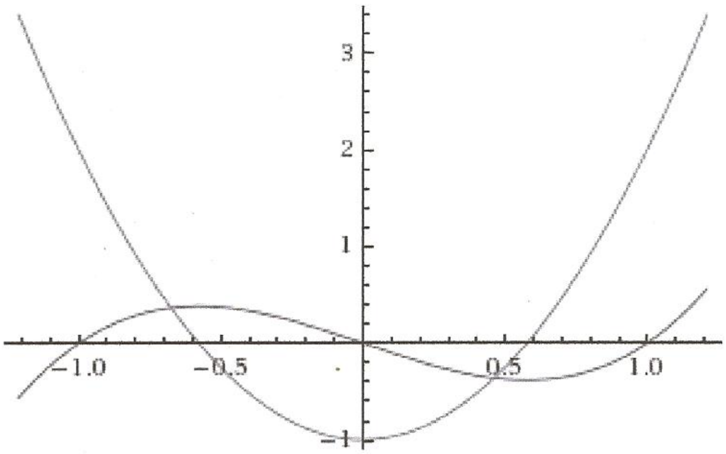
5) 17

6) 5

7) $\lim\limits_{x\to 3-} f(x) = \lim\limits_{x\to 3+} f(x) = 6$ (you need to show why), but $\lim\limits_{x\to 3} f(x) \neq f(3)$

8) You can use the interval $(0,1)$, there are other possible intervals

9)



The parabola is $f'(x)$

10) $y-5=-6(x+1)$

11) $y-\dfrac{1}{3} = -\dfrac{1}{9}(x-3)$

(1a) $$f'(x) = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right] = \lim_{h\to 0}\frac{[3(x+h)^2+4(x+h)-5]-[3x^2+4x-5]}{h}$$

$$= \lim_{h\to 0}\left[\frac{3(x^2+2xh+h^2)+\cancel{4x}+4h\cancel{-5}-3x^2\cancel{-4x}\cancel{+5}}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\cancel{3x^2}+6xh+3h^2+4h-\cancel{3x^2}}{h}\right] = \lim_{h\to 0}\left[\frac{6xh+3h^2+4h}{h}\right]$$

$$= \lim_{h\to 0}\frac{h(6x+3h+4)}{h} = \lim_{h\to 0}(6x+3h+4) = \boxed{6x+4}$$

(1b) $$f'(x) = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right] = \lim_{h\to 0}\left[\frac{1}{h}\cdot[f(x+h)-f(x)]\right]$$

$$= \lim_{h\to 0}\left[\frac{1}{h}\cdot\left(\frac{3(x+h)}{(x+h)+2}-\frac{3x}{x+2}\right)\right] = \lim_{h\to 0}\left[\frac{1}{h}\left(\frac{3x+3h}{x+h+2}-\frac{3x}{x+2}\right)\right]$$

Foil — Distribute

$$= \lim_{h\to 0}\left[\frac{1}{h}\left(\frac{3x+3h}{x+h+2}\cdot\left(\frac{x+2}{x+2}\right)-\frac{3x}{x+2}\cdot\left(\frac{x+h+2}{x+h+2}\right)\right)\right]$$

$$= \lim_{h\to 0}\left[\frac{1}{h}\,\frac{(\cancel{3x^2}+\cancel{6x}+\cancel{3xh}+6h)-\cancel{3x^2}-\cancel{3xh}-\cancel{6x}}{(x+h+2)(x+2)}\right]$$

$$= \lim_{h\to 0}\left[\frac{1}{h}\,\frac{6h}{(x+h+2)(x+2)}\right] = \lim_{h\to 0}\left[\frac{6}{(x+h+2)(x+2)}\right] = \frac{6}{(x+0+2)(x+2)} = \boxed{\frac{6}{(x+2)^2}}$$

(1c) $f'(x) = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right] = \lim_{h\to 0}\left[\frac{\sqrt{(x+h)+3}-\sqrt{x+3}}{h}\right] \to \frac{0}{0}$

multiply by 1 in the form of numerator's conjugate over itself

$$= \lim_{h\to 0}\left[\frac{\sqrt{x+h+3}-\sqrt{x+3}}{h}\left(\frac{\sqrt{x+h+3}+\sqrt{x+3}}{\sqrt{x+h+3}+\sqrt{x+3}}\right)\right]$$

this is one!

