Math 101 — Test 3 **Name:** Key
TUTOR HELP NOT OKAY!!

Directions: DUE FRIDAY, MARCH 27TH BY NOON!!
YOU MAY TURN THIS TEST IN BY EMAIL BY MIDNIGHT ON SAT., MAR. 28TH BUT A 10% LATE PENALTY WILL APPLY IF IT IS TURNED IN AFTER THE FRIDAY DUE DATE. NO EXAMS WILL BE ACCEPTED AFTER MIDNIGHT ON SAT., MAR. 28TH

- You are not to work on this together or get help from mentors/tutors!!
- Show all work.
- Write your solutions on printer paper.
- Only use a calculator if the question specifically says to use one!
- Box off your solutions.
- No credit will be given for sloppy or illegible work.
- Do not write your solutions on this piece of paper.
- Staple your worked out solutions to this piece of paper and use this piece of paper as your top (or cover) sheet.

No Calculator Exercises

1. Do problems 48, 57, 74, 76, 86, 92, 94, 112 and 118 on page 246 — 247 in the textbook.

Calculator Exercises

2. **Section 4.2**

   Do problems 14, 18, 26, 30, 34 on pages 270–271 in the textbook.

3. **Section 4.3**

   Do problems 30 and 44 on page 278 in the textbook.

4. **Section 4.4**

   Do problems 12 and 45 on page 284 in the textbook.

(48) Prove $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\csc\theta$

10 points

$$LHS = \frac{\sin\theta}{1+\cos\theta}\cdot\frac{\sin\theta}{\sin\theta} + \frac{1+\cos\theta}{\sin\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{\sin^2\theta + (1+\cos\theta)(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

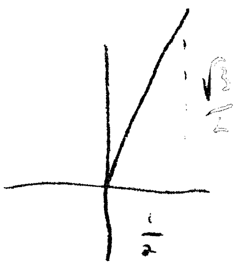
$$= \frac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}} = 2\cdot\frac{1}{\sin\theta} = 2\csc\theta \checkmark$$

57) Prove
$$\frac{1-2\sin^2\theta}{\sin\theta\cos\theta} = \cot\theta + \tan\theta$$
(5 points)

$$LHS = \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta} = \frac{\sin^2\theta}{\sin\theta\cos\theta} - \frac{\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta} = \tan\theta - \cot\theta.$$

74) $$\tan 105^\circ = \tan(60^\circ + 45^\circ) = \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ}$$ (5 points)



$\tan 60^\circ = \sqrt{3}$

$\tan 45^\circ = 1$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{1+2\sqrt{3}+3}{1-3}$$

$$= \frac{4+2\sqrt{3}}{-2} = \frac{4}{-2} + \frac{2\sqrt{3}}{-2}$$

$$= \boxed{-2-\sqrt{3}}$$

One could also use

$$\tan 105^\circ = \tan\tfrac{1}{2}\alpha = -\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

with $\alpha = 210^\circ$

or

$$\tan\tfrac{1}{2}\alpha = \frac{1-\cos\alpha}{\sin\alpha} =$$

or

$$\tan\tfrac{1}{2}\alpha = \frac{\sin\alpha}{1+\cos\alpha}$$

(76) $\sin\left(-\frac{\pi}{12}\right) = -\sin\frac{\pi}{12}$

since $\sin(-\theta) = -\sin\theta$

$-\sin\frac{\pi}{12} = -\sin\left(\frac{1}{2}\cdot\frac{\pi}{6}\right)$

$\frac{\pi}{12}$ in Q1

$\sin\theta > 0$ in Q1

$= -\sin\left(\frac{1}{2}\alpha\right)$

$= -\sqrt{\frac{1-\cos\alpha}{2}}$

$= -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= -\sqrt{\frac{1}{2}\left(1-\frac{\sqrt{3}}{2}\right)} = -\sqrt{\frac{1}{2}-\frac{\sqrt{3}}{4}}$

$= -\sqrt{\frac{2}{4}-\frac{\sqrt{3}}{4}} = -\sqrt{\frac{2-\sqrt{3}}{4}}$

$= \frac{-\sqrt{2-\sqrt{3}}}{\sqrt{4}}$

$= \boxed{\frac{-\sqrt{2-\sqrt{3}}}{2}}$ or

$= \boxed{-\frac{1}{2}\sqrt{2-\sqrt{3}}}$

$\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{3\pi}{12}-\frac{4\pi}{12}\right)$

