**Math 150 – Chapter 4**
**Exam 3 Review Sheet**
**Professor Busken**

Name: Key

**Directions: Work Together! Do not try to cram the full solutions to each question onto this paper!!! Write your extended solutions on different paper! Working only these practice problems is insufficient preparation for the exam. You will need to bring a scientific calculator with you to class on each exam day.**

**For questions 1—4, find the local and absolute extrema of $f$ on the given interval.**

1. $f(x) = x^3 - 6x^2 + 9x + 1, \quad [2, 4]$
2. $f(x) = x\sqrt{1-x}, \quad [-1, 1]$
3. $f(x) = x + \sin(2x), \quad [0, \pi]$
4. $f(x) = \dfrac{\ln(x)}{x^2}, \quad [1, 3]$

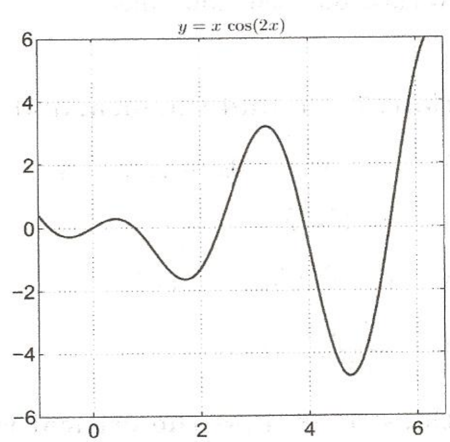
**For questions 5—8, find the critical numbers of $f$.**

5. $f(x) = (x+4)^7(3x-2)^5$
6. $f(x) = (2x-5)\sqrt{x^2-4}$
7. $f(x) = |x^2 - 5|$
8. $f(x) = x + \ln(x^2 - 1)$
9. If $f(x) = x^3 + x^2 + x + 1$, find a number, $c$, that satisfies the conclusion of the Mean Value Theorem (MVT) on the interval $[0, 4]$.
10. A person in a rowboat 2 miles from the nearest point on a straight shoreline wishes to reach a house 6 miles farther down the shore. If a person can row at a rate of 3 mi/hr and walk at a rate of 5 mi/hr, find the least amount of time required to reach the house.
11. Find the maximum volume of a right circular cylinder that can be inscribed in a cone of altitude 12 centimeters, and base radius 4 centimeters, if the axes of the cylinder and cone coincide.
12. A window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 15 feet, find the dimensions that will allow the maximum amount of light to enter.
13. A builder intends to construct a storage shed having a volume of 900 ft$^3$, a flat roof, and a rectangular base whose width is three-fourths the length. The cost per square foot of the materials is \$4 for the floor, \$6 for the sides, and \$3 for the roof. What dimensions will minimize the cost?
14. The gravitational constant for objects near the surface of the moon is 5.3 ft/sec$^2$. If an astronaut on the moon throws a stone directly upward with an initial velocity of 60 ft/sec, find its maximum altitude.

15. Find $y$ if $\dfrac{d^2y}{dx^2} = 3\sin(x) - 4\cos(x)$ and $y = 7$ and $y' = 2$ if $x = 0$.

16. Use Newton's Root Finding Algorithm to approximate $\sqrt[5]{7^4}$ to four decimal places.

17. A dramatic example of the phenomenon of resonance occurs when a singer adjusts the pitch of her voice to shatter a wine glass. Functions given by $f(x) = ax\cos(bx)$ occur in the mathematical analysis of such vibrations.Shown in the figure is a graph of $f(x) = x\cos(2x)$. Use Newton's method to approximate, to three decimal places, the critical number of $f$ that lies between 1 and 2.




18. Sketch the graph of a function that has the following properties:

$f(0) = 0, \qquad f'(-2) = f'(1) = f'(9) = 0,$

$\lim\limits_{x\to\infty} f(x) = 0, \qquad \lim\limits_{x\to 6} f(x) = -\infty$

$f'(x) < 0 \quad \forall\, x \in (-\infty, -2) \cup (1, 6) \cup (9, \infty)$

$f'(x) > 0 \quad \forall\, x \in (-2, 1) \cup (6, 9)$

$f''(x) > 0 \quad \forall\, x \in (-\infty, 0) \cup (12, \infty)$

$f''(x) < 0 \quad \forall\, x \in (0, 6) \cup (6, 12)$

19. Find $f(x)$ if $f'(x) = \dfrac{4}{\sqrt{1-x^2}}$ and $f\left(\dfrac{1}{2}\right) = 1$

20. Find $f(x)$ if $f'(x) = \dfrac{8x-5}{\sqrt[3]{x}}$

21. Find $f(x)$ if $f'(x) = \dfrac{\sec(x)\sin(x)}{\cos(x)}$

22. Factor $f(x) = x^6 - 13x^5 + 67x^4 - 175x^3 + 244x^2 - 172x + 48$

① $f(x) = x^3 - 6x^2 + 9x + 1$ ; $[2,4]$

$$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-3)(x-1)$$

and $f'(x) = 0$ when $x = 1, 3$

$$\begin{array}{ccccccc} + & & \text{odd} & - & \text{odd} & & + \\ \nearrow & & & \searrow & & & \nearrow \\ \hline & 1 & & & & 3 & \to x, f'(x) \\ & \text{max} & & & & \text{min} & \end{array}$$

Now $(1, f(1))$ is a max, but

$x = 1$ is not in $[2,4]$

Absolute Max

$(x\ y) = (4, 5)$

Absolute Min

$(x, y) = (3, 1)$

Endpoint

$f(2) = 3$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 9 & 1 \\ & \downarrow & 2 & -8 & 2 \\ \hline & 1 & -4 & 1 & 3 \end{array}$$

$f(3) = 1$

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 9 & 1 \\ & \downarrow & 3 & -9 & 0 \\ \hline & 1 & -3 & 0 & 1 \end{array}$$

Endpoint

$f(4) = 5$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 9 & 1 \\ & \downarrow & 4 & -8 & 4 \\ \hline & 1 & -2 & 1 & 5 \end{array}$$

Remember, we can quickly compute functional values of polynomials with synthetic division. This is usually quicker than punching a calculator!