$$= \lim_{h\to 0}\left[\frac{(x+h+3)-(x+3)}{h\left(\sqrt{x+h+3}+\sqrt{x+3}\right)}\right]$$

unnecessary here to distribute h over the sum in the denominator

$$= \lim_{h\to 0}\left(\frac{h}{h\left[\sqrt{x+h+3}+\sqrt{x+3}\right]}\right) = \lim_{h\to 0}\left(\frac{1}{\sqrt{x+h+3}+\sqrt{x+3}}\right)$$

$\frac{1}{2}(x+3)^{-1/2}$ would also be an acceptable answer

$$= \frac{1}{\sqrt{x+0+3}+\sqrt{x+3}} = \frac{1}{\sqrt{x+3}+\sqrt{x+3}} = \boxed{\frac{1}{2\sqrt{x+3}}}$$

(2a) $\lim_{x\to\infty}\left(\frac{\sqrt{3x^2+2x}}{x+2}\right) \to \frac{\infty}{\infty}$, which is indeterminate form. So, we algebraically manipulate the limiting expression by dividing both numerator and denominator by x raised to the highest power in the denominator (not the given expression)

Note: this maneuver is equivalent to multiplying the expression by 1 in the form of $\left(\frac{\frac{1}{x}}{\frac{1}{x}}\right)$

$$= \lim_{x\to\infty}\left[\frac{\sqrt{3x^2+2x}}{x+2}\cdot\frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}\right]$$

$$= \lim_{x\to\infty}\left[\frac{\frac{\sqrt{3x^2+2x}}{x}}{\left(\frac{x+2}{x}\right)}\right]$$

use $x=\sqrt{x^2}$ and $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

$$= \lim_{x\to\infty}\left[\frac{\frac{\sqrt{3x^2+2x}}{\sqrt{x^2}}}{\frac{x}{x}+\frac{2}{x}}\right] = \lim_{x\to\infty}\left[\frac{\sqrt{\frac{3x^2+2x}{x^2}}}{1+\frac{2}{x}}\right] = \lim_{x\to\infty}\left[\frac{\sqrt{3+\frac{2}{x}}}{1+\frac{2}{x}}\right]$$

next page

$$= \lim_{x\to\infty}\left[\frac{\sqrt{3+\frac{2}{x}}}{1+\frac{2}{x}}\right]$$

(2a) continued

Now, recall properties of limits of continuous functions, $f(x)$ & $g(x)$. We use those properties and the following theorem next.

Thm: Suppose c is a particular, but arbitrarily chosen rational number. Then, $\lim_{x\to\infty}\left(\frac{1}{x^c}\right) = 0$, and, if $\frac{1}{x^c}$ is defined for x on the interval $(-\infty, 0)$, then $\lim_{x\to-\infty}\left(\frac{1}{x^c}\right) = 0$.

$$= \frac{\lim_{x\to\infty}\sqrt{3+\frac{2}{x}}}{\lim_{x\to\infty}\left[1+\frac{2}{x}\right]}$$

(since $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$)

$$= \frac{\sqrt{\lim_{x\to\infty}\left(3+\frac{2}{x}\right)}}{\lim_{x\to\infty}\left(1+\frac{2}{x}\right)}$$

since $\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ whenever Either

n is an odd positive integer, or

n is an even positive integer and $\lim_{x\to a} f(x) > 0$.

$$= \frac{\sqrt{\lim_{x\to\infty}(3) + \lim_{x\to\infty}\left(\frac{2}{x}\right)}}{\lim_{x\to\infty}(1) + \lim_{x\to\infty}\left(\frac{2}{x}\right)}$$

(since $\lim_{x\to a}\left[f(x) \pm g(x)\right] = \lim_{x\to a}\left[f(x)\right] \pm \lim_{x\to a}\left[g(x)\right]$

$$= \frac{\sqrt{3 + 2\cdot\lim_{x\to\infty}\left(\frac{1}{x}\right)}}{1 + 2\lim_{x\to\infty}\left(\frac{1}{x}\right)}$$

(since $\lim_{x\to a}\left[c\cdot f(x)\right] = c\cdot\lim_{x\to a}\left[f(x)\right]$ for any constant c.

and we use $\lim_{x\to a}(c) = c$.

$$= \frac{\sqrt{3+2\cdot 0}}{1+2\cdot 0}$$

since $\lim_{x\to\infty}\left(\frac{1}{x^c}\right) = 0$ by the above theorem. So, the answer is $\sqrt{3}$. Now, we are allowed to skip steps and just say $\lim_{x\to\infty}\left[\frac{\sqrt{3+\frac{2}{x}}}{1+\frac{2}{x}}\right] = \lim_{x\to\infty}\frac{\sqrt{3+\frac{2}{x}}}{1+\frac{2}{x}}$ so long as you know why. (ie, prop's of limits & above thm.)