5 points

$= \sin\left(\frac{\pi}{4}-\frac{\pi}{3}\right)$

$= \sin(\alpha-\beta)$

$= \sin\alpha\cdot\cos\beta - \sin\beta\cdot\cos\alpha$

$= \sin\frac{\pi}{4}\cdot\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cdot\cos\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

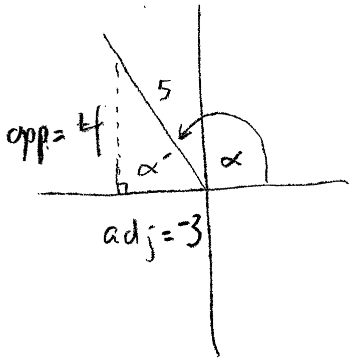
$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

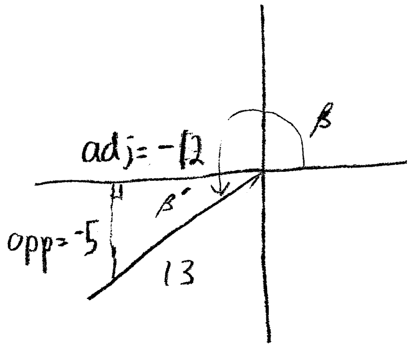
$= \boxed{\frac{1}{4}\left(\sqrt{2}-\sqrt{6}\right)}$

$\approx -0,259$

(86) $\tan\alpha = -\frac{4}{3}$, $\frac{\pi}{2} < \alpha < \pi$

$\cot\beta = \frac{12}{5}$, $\pi < \beta < \frac{3\pi}{2}$





$\cos\alpha = \frac{-3}{5}$	$\cos\beta = \frac{-12}{13}$
$\sin\alpha = \frac{4}{5}$	$\sin\beta = \frac{-5}{13}$
$\tan\alpha = -\frac{4}{3}$	$\tan\beta = \frac{5}{12}$

15 points

(a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$$= \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right) + \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right) = \frac{-48}{65} + \frac{-15}{65} = \boxed{\frac{-33}{65}}$$

(b) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \left(\frac{-3}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{-5}{13}\right) = \frac{36}{65} + \frac{20}{65} = \boxed{\frac{56}{65}}$$

(c) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$$= \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right) = \frac{-48}{65} - \frac{15}{65} = \boxed{\frac{-63}{65}}$$

(d) $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{-\frac{4}{3} + \frac{5}{12}}{1 - \frac{-4}{3}\cdot\frac{5}{12}} = \frac{\frac{-16}{12} + \frac{5}{12}}{1 + \frac{20}{36}} = \frac{-11}{\cancel{12}_{1}} \cdot \frac{\cancel{36}^{3}}{56} = \boxed{\frac{-33}{56}}$

86 continued

(e) $\sin(2\alpha) = 2\sin\alpha\cos\alpha = 2 \cdot \frac{4}{5} \cdot \frac{-3}{5} = \boxed{\frac{-24}{25}}$

(f) $\cos 2\beta = \cos^2\beta - \sin^2\beta = \left(-\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

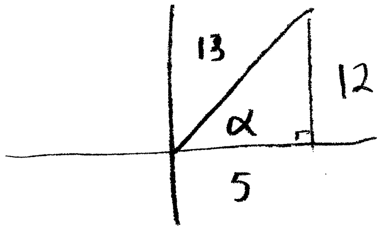
(g) $\sin\frac{1}{2}\beta = \sqrt{\frac{1-\cos\beta}{2}} = \sqrt{\frac{1}{2}\left(1+\frac{12}{13}\right)} = \sqrt{\frac{1}{2}+\frac{12}{26}} = \sqrt{\frac{25}{26}}$

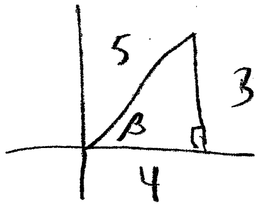
$= \frac{\sqrt{25}}{\sqrt{26}} = \frac{5}{\sqrt{26}} = \boxed{\frac{5\sqrt{26}}{26}}$

(h) $\cos\frac{1}{2}\alpha = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1}{2}\left(1-\frac{3}{5}\right)} = \sqrt{\frac{1}{2}\left(\frac{2}{5}\right)} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$

(92) $\sin\left(\cos^{-1}\left(\frac{5}{13}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right)$ (5 points)

Let $\alpha = \cos^{-1}\left(\frac{5}{13}\right)$ and $\beta = \cos^{-1}\left(\frac{4}{5}\right)$. Then both α and β are in Q1; and $\cos\alpha = \frac{5}{13}$ and $\cos\beta = \frac{4}{5}$.