(2) $f(x) = x\sqrt{1-x} = x(1-x)^{1/2}$ ; $[-1, 1]$

$$f'(x) = 1\,(1-x)^{1/2} + x \cdot \tfrac{1}{2}(1-x)^{-1/2} \cdot (-1)$$

$$= (1-x)^{1/2} - \frac{x}{2}(1-x)^{-1/2} = \tfrac{1}{2}(1-x)^{-1/2}\left[2(1-x) - x\right] = \frac{2-3x}{2\sqrt{1-x}}$$

CN's satisfy $2-3x=0$ or $2\sqrt{1-x}=0$, or $x = \frac{2}{3}, 1$

$\nearrow$ $\searrow$

$+$ odd $-$ not in dom(f) $\rightarrow$ $x, f'(x)$

$\frac{2}{3}$ $\quad$ $1$

max

$f(-1) = -\sqrt{2}$

$f(1) = 0$

$f\left(\frac{2}{3}\right) = \frac{2}{3}\sqrt{\frac{1}{3}}$

absolute max

$(x,y) = \left(\frac{2}{3}, \frac{2}{3}\sqrt{\frac{1}{3}}\right)$

absolute min

$(x,y) = (-1, -\sqrt{2})$

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(3) $f(x) = x + \sin(2x)$ ; $[0, \pi]$

$\frac{2\pi}{3}$ $\quad$ $\frac{4\pi}{3}$

$f'(x) = 1 + 2\cos(2x) = 0$ when $\cos(2x) = -\frac{1}{2}$

Let $\theta = 2x$. Then $\cos(\theta) = -\frac{1}{2}$ when $\theta = \frac{2\pi}{3} + k2\pi$ or $\theta = \frac{4\pi}{3} + k2\pi$

So, $2x = \frac{2\pi}{3} + 2\pi k$, or $2x = \frac{4\pi}{3} + 2\pi k$, or when

$x = \frac{\pi}{3} + \pi k$ or $x = \frac{2\pi}{3} + \pi k$

$\nearrow$ $\searrow$ $\nearrow$

$+$ $-$ $+$ $\rightarrow$ $x, f'(x)$

$\frac{\pi}{3}$ $\quad$ $\frac{2\pi}{3}$

max $\quad$ min

$f(0) = 0$

$f(\pi) = \pi$

$f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} + \sin\left(\frac{2\pi}{3}\right) = \frac{\pi}{3} + \frac{\sqrt{3}}{2} \approx 1.91$

$f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} + \sin\left(\frac{4\pi}{3}\right) = \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \approx 1.22$

| | |
|---|---|
| Abs max | $(\pi, \pi)$ |
| Abs min | $(0, 0)$ |
| Local max | $\left(\frac{\pi}{3}, \frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$ |
| Local min | $\left(\frac{2\pi}{3}, \frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ |

④ $f(x) = x^{-2}\ln x$ ; $[1,3]$

$$f'(x) = -2x^{-3}\ln x + x^{-2}\cdot\frac{1}{x} = -2x^{-3}\ln x + x^{-3} = x^{-3}(-2\ln x + 1)$$

so $f'(x) = \dfrac{1-2\ln x}{x^3}$, the only CN of f satisfies $1-2\ln x = 0$, or

$\ln x = \frac{1}{2}$ or $x = e^{1/2} \approx 1.64.$ $\quad \overset{\nearrow\ \ \searrow}{\underset{e^{1/2}}{+\ \big|\ -}} \to x, f'(x)$

$f(1) = \dfrac{\ln 1}{1^2} = 0$

$f(e^{1/2}) = (e^{1/2})^{-2}\ln e^{1/2} = e^{-1}\cdot\frac{1}{2}\ln e = \dfrac{1}{2e} \approx 0.18$

$f(3) = \dfrac{\ln 3}{9} \approx 0.122$

abs max
$(x,y) = \left(e^{1/2}, \frac{1}{2e}\right)$

abs min
$(1,0)$

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⑤ $f(x) = (x+4)^3(3x-2)^4$;

$$f'(x) = 3(x+4)^2(3x-2)^4 + (x+4)^3\cdot 4(3x-2)^3\cdot 3$$

$$= 3(x+4)^2(3x-2)^3\left[(3x-2) + 4(x+4)\right] = 21(x+4)(3x-2)^3(x+2)$$

$\Rightarrow$ $x = -4, \frac{2}{3}, -2$ are CN's of f.