(2b) $\lim\limits_{x\to\infty}\left(\dfrac{x^2-5}{\sqrt{x^4+3x}}\right)$

For all the limiting expressions in questions 2a through 2e, we deploy the same "trick", which is multiplying numerator and denom. by x raised to the highest power appearing in the denominator. In this case, we argue that in the long run (ie, for large x) $\sqrt{x^4+3x}\approx\sqrt{x^4}$, or $\sqrt{x^4+3x}\approx x^2$ since $\sqrt{x^4}=x^2$. This tells us that the highest power of x appearing in the denominator is well approximated by 2 (or x^2) for infinitely large x.

$$=\lim_{x\to\infty}\left[\frac{x^2-5}{\sqrt{x^4+3x}}\cdot\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right)\right]$$

$$=\lim_{x\to\infty}\left[\frac{\frac{x^2-5}{x^2}}{\frac{\sqrt{x^4+3x}}{x^2}}\right]$$

$$=\lim_{x\to\infty}\left[\frac{\frac{x^2}{x^2}-\frac{5}{x^2}}{\frac{\sqrt{x^4+3x}}{\sqrt{x^4}}}\right]=\lim_{x\to\infty}\left[\frac{1-\frac{5}{x^2}}{\sqrt{\frac{x^4+3x}{x^4}}}\right]=\lim_{x\to\infty}\left[\frac{1-\overset{\to 0}{\frac{5}{x^2}}}{\sqrt{1+\underset{\to 0}{\frac{3}{x^3}}}}\right]=\frac{1}{\sqrt{1}}=1$$

You dont have to show this many steps, but it doesn't hurt to write things out so you are more sure what you are writing is correct.

(2c) $\lim\limits_{x\to\infty}\left(x-\sqrt{x^2+3x-5}\right)\to(\infty-\infty)$ and we cannot conclude that $\infty-\infty=0$. So, we multiply by 1 again.

$$=\lim_{x\to\infty}\left[\frac{x-\sqrt{x^2+3x-5}}{1}\cdot\left(\frac{x+\sqrt{x^2+3x-5}}{x+\sqrt{x^2+3x-5}}\right)\right]=\lim_{x\to\infty}\left[\frac{x^2-(x^2+3x-5)}{x+\sqrt{x^2+3x-5}}\right]$$

$$=\lim_{x\to\infty}\left(\frac{x^2-x^2-3x+5}{x+\sqrt{x^2+3x-5}}\right)=\lim_{x\to\infty}\left[\frac{-3x+5}{x+\sqrt{x^2+3x-5}}\right]$$

now, this seems utterly fruitless, but for lack of a better trick to try, We again divide both numerator & denom. by x raised to the highest power appearing in the denominator. In this case that will be x, since we again argue that when $x \to \infty$, $\sqrt{x^2+3x-5} \approx \sqrt{x^2} = x$. So,

this is one!

$$\lim_{x\to\infty}\left[\frac{-3x+5}{x+\sqrt{x^2+3x-5}}\right] = \lim_{x\to\infty}\left[\frac{-3x+5}{x+\sqrt{x^2+3x-5}}\cdot\left(\frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}\right)\right]$$

$$= \lim_{x\to\infty}\left[\frac{\frac{-3x}{x}+\frac{5}{x}}{\frac{x}{x}+\frac{\sqrt{x^2+3x-5}}{x}}\right] = \lim_{x\to\infty}\left[\frac{-3+\frac{5}{x}}{1+\sqrt{\frac{x^2+3x-5}{x^2}}}\right]$$

$$= \lim_{x\to\infty}\left[\frac{-3+\frac{5}{x}}{1+\sqrt{1+\frac{3}{x^2}-\frac{5}{x^3}}}\right] = \frac{-3+0}{1+\sqrt{1+0-0}} = \frac{-3}{1+\sqrt{1}}$$

$= \frac{-3}{2}$ we skipped a bunch of steps using those limit properties and the theorem from section 7.