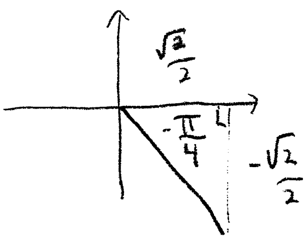


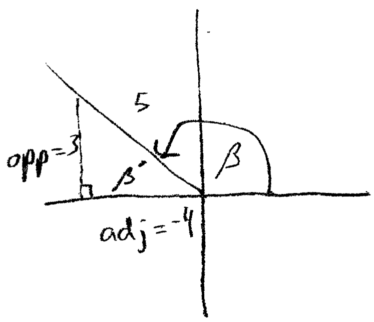
$\cos\alpha = \frac{5}{13}$	$\cos\beta = \frac{4}{5}$
$\sin\alpha = \frac{12}{13}$	$\sin\beta = \frac{3}{5}$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$$
$$= \left(\frac{12}{13}\right)\cdot\left(\frac{4}{5}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) = \frac{48}{65} - \frac{15}{65} = \boxed{\frac{33}{65}}$$

(94) $\cos\left[\tan(-1) + \cos^{-1}\left(-\frac{4}{5}\right)\right]$ (5 points)

Let $\alpha = \tan^{-1}(-1)$ and $\beta = \cos^{-1}\left(-\frac{4}{5}\right)$ then $\alpha = -\frac{\pi}{4}$ and $\cos\beta = -\frac{4}{5}$ with β in Q2.





α

$\cos\alpha = \frac{\sqrt{2}}{2}$

$\sin\alpha = -\frac{\sqrt{2}}{2}$

β

$\cos\beta = -\frac{4}{5}$

$\sin\beta = \frac{3}{5}$

$$\cos\left[\frac{\pi}{4} + \cos^{-1}\left(-\frac{4}{5}\right)\right]$$
$$= \cos(\alpha+\beta)$$
$$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{-4}{5} - \left(-\frac{\sqrt{2}}{2}\right)\cdot\frac{3}{5}$$
$$= \frac{-4\sqrt{2}}{10} + \frac{3\sqrt{2}}{10}$$
$$= \boxed{-\frac{1}{10}\sqrt{2}}$$

(112) Solve $\sin 2\theta - \sin\theta - 2\cos\theta + 1 = 0$ over $[0, 2\pi)$

(5 points)

$$2\sin\theta\cos\theta - \sin\theta - 2\cos\theta + 1 = 0$$

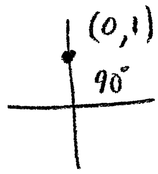
$$\sin\theta\left(2\cos\theta - 1\right) - 1\left(2\cos\theta - 1\right) = 0$$

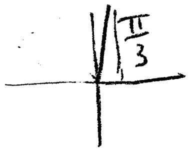
$$\left(\sin\theta - 1\right)\left(2\cos\theta - 1\right) = 0$$

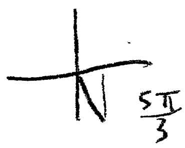
$\sin\theta - 1 = 0$ or $2\cos\theta - 1 = 0$

$\sin\theta = 1$ or $\cos\theta = \frac{1}{2}$

$$\boxed{\begin{gathered}\{60^\circ, 90^\circ, 300^\circ\} \\ \left\{\frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{3}\right\}\end{gathered}}$$







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(118) Solve $1 + \sqrt{3}\cos\theta + \cos 2\theta = 0$ over $[0, 2\pi)$

(5 points)

$$1 + \sqrt{3}\cos\theta + 2\cos^2\theta - 1 = 0$$

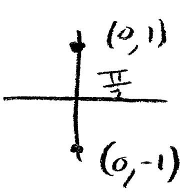
$$2\cos^2\theta + \sqrt{3}\cos\theta = 0$$

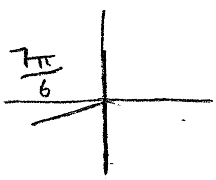
$$\cos\theta\left(2\cos\theta + \sqrt{3}\right) = 0$$