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⑥ $f(x) = (2x-5)(x^2-4)^{1/2}$ and $\mathrm{dom}(f) \equiv (-\infty,-2] \cup [2,\infty)$

$$f'(x) = 2(x^2-4)^{1/2} + (2x-5)\tfrac{1}{2}(x^2-4)^{-1/2}\cdot 2x$$

$$= (x^2-4)^{-1/2}\left[2(x^2-4) + x(2x-5)\right]$$

$$= (x^2-4)^{-1/2}\left[4x^2-5x-8\right]$$

$$= \frac{4x^2-5x-8}{\sqrt{x^2-4}}$$

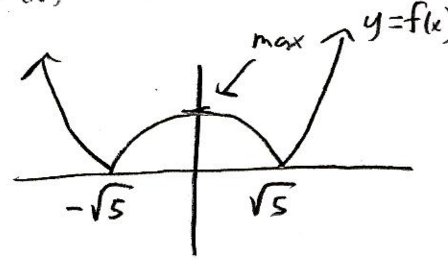
CN's of satisfy $4x^2-5x-8=0$ or $\sqrt{x^2-4}=0$, so long as they are in dom(f). But $[4x^2-5x-8=0] \Leftrightarrow \left[x = \frac{5\pm\sqrt{153}}{8} \approx \{-0.93, 2.17\}\right]$, and $\left[\sqrt{x^2-4}=0\right]$ when $x=\pm 2$. But $x = \frac{5-\sqrt{153}}{8} \approx -0.93$ is not in dom f, so the CN's of f(x) are $\boxed{\pm 2, \frac{5+\sqrt{153}}{8}}$

(7) $f(x) = |x^2-5| = \begin{cases} x^2-5, & \text{if } x \in (-\infty,-\sqrt{5}] \cup [\sqrt{5},\infty) \\ -(x^2-5), & \text{if } x \in (-\sqrt{5},\sqrt{5}) \end{cases}$

$f'(x) = \begin{cases} 2x, & \text{if } x \in (-\infty,-\sqrt{5}) \cup (\sqrt{5},\infty) \\ -2x, & \text{if } x \in (-\sqrt{5},\sqrt{5}) \end{cases}$

So, the CN's of f are $x=0$, and $x = \pm\sqrt{5}$, since $f'$ DNE at $x=\pm\sqrt{5}$, i.e., it's left hand derivative $\neq$ it's right-hand derivative. Note that $f(x)$ has corners at

$x = \pm\sqrt{5}$ since $\lim\limits_{x\to-\sqrt{5}^-} f'(x) = -2\sqrt{5}$ and $\lim\limits_{x\to-\sqrt{5}^+} f'(x) = 2\sqrt{5}$ and




Since $\lim\limits_{x\to\sqrt{5}^-} f'(x) = -2\sqrt{5}$ and $\lim\limits_{x\to\sqrt{5}^+} f'(x) = 2\sqrt{5}$.

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(8) $f(x) = x + \ln(x^2-1)$ and $\text{dom}(f) \equiv (-\infty,-1) \cup (1,\infty)$

$$f'(x) = 1 + \frac{1}{x^2-1} \cdot 2x = 1 + \frac{2x}{x^2-1} = \frac{x^2-1+2x}{x^2-1} = \frac{x^2+2x-1}{x^2-1}$$

and CN's of f satisfy $x^2+2x-1=0$ and $x^2-1=0$, so long as they are values in dom(f). But $\left[x^2+2x-1=0\right] \Leftrightarrow \left[x = \frac{-2\pm\sqrt{4-4(1)(-1)}}{2} = \frac{-2\pm2\sqrt{2}}{2}\right]$

$\Leftrightarrow \left[x = -1 \pm \sqrt{2} \approx -1 \pm 1.4 \equiv \{0.4, -2.8\}\right]$.

So, the only CN of f is $x = -1-\sqrt{2} \approx -2.8$, since

$x = -1+\sqrt{2} \approx 0.4$ and $x = \pm 1$ are not in ($\notin$) dom(f).

(9) MVT If $f$ is cont. for $x \in [a,b]$ and differentiable for $x \in (a,b)$, then $\exists$ a number $x=c$ in $(a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$f(x) = x^3 + x^2 + x + 1$ is cont on $[0,4]$ & differentiable on $(0,4)$ since polynomials are cont. & diff'ble on $\mathbb{R}$. So there exist a number $x=c$ in $(0,4)$ that satisfies $f'(c) = \frac{f(4)-f(0)}{4-0}$.

Now, $f'(x) = 3x^2 + 2x + 1$ and $f'(c) = 3c^2 + 2c + 1$, so $f'(c) = \frac{f(4)-f(0)}{4-0}$ is equivalent to $3c^2 + 2c + 1 = \frac{85-1}{4-0}$, or $3c^2 + 2c + 1 = \frac{84}{4}$,

or $3c^2 + 2c + 1 = 21$, or $3c^2 + 2c - 20 = 0$

$$c = \frac{-2 \pm \sqrt{2^2 - 4(3)(-20)}}{2 \cdot 3} = \frac{-2 \pm \sqrt{4+240}}{6} = \frac{-2 \pm \sqrt{244}}{6}$$

$$= \frac{-2 \pm \sqrt{4 \cdot 61}}{6} = \frac{-2 \pm 2\sqrt{61}}{6} = \frac{-1 \pm \sqrt{61}}{3}$$

but $c \neq \frac{-1-\sqrt{61}}{3} \approx -2.9$. So $\boxed{c = \frac{-1+\sqrt{61}}{3}}$

*We wish to find the time $t$ at which $d$ has its smallest value.* This will occur when the expression under the radical is minimal because $d$ increases if and only if $4 - 200t + 2900t^2$ increases. Thus, we may simplify our work by letting

$$f(t) = 4 - 200t + 2900t^2$$

and finding the value of $t$ for which $f$ has a minimum. Since

$$f'(t) = -200 + 5800t,$$

the only critical number for $f$ is

$$t = \frac{200}{5800} = \frac{1}{29}.$$

Moreover, $f''(t) = 5800$, so the second derivative is always positive. Therefore, $f$ has a local minimum at $t = \frac{1}{29}$, and $f(\frac{1}{29}) = \frac{16}{29}$. Since the domain of $t$ is $[0, \infty)$ and since $f(0) = 4$, there is no endpoint extremum. Consequently, the automobiles will be closest at $\frac{1}{29}$ hour (or approximately 2.07 minutes) after 10:00 A.M. The minimal distance is