(2d) $\lim_{x\to -\infty}\left[\frac{x^3+5x-4}{4x^3+2x^2+x-6}\right] \to \frac{\infty}{\infty}$, which is indeterminate form.

$$= \lim_{x\to -\infty}\left[\frac{\frac{x^3+5x-4}{x^3}}{\frac{4x^3+2x^2+x-6}{x^3}}\right] = \lim_{x\to -\infty}\left[\frac{1+\cancel{\frac{5}{x^2}}-\cancel{\frac{4}{x^3}}}{4+\frac{2}{x}+\frac{1}{x^2}-\frac{6}{x^3}}\right] = \frac{1}{4}$$

by prop's of limits and since $\lim_{x\to a}\left(\frac{1}{x^c}\right) = 0$.

(2e) $$\lim_{x\to\infty}\left[\left(\frac{x^2+4}{x^3+3x^2-7}\right)\cdot\left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right)\right] = \lim_{x\to\infty}\left[\frac{\frac{x^2+4}{x^3}}{\frac{x^3+3x^2-7}{x^3}}\right] = \lim_{x\to\infty}\left[\frac{\frac{1}{x}+\frac{4}{x^3}}{1+\frac{3}{x}-\frac{7}{x^3}}\right]$$

$= \frac{0}{1} = 0$ by prop's of limits (see 2a solns) and b.c. $\lim_{x\to a}\frac{1}{x^c}=0$

(3a) $$\lim_{x\to 0}\left[\frac{\frac{1}{x+3}-\frac{1}{3}}{x}\right] = \lim_{x\to 0}\left[\frac{1}{x}\left(\frac{1}{x+3}-\frac{1}{3}\right)\right]$$

$$= \lim_{x\to 0}\left[\frac{1}{x}\left[\frac{1}{x+3}\cdot\left(\frac{3}{3}\right) - \frac{1}{3}\cdot\left(\frac{x+3}{x+3}\right)\right]\right] = \lim_{x\to 0}\left[\frac{1}{x}\left(\frac{3-(x+3)}{3(x+3)}\right)\right]$$

$$= \lim_{x\to 0}\left[\frac{1}{x}\left(\frac{3-x-3}{3(x+3)}\right)\right] = \lim_{x\to 0}\left[\frac{-x}{x(3\cdot(x+3))}\right] = \lim_{x\to 0}\left[\frac{-1}{3(x+3)}\right]$$

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$$= \lim_{x\to 0}\left(\frac{-1}{3x+9}\right) = \frac{-1}{3\cdot 0+9} = \frac{-1}{0+9} = -\frac{1}{9}.$$

3b $\lim_{h\to 0}\left[\frac{\sqrt{2+h}-\sqrt{2}}{h}\right] \to \frac{0}{0}$, which is indeterminate form.

common trick

$$= \lim_{h\to 0}\left[\frac{\sqrt{2+h}-\sqrt{2}}{h}\cdot\left(\frac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}}\right)\right]$$

$$= \lim_{h\to 0}\left[\frac{(2+h)-(2)}{h\{\sqrt{2+h}+\sqrt{2}}\right] = \lim_{h\to 0}\left(\frac{h}{h[\sqrt{2+h}+\sqrt{2}]}\right) = \lim_{h\to 0}\frac{1}{\sqrt{2+h}+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2+0}+\sqrt{2}} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}}\,\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

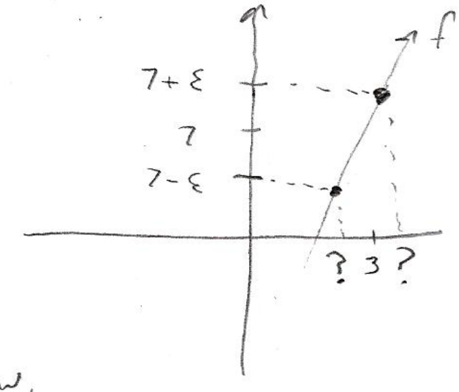
3c

$$\lim_{t\to 0}\left[\frac{1}{t}-\frac{1}{t^2+t}\right] = \lim_{t\to 0}\left[\frac{1}{t}-\frac{1}{t(t+1)}\right]$$

$$= \lim_{t\to 0}\left[\frac{1}{t}\cdot\left(\frac{t+1}{t+1}\right)-\frac{1}{t(t+1)}\right] = \lim_{t\to 0}\left[\frac{t+1-1}{t(t+1)}\right] = \lim_{t\to 0}\left[\frac{t}{t(t+1)}\right]$$

$$= \lim_{t\to 0}\left(\frac{1}{t+1}\right) = \frac{1}{0+1} = 1$$

(4) Prove $\lim_{x \to 3} (3x-2) = 7$



Proof Suppose $\varepsilon > 0$, $\delta = \frac{\varepsilon}{3}$ and that $|x-3| < \delta$. Notice that

$$|f(x) - 7| = |(3x-2)-7| = |3x-9| = 3|x-3|.$$

We n.t.s. that $\left[\,|x-3| < \delta\,\right] \Rightarrow \left[\,|f(x)-7| < \varepsilon\,\right]$.