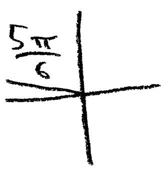
$\cos\theta = 0$ or $2\cos\theta + \sqrt{3} = 0$

$\cos\theta = 0$ or $\cos\theta = \frac{-\sqrt{3}}{2}$

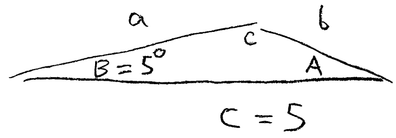
$$\boxed{\begin{gathered}\{90^\circ, 150^\circ, 210^\circ, 270^\circ\} \\ \left\{\frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}\right\}\end{gathered}}$$



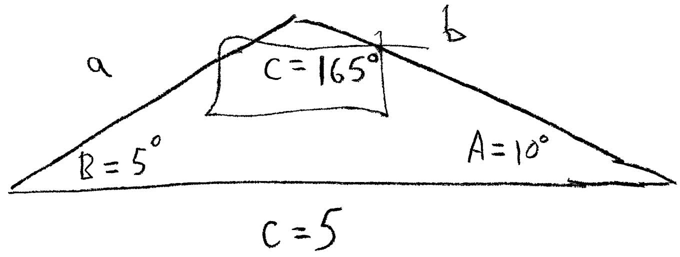




(14)



ASA



$$\left[\frac{\sin 165^\circ}{5} = \frac{\sin 5^\circ}{b}\right] \Rightarrow b = \frac{5\sin 5^\circ}{\sin 165^\circ} \doteq \boxed{1.68}$$

$$\left[\frac{\sin 165^\circ}{5} = \frac{\sin 10^\circ}{a}\right] \Rightarrow a = \frac{5\sin 10^\circ}{\sin 165^\circ} \doteq \boxed{3.35}$$

(5 points)

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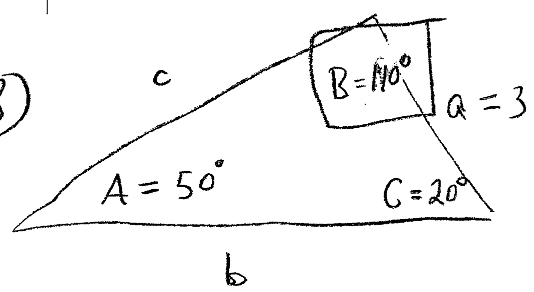
(18)



AAS

(5 points)

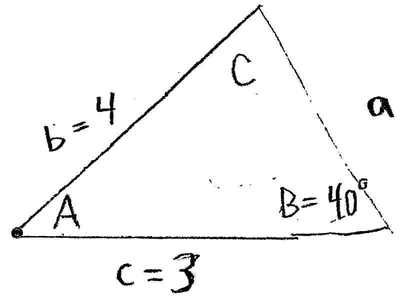
$\boxed{B = 110^\circ}$

$$\left[\frac{\sin 50^\circ}{3} = \frac{\sin 80^\circ}{b}\right] \Rightarrow b = \frac{3\sin 80^\circ}{\sin 50^\circ} \doteq \boxed{3.86}$$

$$\left[\frac{\sin 50^\circ}{3} = \frac{\sin 20^\circ}{c}\right] \Rightarrow c = \frac{3\sin 20^\circ}{\sin 50^\circ} \doteq \boxed{1.34}$$

(26)

(5 points)



SSA

The Ambiguous Case

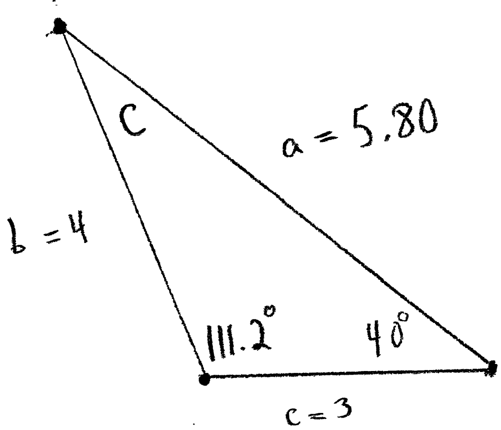
$$\left[\frac{\sin 40^\circ}{4} = \frac{\sin C}{3}\right] \Rightarrow \left[\sin C = \frac{3}{4}\sin 40^\circ\right] \Rightarrow \left[C = \sin^{-1}\left(\frac{3}{4}\sin 40^\circ\right)\right]$$

$$\boxed{C \doteq 28.8^\circ}$$

The other angle in $[0, 180^\circ]$ whose sine is $\frac{3}{4}\sin 40^\circ$ is $180^\circ - 28.8^\circ = 151.2^\circ$. However, $151.2^\circ + 40^\circ = 191.2^\circ$ which is greater than 180°. So, there is only 1 triangle.