$$\sqrt{f(\tfrac{1}{29})} = \sqrt{\tfrac{16}{29}} \approx 0.74 \text{ mi.}$$

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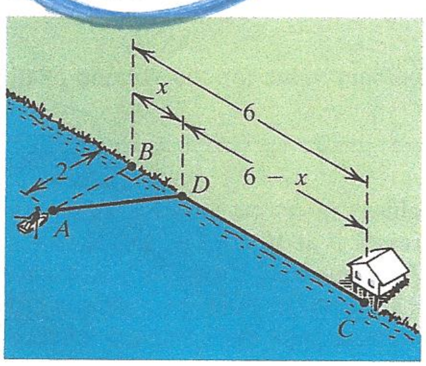
**EXAMPLE 6** A person in a rowboat 2 miles from the nearest point on a straight shoreline wishes to reach a house 6 miles farther down the shore. If the person can row at a rate of 3 mi/hr and walk at a rate of 5 mi/hr, find the least amount of time required to reach the house.

FIGURE 4.50




SOLUTION Figure 4.50 illustrates the problem: $A$ denotes the position of the boat, $B$ the nearest point on shore, $C$ the house, $D$ the point at which the boat reaches shore, and $x$ the distance between $B$ and $D$. By the Pythagorean theorem, the distance between $A$ and $D$ is $\sqrt{x^2 + 4}$, where $0 \le x \le 6$. Using the formula

$$\text{time} = \frac{\text{distance}}{\text{rate}},$$

we obtain

$$\text{time to row from } A \text{ to } D = \frac{\text{distance from } A \text{ to } D}{\text{rowing rate}} = \frac{\sqrt{x^2 + 4}}{3}$$

$$\text{time to walk from } D \text{ to } C = \frac{\text{distance from } D \text{ to } C}{\text{walking rate}} = \frac{6 - x}{5}.$$

Hence the total time $T$ for the trip is

$$T = \frac{\sqrt{x^2 + 4}}{3} + \frac{6 - x}{5},$$

or, equivalently,
$$T = \tfrac{1}{3}(x^2 + 4)^{1/2} + \tfrac{6}{5} - \tfrac{1}{5}x.$$

*We wish to find the minimum value for $T$.* Note that $x = 0$ corresponds to the extreme situation in which the person rows directly to $B$ and then

10 cont

walks the entire distance from $B$ to $C$. If $x = 6$, then the person rows directly from $A$ to $C$. These numbers may be considered as endpoints of the domain of $T$. If $x = 0$, then, from the formula for $T$,

$$T = \frac{\sqrt{4}}{3} + \frac{6}{5} - 0 = \frac{28}{15},$$

which is 1 hour 52 minutes. If $x = 6$, then

$$T = \frac{\sqrt{40}}{3} + \frac{6}{5} - \frac{6}{5} = \frac{2\sqrt{10}}{3} \approx 2.11,$$

or approximately 2 hours 7 minutes.

Differentiating the general formula for $T$, we see that

$$D_x T = \tfrac{1}{3} \cdot \tfrac{1}{2}(x^2 + 4)^{-1/2}(2x) - \tfrac{1}{5},$$

or

$$D_x T = \frac{x}{3(x^2 + 4)^{1/2}} - \frac{1}{5}.$$

In order to find the critical numbers, we let $D_x T = 0$, obtaining the following equations:

$$\frac{x}{3(x^2 + 4)^{1/2}} = \frac{1}{5}$$

$$5x = 3(x^2 + 4)^{1/2}$$

$$25x^2 = 9(x^2 + 4)$$

$$x^2 = \tfrac{36}{16}$$

$$x = \tfrac{6}{4} = \tfrac{3}{2}$$

Thus, $\frac{3}{2}$ is the only critical number. The time $T$ that corresponds to $x = \frac{3}{2}$ is

$$T = \tfrac{1}{3}(\tfrac{9}{4} + 4)^{1/2} + \tfrac{6}{5} - \tfrac{3}{10} = \tfrac{26}{15},$$

or, equivalently, 1 hour 44 minutes.

We have already examined the values of $T$ at the endpoints of the domain, obtaining 1 hour 52 minutes and approximately 2 hours 7 minutes, respectively. Hence the minimum time of 1 hour 44 minutes occurs at $x = \frac{3}{2}$. Therefore, the boat should land at $D$, $1\frac{1}{2}$ miles from $B$, in order to minimize $T$. For a similar problem, but one in which the endpoints of the domain lead to minimum time, see Exercise 6.

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EXAMPLE 7 A wire 60 inches long is to be cut into two pieces. One of the pieces will be bent into the shape of a circle and the other into the shape of an equilateral triangle. Where should the wire be cut so that the sum of the areas of the circle and triangle is minimized? maximized?

SOLUTION If $x$ denotes the length of one of the cut pieces of wire, then the length of the other piece is $60 - x$. Let the piece of length $x$ be bent to form a circle of radius $r$ so that $2\pi r = x$, or $r = x/(2\pi)$ (see Figure 4.51).

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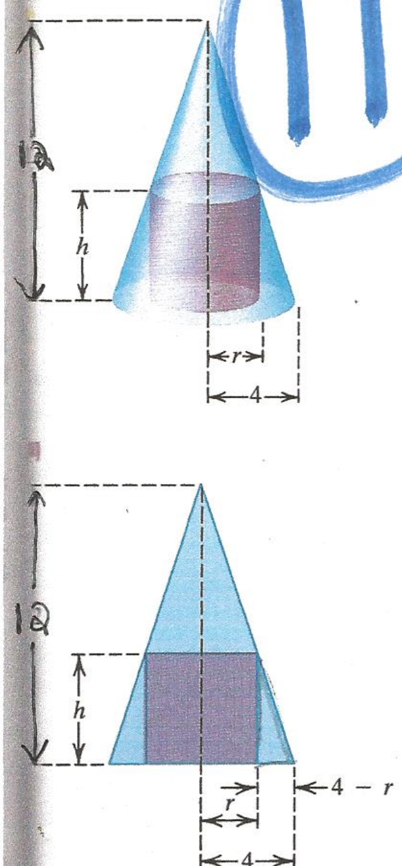
FIGURE 4.48




**EXAMPLE 4** Find the maximum volume of a right circular cylinder that can be inscribed in a cone of altitude 12 centimeters, and base radius 4 centimeters, if the axes of the cylinder and cone coincide.