But, $\left[\,|x-3|<\delta\,\right] \Leftrightarrow \left[\,|x-3| < \frac{\varepsilon}{3}\,\right] \Leftrightarrow \left[\,3|x-3| < \varepsilon\,\right] \Leftrightarrow \left[\,|f(x)-7| < \varepsilon\,\right]$

Q.E.D.

Now,

$$\left[7+\varepsilon = 3x-2\right] \Rightarrow \left[x = 3 + \frac{\varepsilon}{3}\right]$$

and

$$\left[7-\varepsilon = 3x-2\right] \Rightarrow \left[x = 3 - \frac{\varepsilon}{3}\right].$$

This $\Rightarrow$ we should take $\delta = \frac{\varepsilon}{3}$ in our limit proof.

(5) $$\frac{f(b)-f(a)}{b-a} = \frac{f(3)-f(2)}{3-2} = \frac{(3^3 - 2\cdot 3) - (2^3 - 2\cdot 2)}{1} = 27-6-4 = 17.$$

(6) $$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h \to 0}\left[\frac{\left[(3+h)^2 - (3+h)\right] - \left[3^2 - 3\right]}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\left[\cancel{9} + 6h + h^2 \cancel{-3} - h \cancel{-6}\right]}{h}\right] = \lim_{h\to 0}\left[\frac{5h+h^2}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{h\cdot(5+h)}{h}\right] = \lim_{h\to 0}(5+h) = 5$$

(7) To show that a function is continuous at $x=a$, we n.t.s. that

1) $x=a$ is in the dom(f)

2) $\lim_{x\to a^-}[f(x)] = \lim_{x\to a^+}[f(x)]$ (i.e, that LHL = RHL)

3) $\lim_{x\to a} f(x) = f(a)$

But, we only n.t.s. that one of these conditions is violated to show that f is discontinuous at $x=a$. We do the latter, in this case.

Soln: First, note that $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x\neq 3 \\ 4, & x=3 \end{cases} = \begin{cases} x+3, & x\neq 3 \\ 4, & x=3 \end{cases}$.

Observe that $LHL = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(x+3) = 6$, and that

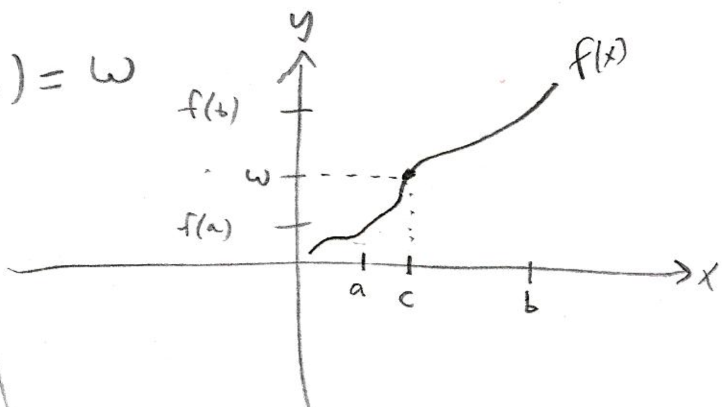
$RHL = \lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(x+3) = 6$. $\therefore$, $\lim_{x\to 3} f(x) = 3$.

But, by defn of the function, f, $f(3)=4$.

So, condition 3 is violated. That is, $\lim_{x\to 3} f(x) \neq f(3)$.

Thus, f is discontinuous at $x=3$.

8 <u>IVT</u> Suppose f is continuous on a closed ~~x~~ interval, $[a,b]$. In addn, suppose w is a number between $f(a)$ and $f(b)$.
(If All this IS TRUE) Then there exists a number $x=c$ in $[a,b]$ such that $f(c)=w$



Proof

$f(x)$ is continuous, b.c. its a polynomial function. We have to recognize that 0 is playing the role of w in the statement of the thm. We have to find numbers a and b so that $w=0$ is between $f(a)$ and $f(b)$.
Then, we can conclude $x=c$ exists and finished.
Let $a=0$ and $b=1$. Then, using the defn of f,
$f(a) = f(0) = 1$ and $f(b) = f(1) = -1$. Therefore,

[It is true that $f(x)$ is continuous on the x interval $[0,1]$ and $f(b) \le 0 \le f(a)$.] $\xRightarrow{\text{By the IVT}}$ [There exists ($\exists$) a number $x=c$ in $[0,1]$ Such that (s.t.) $f(c)=0$].

