

① removable, $x = 2\pi$

$$\textcircled{2} \quad \frac{dy}{dx} = 3 \frac{d}{dx}(e^{2x}) + 5 \frac{d}{dx}(x^{-4}) - \frac{d}{dx}(x^{3/5})$$

$$= 3 \cdot e^{2x} \cdot 2 + 5 \cdot (-4) x^{-5} - \frac{3}{5} x^{\frac{3}{5} - \frac{5}{5}}$$

$$= 6e^{2x} - 20x^{-5} - \frac{3}{5} x^{-2/5}$$

$$\textcircled{3} \quad \ln y = \ln[(1+4x)^5(3+x-x^2)^8]$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx} \left[5 \ln(1+4x) + 8 \ln(3+x-x^2) \right]$$

$$\frac{y'}{y} = 5 \cdot \frac{1}{1+4x} \cdot (4) + 8 \frac{1}{3+x-x^2} \cdot (1-2x)$$

$$\frac{dy}{dx} = y \left(\frac{20}{1+4x} + \frac{8(1-2x)}{3+x-x^2} \right)$$

$$= (1+4x)^5 (3+x-x^2)^8 \left[\frac{20}{1+4x} + \frac{8(1-2x)}{3+x-x^2} \right]$$

$$= 4(1+4x)^4 (3+x-x^2)^7 \left[5(3+x-x^2) + 2(1-2x)(1+4x) \right]$$

$$= 4(1+4x)^4 (3+x-x^2)^7 \left[15 + 5x - 5x^2 + 2[1 + 2x - 8x^2] \right]$$

$$= 4(1+4x)^4 (3+x-x^2)^7 \left[15 + 5x - 5x^2 + 2 + 4x - 16x^2 \right]$$

$$= 4(1+4x)^4 (3+x-x^2)^7 (-21x^2 + 9x + 17)$$

$$= -4(1+4x)^4 (3+x-x^2)^7 (21x^2 - 9x - 17)$$

3

$$\frac{dy}{dx} = \left[\frac{d}{dx} (1+4x)^5 \right] \cdot (3+x-x^2)^8 + (1+4x)^5 \frac{d}{dx} (3+x-x^2)^8$$

$$= \left[5(1+4x)^4 \cdot \frac{d}{dx} (1+4x) \right] (3+x-x^2)^8 + (1+4x)^5 \cdot 8(3+x-x^2)^7 \cdot \frac{d}{dx} (3+x-x^2)$$

$$= 5(1+4x)^4 \cdot 4(3+x-x^2)^8 + 8(1+4x)^5 (3+x-x^2)^7 (1-2x)$$

$$= 4(1+4x)^4 (3+x-x^2)^7 \left[5(3+x-x^2) + 2(1+4x)(1-2x) \right]$$

$$= -4(1+4x)^4 (3+x-x^2)^7 (21x^2 - 9x - 17)$$

4

$$\frac{dy}{dx} = \frac{d}{dx} (\cos(\tan x)) = -\sin(\tan x) \cdot \frac{d}{dx} (\tan x)$$

$$= -\sin(\tan x) \sec^2(x)$$

5

$$\frac{dy}{dx} = \frac{(2x-1)^{1/2} \cdot 3 - (3x-2) \cdot \frac{d}{dx} (2x-1)^{1/2}}{2x-1}$$

$$= \frac{3(2x-1)^{1/2} - (3x-2) \cdot \frac{1}{2} (2x-1)^{-1/2} \cdot \frac{d}{dx} (2x-1)}{2x-1}$$

$$= \frac{3(2x-1)^{1/2} - (3x-2)(2x-1)^{-1/2}}{2x-1}$$

$$= \frac{(2x-1)^{-1/2} [3(2x-1) - (3x-2)]}{2x-1} = \frac{6x-3-3x+2}{(2x-1)^{3/2}} = \frac{3x-1}{(2x-1)^{3/2}}$$

$$\begin{aligned} \textcircled{6} \quad \frac{dy}{dx} &= \frac{d}{dx} (10^{1-x^2}) = 10^{1-x^2} \cdot \ln 10 \cdot \frac{d}{dx} (1-x^2) \\ &= -2x (\ln 10) \cdot 10^{1-x^2} \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad \frac{dy}{dx} &= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right) \cdot \frac{d}{dx} \left(\frac{1-e^{2x}}{1+e^{2x}}\right) \\ &= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right) \cdot \left[\frac{(1+e^{2x})(-2e^{2x}) - (1-e^{2x})(2e^{2x})}{(1+e^{2x})^2} \right] \\ &= 2e^{2x} \sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right) \left[\frac{1+e^{2x} + 1-e^{2x}}{(1+e^{2x})^2} \right] \\ &= \frac{4e^{2x} \sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)}{(1+e^{2x})^2} \end{aligned}$$

$$\begin{aligned} \textcircled{8} \quad \frac{dy}{dx} &= \frac{1}{\csc(5x)} \cdot \frac{d}{dx} (\csc(5x)) = \frac{1}{\csc(5x)} \cdot -\csc(5x) \cot(5x) \cdot \frac{d}{dx} (5x) \\ &= -5 \cot(5x) \end{aligned}$$