SOLUTION The problem is sketched in Figure 4.48, where (ii) represents a cross section through the axes of the cone and cylinder. *The quantity we wish to maximize is the volume $V$ of the cylinder.* From geometry,

$$V = \pi r^2 h.$$

Next we express $V$ in terms of one variable by finding a relationship between $r$ and $h$. Referring to Figure 4.48(ii) and using similar triangles, we see that

$$\frac{h}{4-r} = \frac{12}{4} = 3, \quad \text{or} \quad h = 3(4-r).$$

Consequently,

$$V = \pi r^2 h = \pi r^2 \cdot 3(4-r) = 3\pi r^2(4-r).$$

The domain of $V$ is $0 \le r \le 4$.

If either $r = 0$ or $r = 4$, we see that $V = 0$, and hence the maximum volume is not an endpoint extremum. It is sufficient, therefore, to search for local maxima. Since $V = 3\pi(4r^2 - r^3)$,

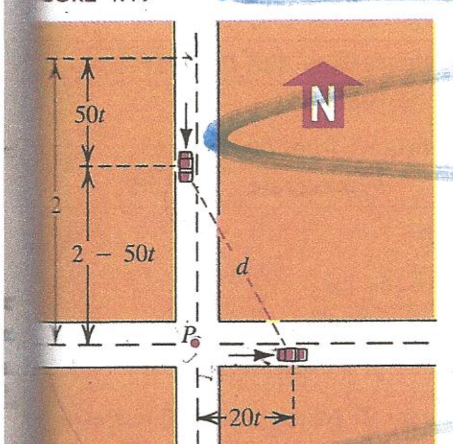
$$D_r V = 3\pi(8r - 3r^2) = 3\pi r(8 - 3r).$$

Thus, the critical numbers for $V$ are $r = 0$ and $r = \frac{8}{3}$. At $r = \frac{8}{3}$, we have

$$V = \pi\left(\frac{8}{3}\right)^2(4) = \frac{256\pi}{9} \approx 89.4 \text{ cm}^3,$$

which, by Guidelines (4.9), is a maximum value for the volume of the inscribed cylinder.

FIGURE 4.49

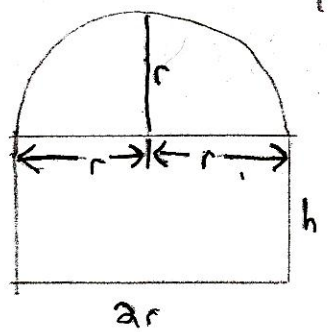



**EXAMPLE 5** A North-South highway intersects an East-West highway at a point $P$. An automobile crosses $P$ at 10:00 A.M., traveling east at a constant speed of 20 mi/hr. At that same instant another automobile is 2 miles north of $P$, traveling south at 50 mi/hr. Find the time at which they are closest to each other and approximate the minimum distance between the automobiles.

SOLUTION Typical positions of the automobiles are illustrated in Figure 4.49. If $t$ denotes the number of hours after 10:00 A.M., then the slower automobile is $20t$ miles east of $P$. The faster automobile is $50t$ miles south of its position at 10:00 A.M., and hence its distance from $P$ is $2 - 50t$. By the Pythagorean theorem, the distance $d$ between the automobiles is

$$\begin{aligned} d &= \sqrt{(2-50t)^2 + (20t)^2} \\ &= \sqrt{4 - 200t + 2500t^2 + 400t^2} \\ &= \sqrt{4 - 200t + 2900t^2}. \end{aligned}$$

(12)




The question asks us to find the dimensions of r and h that will **maximize** the **surface area** of the window subject to the constraint: the perimeter of the window is to be equal to 15 Ft.

$$\text{Area} = A = \frac{1}{2}\pi r^2 + 2rh \quad (1)$$

This is the equation that must be maximized. We need the Area equation in terms of one variable. Then we can take the derivative. We will use the constraint equation, Perimeter $= P = 2h + 2r + \pi r = 15$ ft to make a substitution into the Area function.

$$2h + 2r + \pi r = 15$$

solving for h, then substituting the result into (1)

$$2h = 15 - 2r - \pi r$$

$$h = \frac{15 - 2r - \pi r}{2} = \frac{15}{2} - r - \frac{\pi}{2} r$$

$A(r)$ = Area is a function of the radius becomes:

$$A(r) = \frac{1}{2}\pi r^2 + 2r\left(\frac{15}{2} - r - \frac{\pi}{2} r\right)$$

$$A(r) = \frac{1}{2}\pi r^2 + 15r - 2r^2 - \pi r^2$$

12 cont

Now we can take the 1st derivative of the area function, set it equal to zero, then solve for the value of r at which the area function is at maximum.

