3.1 Hw Note For some hw problems you are asked to find the exact value, without a calculator, of expressions like

$\cos(\cos^{-1}(x))$, $\sin(\sin^{-1}(x))$ and $\tan(\tan^{-1}(x))$, or

$\cos^{-1}(\cos(x))$, $\sin^{-1}(\sin(x))$ and $\tan^{-1}(\tan(x))$ with

a specific value of x. For these problem types we are attempting to use one of the properties that defines inverse functions, either $f(f^{-1}(x)) = x$ or $f^{-1}(f(x)) = x$. There are 3 cases for a problem of this kind:

case 1 We can use either $f(f^{-1}(x)) = x$ or $f^{-1}(f(x)) = x$ directly and just say that the expression is equal to the input value, x.

Ex: $\sin^{-1}(\sin(\frac{\pi}{4})) = \frac{\pi}{4}$ and $\tan(\tan^{-1}(-1)) = -1$. We can only do this when the input value, x, is a value in the domain of the inner function and the range of the outer function.

case 2 Another possible answer is that the exact value of the expression is "not defined." We answer "not defined" whenever the input value, x, is not in the domain of the innermost function.

Ex $\cos(\cos^{-1}(2))$ is not defined since 2 is not a value in the domain of $f(x) = \cos^{-1}(x)$, which is the interval $[-1,1]$.

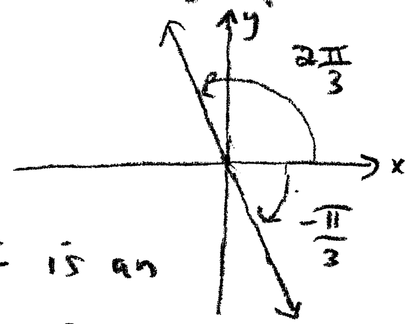
Ex $\tan^{-1}(\tan(\frac{3\pi}{2}))$ is not defined since $\frac{3\pi}{2}$ is not in the domain of tangent, which is $\{\theta \mid \theta \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$.

Case 3 Another possible answer is that the exact value of the expression is a real number different from the input value, x. This comes up whenever the input value, x is a value in the domain of the innermost function AND NOT a value in the range of the outermost function.

Ex $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right) = -\frac{\pi}{3}$ and $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right) = \frac{\pi}{4}$

and $\cos^{-1}\left(\cos\left(\frac{3\pi}{2}\right)\right) = \frac{\pi}{2}$.

Ex $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right) = -\frac{\pi}{3}$ since although $\frac{2\pi}{3}$ is in the domain of $f(x) = \tan(x)$, it is not an angle in the range of $f(x) = \tan^{-1}(x)$, which is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. But $-\frac{\pi}{3}$ is an angle in the range of $\tan^{-1}(x)$ that has the same tangent as $\frac{2\pi}{3}$, so $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right) = -\frac{\pi}{3}$.



Ex $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right) = \frac{\pi}{4}$ since although $\frac{3\pi}{4}$ is an angle in dom$(\sin(x))$, it is not an angle in rng$(\sin^{-1}(x))$, which is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Instead, we find an angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that has the same sine as $\frac{3\pi}{4}$.