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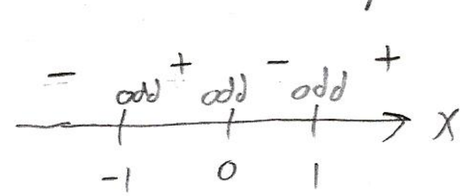
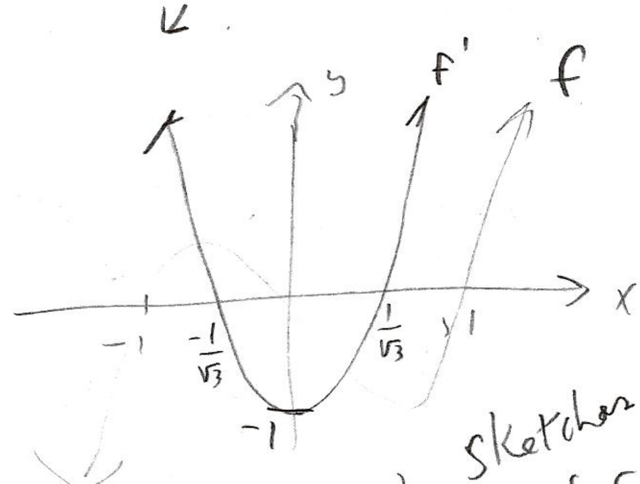
$f(x) = x^3 - x = x(x-1)(x+1)$



x-ints $x = 0, 1, -1$

y-int $x = 0$



$f'(x) = 3x^2 - 1$

x-ints $x = \pm\frac{1}{\sqrt{3}}$

Rough sketches will do for now, but they must look sane to get full credit.

$$f'(x) = \lim_{h\to 0}\left[\frac{f(x+h) - f(x)}{h}\right] = \lim_{h\to 0}\left[\frac{[(x+h)^3 - (x+h)] - [x^3 - x]}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{1}{h}\left[(x^3 + 3x^2h + 3xh^2 + h^3) - (x+h) - [x^3 - x]\right]\right]$$

$$= \lim_{h\to 0}\left[\frac{1}{h}\left(\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x} - h - \cancel{x^3} + \cancel{x}\right)\right]$$

$$= \lim_{h\to 0}\left(\frac{1}{h}\left(3x^2h + 3xh^2 + h^3 - h\right)\right) = \lim_{h\to 0}\frac{h}{h}\cdot(3x^2 + 3xh + h^2 - 1)$$

$$= \lim_{h\to 0}(3x^2 + 3xh + h^2 - 1) = 3x^2 - 1$$

(10) Use $y - y_1 = m(x - x_1)$ with $m = f'(-1)$ and

$(x_1, y_1) = (-1, f(-1))$. Use $f(x) = 3x^2 + 2$

$$f'(-1) = \lim_{h \to 0} \left[\frac{f(-1+h) - f(-1)}{h} \right] = \lim_{h \to 0} \left[\frac{1}{h} \cdot \left[\left(3(-1+h)^2 + 2\right) - \left(3 \cdot (-1)^2 + 2\right) \right] \right]$$

$$= \lim_{h \to 0} \left[\frac{1}{h} \cdot \left(3(1 - 2h + h^2) + 2 - 5 \right) \right] = \lim_{h \to 0} \left[\frac{1}{h} \cdot \left(3 + 6h + 3h^2 - 3 \right) \right]$$

$$= \lim_{h \to 0} \left[\frac{1}{h} \cdot (6h + 3h^2) \right] = \lim_{h \to 0} \left[\frac{h}{h} (6 + 3h) \right]$$

$$= \lim_{h \to 0} (6 + 3h) = \lim_{h \to 0} (6) + 3 \lim_{h \to 0} (h) = 6 + 3 \cdot 0 = 6$$

you can also just find $f'(x)$ then evaluate it for $x = -1$ instead of doing the above.

So, we need the eqn of the line tangent to f at $(x, y) = (-1, f(-1)) = (-1, 5)$, which has slope $m = f(-1) = 6$.

We use $y - y_1 = m(x - x_1)$, or that

$y - 5 = 6(x - (-1))$, or $y = 6x + 6 + 5$,

or $y = 6x + 11$