$$A'(r) = \pi r + 15 - 4r - 2\pi r = -\pi r - 4r + 15 = 0$$

$$-r(\pi + 4) = -15 \Rightarrow \boxed{r = \frac{15}{\pi + 4}}$$

the corresponding value of h, when $r = \frac{15}{\pi+4}$, (using the perimeter equation)

is then:

$$2h + 2r + \pi r = 15$$

$$2h = 15 - r(2+\pi)$$

$$h = \frac{15}{2} - \frac{1}{2} r(2+\pi) \Leftrightarrow h = \frac{15}{2} - \frac{15}{2}\left(\frac{2+\pi}{\pi+4}\right)$$

or $\boxed{h = \frac{15}{2}\left(1 - \frac{2+\pi}{4+\pi}\right)}$

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$A''(r) = \pi - 4 - 2\pi < 0$, therefore since the area function is negative, it is then concave up and we do in fact have max dimensions at $r = \frac{15}{\pi+4}$, $h = \frac{15}{2}\left(1 - \frac{2+\pi}{4+\pi}\right)$

13) The question asks us to find the dimensions that minimize the cost of the building, subject to the constraint that the volume of the building is $900\ ft^3$ and $W = \frac{3}{4}L$

$$V = 900ft^3 \quad ①$$

These equations are all related.

$$V = LWH \quad ②$$

$$W = \frac{3}{4}L \quad ③$$

④ Cost = \$4(Area of the floor) + \$6(Area of the sides) + \$3(Area of Roof)

Putting equations ①, ②, ③ together gives

$$V = \frac{3}{4}L^2H = 900 \quad \text{or} \quad \frac{3}{4}L^2H = 900$$

solving for H gives: $\boxed{H = \frac{1200}{L^2}}$ This is a new

parameter equation.

(13) cont

Area of the floor $= LW$

Area of the sides $= 2LH + 2HW$

Area of the roof $= LW$


So, we have

$$\text{Cost} = \$4(LW) + \$6(2LH + 2HW) + \$3(LW)$$

$$= \$4\left[L\left(\frac{3}{4}L\right)\right] + \$6\left[2L\left(\frac{1200}{L^2}\right) + 2\left(\frac{1200}{L^2}\right)\left(\frac{3}{4}L\right)\right] + \$3\left(\frac{3}{4}L\right)$$

$$= \$3L^2 + \$6\left[2400L^{-1} + 1800L^{-1}\right] + \$3\left[\frac{3}{4}L^2\right]$$

$$= \$3L^2 + \$25{,}200L^{-1} + \$\frac{9}{4}L^2$$

$$= \$5.25L^2 + \$25{,}200L^{-1} = C(L)$$

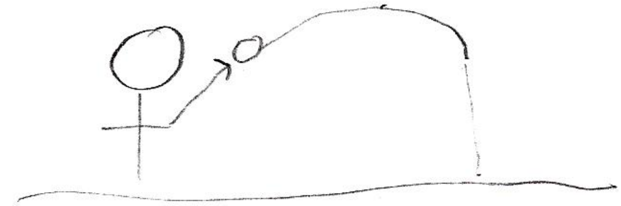
13 cont It follows that our values are the minimum values, because

$$c''(L) = 10.5 + \frac{50{,}400}{L^3} > 0$$ for our values!

Since $c''(L) > 0$, the cost function is concave up, and we have a minimum.

(14) $a(t) = -5.3 \text{ ft/sec}^2$

$v(0) = 60 \text{ ft/sec}$



$v = -5.3t + C$

$v(t) = -5.3t + 60$

$s(t) = \frac{-5.3t^2}{2} + 60t + C$

assume astronaut is 6 ft tall, then $s(0) = 6$

$s(t) = -2.65t^2 + 60t + 6$

$v(t) = 0$ when $-5.3t + 60 = 0$, or when $t = \frac{60}{5.3} \approx 11.32$

Then the maximum altitude is $s\left(\frac{60}{5.3}\right) = -2.65\left(\frac{60}{5.3}\right)^2 + 60\left(\frac{60}{5.3}\right) + 6$

$\approx \boxed{346 \text{ ft}}$

(15) $y'' = 3\sin(x) - 4\cos(x)$, $y = 7$ and $y' = 2$ if $x = 0$.

$$y' = -3\cos(x) - 4\sin(x) + C$$

$$[y'(0) = 2] \Leftrightarrow [2 = -3\cos(0) - 4\sin(0) + c] \Leftrightarrow [2 = -3 + c] \Leftrightarrow [c = 5].$$

So $y' = -3\cos(x) - 4\sin(x) + 5$ and $y = -3\sin(x) + 4\cos(x) + 5x + C$.

But, $[y(0) = 7] \Leftrightarrow [7 = -3\sin(0) + 4\cos(0) + c] \Leftrightarrow [7 = 4 + c] \Leftrightarrow [c = 3]$.

So $\boxed{y(x) = -3\sin(x) + 4\cos(x) + 5x + 3}$

(16) We need to approximate $x = 7^{4/5}$. But,

$[x = 7^{4/5}] \Leftrightarrow [x^{5/4} = 7] \Leftrightarrow [x^{5/4} - 7 = 0]$. this suggests we should use $f(x) = x^{5/4} - 7$ in the Newton Algorithm.

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)} = X_n - \frac{X_n^{5/4} - 7}{\frac{5}{4} X_n^{1/4}} = X_n - \frac{4X_n^{5/4} - 28}{5X_n^{1/4}} = X_n - \frac{4}{5}X_n + \frac{28}{5}X_n^{-1/4}$$

or $X_{n+1} = \frac{1}{5}X_n + \frac{28}{5}X_n^{-1/4}$, using $X_1 = 4$

$X_2 = \frac{1}{5}X_1 + \frac{28}{5}X_1^{-1/4} = \frac{1}{5}(4) + \frac{28}{5}(4)^{-1/4} \approx 4.759797464\!4$

$X_3 = \frac{1}{5}X_2 + \frac{28}{5}X_2^{-1/4} \approx 4.7432835\!6847$

$X_4 = \frac{1}{5}X_3 + \frac{28}{5}X_3^{-1/4} \approx 4.7427639804$

$X_5 = \frac{1}{5}X_4 + \frac{28}{5}X_4^{-1/4} \approx 4.74276\ldots$

(17) $f'(x) = \cos(2x) - 2x\sin(2x)$

We want to estimate where $f'(x) = 0$ for $x \in [1, 2]$.