$$\textcircled{9} \quad \frac{d}{dx} (x^2 \cos(y)) + \frac{d}{dx} (\sin(2y)) = \frac{d}{dx} (xy)$$

$$[2x \cos y + x^2 (-\sin y) y'] + 2 \cos(2y) - y' = y + xy'$$

$$-xy' + x^2 \sin(y) y' - y' 2 \cos(2y) = y - 2x \cos y$$

$$y' \cdot (-1) \cdot (x + x^2 \sin y + 2 \cos 2y) = y - 2x \cos y$$

$$y' = \frac{2x \cos y - y}{x + x^2 \sin y + 2 \cos(2y)}$$

$$\begin{aligned} \textcircled{10} \quad \frac{dy}{dx} &= \frac{d}{dx} [2 \ln x + 3x \ln e] \\ &= 2 \cdot \frac{d}{dx} (\ln x) + \frac{d}{dx} (3x) = \frac{2}{x} + 3 \end{aligned}$$

$$\begin{aligned} \textcircled{11} \quad y &= \left(\tan^{-1}(x^2) \right)^{1/2} \\ \frac{dy}{dx} &= \frac{1}{2} \left(\tan^{-1}(x^2) \right)^{-1/2} \cdot \frac{d}{dx} \left(\tan^{-1}(x^2) \right) \\ &= \frac{1}{2} \left(\tan^{-1}(x^2) \right)^{-1/2} \cdot \frac{1}{1+(x^2)^2} \cdot \frac{d}{dx} (x^2) \\ &= \frac{1}{2} \left(\tan^{-1}(x^2) \right)^{-1/2} \cdot \frac{1}{1+x^4} \cdot 2x \\ &= \frac{x}{(1+x^4) \sqrt{\tan^{-1}(x^2)}} \end{aligned}$$

$$\begin{aligned} \textcircled{12} \quad \frac{dy}{dx} &= \frac{d}{dx} (3 \ln(2x+1) - 4 \ln(3x-1)) \\ &= 3 \cdot \frac{1}{2x+1} \cdot 2 - 4 \cdot \frac{1}{3x-1} \cdot 3 \\ &= 6 \left(\frac{1}{2x+1} - \frac{2}{3x-1} \right) \\ &= 6 \left(\frac{3x-1 - 2(2x+1)}{(2x+1)(3x-1)} \right) = \frac{6(3x-1-4x-2)}{(2x+1)(3x-1)} = \frac{6(-x-3)}{(2x+1)(3x-1)} = \boxed{\frac{-6(x+3)}{(2x+1)(3x-1)}} \end{aligned}$$

$$(13) \ln y = \ln e^{x^2} + \ln x^{1/2} + 10 \ln(x^2+1)$$

$$\frac{y'}{y} = \frac{d}{dx}(x^2) + \frac{1}{2} \frac{d}{dx}(\ln x) + 10 \frac{d}{dx} \ln(x^2+1)$$

$$\frac{dy}{dx} = y \left(2x + \frac{1}{2x} + 10 \cdot \frac{1}{x^2+1} \cdot 2x \right)$$

$$= e^{x^2} \sqrt{x} (x^2+1)^{10} \left(2x + \frac{1}{2x} + \frac{20x}{x^2+1} \right)$$

$$(14) \ln y = \cos(x) \ln(\ln x)$$

$$\frac{y'}{y} = \frac{d}{dx}(\cos(x) \ln(\ln x))$$

$$\frac{1}{y} \frac{dy}{dx} = -\sin(x) \ln(\ln x) + \cos x \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = (\ln x)^{\cos(x)} \left(\frac{\cos x}{x \ln x} - \sin(x) \ln(\ln x) \right)$$

$$(15) \quad f'(x) = 2ax + b$$

$$[f'(-1) = 6] \Rightarrow [-2a + b = 6]$$

$$[f'(5) = -2] \Rightarrow [10a + b = -2]$$

$$[f(1) = 4] \Rightarrow [4 = a + b + c]$$

$$\left. \begin{array}{l} [f'(-1) = 6] \Rightarrow [-2a + b = 6] \\ [f'(5) = -2] \Rightarrow [10a + b = -2] \\ [f(1) = 4] \Rightarrow [4 = a + b + c] \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} a + b + c = 4 \\ -2a + b = 6 \\ 10a + b = -2 \end{array} \right\}$$

$$\Leftrightarrow \left\{ \begin{bmatrix} 1 & 1 & 1 \\ -2 & 1 & 0 \\ 10 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -2 \end{bmatrix} \right\} \Leftrightarrow \left[(a, b, c) = \left(-\frac{2}{3}, \frac{4}{3}, 0 \right) \right]$$

$$\Rightarrow \left[f(x) = -\frac{2}{3}x^2 + \frac{14}{3}x \right]$$

$$(16) \quad \frac{dK}{dt} = \frac{1}{2} m \frac{d}{dt} v^2$$

$$= \frac{1}{2} m 2v \frac{dv}{dt}$$

$$\frac{dK}{dt} = mv \frac{dv}{dt}$$