We use

$$x_{n+1} = x_n - \frac{f'(x)}{f''(x)} = x_n - \frac{\cos(2x_n) - 2x_n\sin(2x_n)}{f''(x)}$$

Now, $f''(x) = -2\sin(2x) - 2\sin(2x) - 4x\cos(2x)$

$$= -4\sin(2x) - 4x\cos(2x).$$

So, $$x_{n+1} = x_n + \frac{\cos(2x_n) - 2x_n\sin(2x_n)}{4\sin(2x_n) + 4x_n\cos(2x_n)}$$

using $x_1 = 2$ as an initial guess for the newton method.

$x_2 = 1.712516573555387$

$x_3 = 1.712809272514480$

$x_4 = 1.712809229740065$

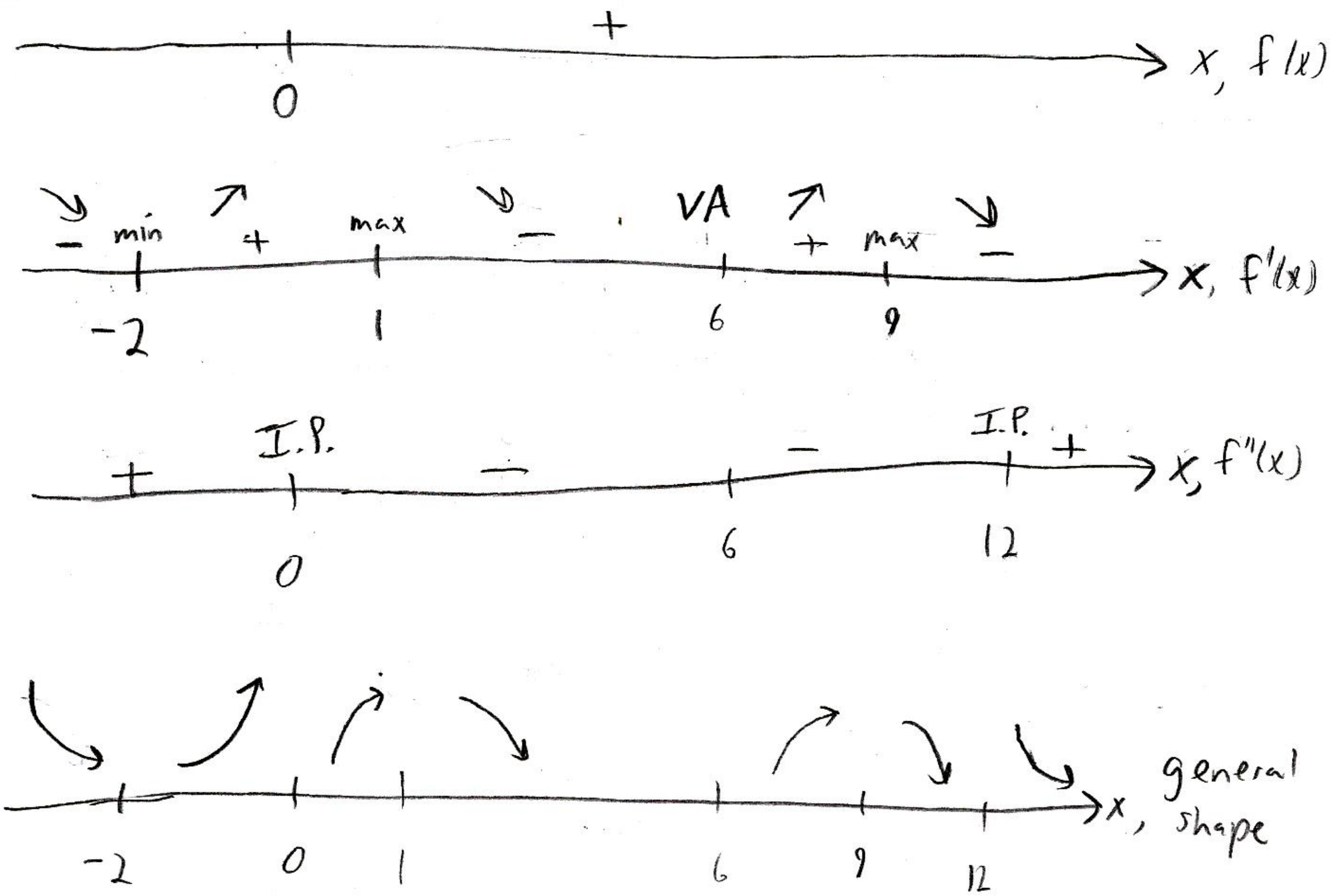
So $x \approx 1.712$ is the CN of $f$

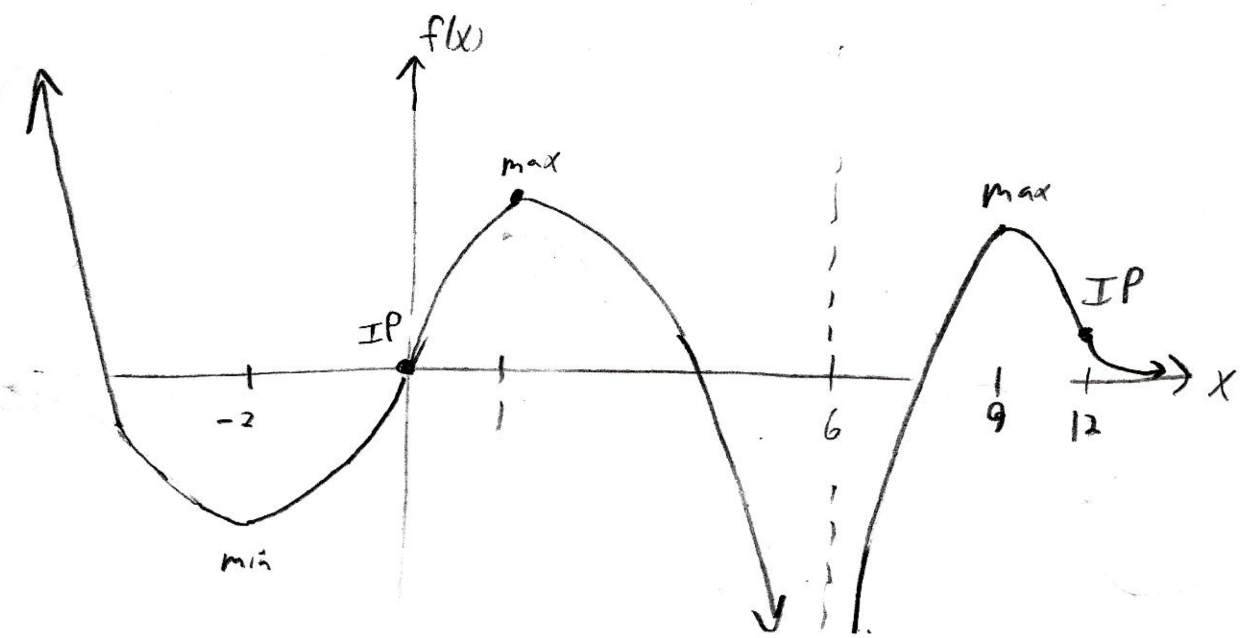
that lies between 1 and 2.

$\left[\lim_{x\to\infty} f(x) = 0\right] \Rightarrow$ [f has a H.A. at $y=0$]

18

$\left[\lim_{x\to 6} f(x) = -\infty\right] \Rightarrow$ [$x=6$ is an even vertical asymptote]



19 $f'(x) = 4 \cdot \left(\frac{1}{\sqrt{1-x^2}}\right)$ and $f\left(\frac{1}{2}\right) = 1$

$f(x) = 4\sin^{-1}(x) + c$

$1 = 4\sin^{-1}\left(\frac{1}{2}\right) + C$

$\frac{1}{4} = \sin^{-1}\left(\frac{1}{2}\right) + C$

and $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\left[\frac{1}{4} = \frac{\pi}{6} + C\right] \Rightarrow \left[c = \frac{1}{4} - \frac{\pi}{6}\right]$

$\boxed{f(x) = 4\sin^{-1}(x) + \frac{1}{4} - \frac{\pi}{6}}$

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20 $f'(x) = \frac{8x-5}{\sqrt[3]{x}} = \frac{8x}{x^{1/3}} - \frac{5}{x^{1/3}} = 8x^{2/3} - 5x^{-1/3}$

$f(x) = 8\left(\frac{1}{\frac{5}{3}}x^{2/3+1}\right) - 5\left(\frac{x^{-1/3+1}}{-1/3+1}\right) + C$

$= 8 \cdot \frac{3}{5}x^{5/3} - \frac{5x^{2/3}}{\frac{2}{3}} + C = \frac{24}{5}x^{5/3} - \frac{15}{2}x^{2/3} + C$

$\boxed{f(x) = \frac{3}{10}x^{2/3}(16x - 25) + C}$

(21) $f'(x) = \dfrac{\sec(x)\sin(x)}{\cos(x)} = \sec(x) \cdot \dfrac{\sin(x)}{\cos(x)} = \sec(x)\tan(x)$

$f(x) = \sec(x) + C$

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$f(x) = x^6 - 13x^5 + 67x^4 - 175x^3 + 244x^2 - 172x + 48$

(22) possible rational zeros of $f(x)$ are $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 32, \pm 48\}$

$$\begin{array}{r|rrrrrrr} 1 & 1 & -13 & 67 & -175 & 244 & -172 & 48 \\ & \downarrow & 1 & -12 & 55 & -120 & 124 & -48 \\ \hline & 1 & -12 & 55 & -120 & 124 & -48 & 0 \end{array}$$

$f(x) = (x^5 - 12x^4 + 55x^3 - 120x^2 + 124x - 48)(x-1)$

$$\begin{array}{r|rrrrrr} 1 & 1 & -12 & 55 & -120 & 124 & -48 \\ & \downarrow & 1 & -11 & 44 & -76 & 48 \\ \hline & 1 & -11 & 44 & -76 & 48 & 0 \end{array}$$

$f(x) = (x^4 - 11x^3 + 44x^2 - 76x + 48)(x-1)^2$

$$\begin{array}{r|rrrrr} 2 & 1 & -11 & 44 & -76 & 48 \\ & \downarrow & 2 & -18 & 52 & -48 \\ \hline & 1 & -9 & 26 & -24 & 0 \end{array}$$

$f(x) = (x^3 - 9x^2 + 26x - 24)(x-2)(x-1)^2$

$$\begin{array}{r|rrrr} 3 & 1 & -9 & 26 & -24 \\ & & 3 & -18 & 24 \\ \hline & 1 & -6 & 8 & 0 \end{array}$$

$f(x) = (x^2 - 6x + 8)(x-3)(x-2)(x-1)^2$

$= (x-2)(x-4)(x-3)(x-2)(x-1)^2$

$$\boxed{f(x) = (x-1)^2(x-2)^2(x-3)(x-4)}$$