Then, $A = 180^\circ - 40^\circ - 28.8^\circ = 111.2^\circ$, and

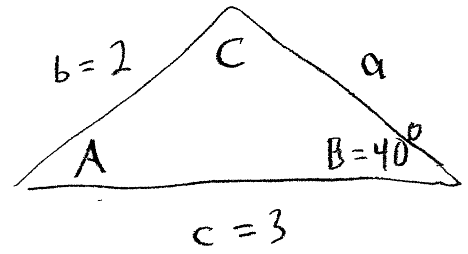
$$\left[\frac{\sin 40^\circ}{4} = \frac{\sin 111.2^\circ}{a}\right] \Rightarrow \left[a = \frac{4\sin 111.2^\circ}{\sin 40^\circ} \doteq 5.80\right]$$



$$\boxed{\begin{aligned} a &= 5.80 \\ A &= 111.2^\circ \\ C &= 28.8^\circ \end{aligned}}$$

(9)

(30)



SSA The Ambiguous Case

(5 points)

$$\left[\frac{\sin 40^\circ}{2} = \frac{\sin C}{3}\right] \Rightarrow \left[\sin C = \frac{3}{2}\sin 40^\circ\right] \Rightarrow \left[C = \sin^{-1}\left(\frac{3}{2}\sin 40^\circ\right) \doteq 74.6^\circ\right]$$

The other angle in $[0^\circ, 180^\circ]$ whose sine is $\frac{3}{2}\sin 40^\circ$ is $180^\circ - 74.6^\circ = 105.4^\circ$.

Then, since $105.4^\circ + B = 105.4^\circ + 40^\circ = 145.4^\circ$ and $145.4^\circ < 180^\circ$,

there are 2 triangles.

Triangle 1

$$\boxed{C = 74.6^\circ}$$

$$A = 180^\circ - 40^\circ - 74.6^\circ =$$

$$\boxed{A = 65.4^\circ}$$

$$\frac{\sin 40^\circ}{2} = \frac{\sin 65.4^\circ}{a}$$

$$a = \frac{2\sin 65.4^\circ}{\sin 40^\circ}$$

$$\boxed{a \doteq 2.83}$$

Triangle 2

$$\boxed{C = 105.4^\circ}$$

$$A = 180^\circ - 40^\circ - 105.4^\circ$$

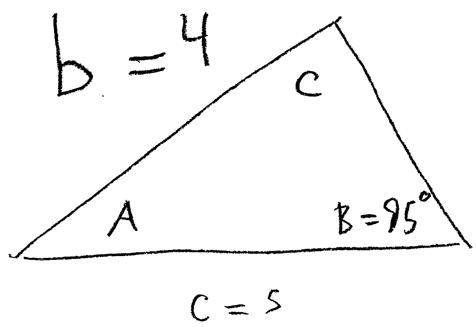
$$\boxed{A = 34.6^\circ}$$

$$\frac{\sin 40^\circ}{2} = \frac{\sin 34.6^\circ}{a}$$

$$a = \frac{2\sin 34.6^\circ}{\sin 40^\circ}$$

$$\boxed{a \doteq 1.77}$$

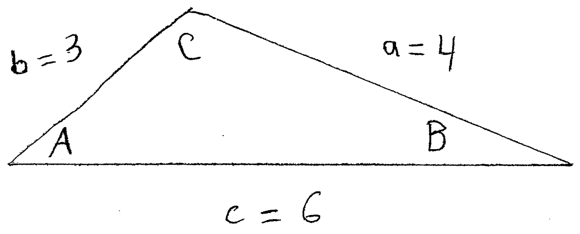
(34) (5 points)



$$\left[\frac{\sin 95^\circ}{4} = \frac{\sin C}{5}\right] \Rightarrow \left[\sin C = \frac{5}{4}\sin 95^\circ\right] \Rightarrow \left[C = \sin^{-1}\left(\frac{5}{4}\sin 95^\circ\right)\right]$$

since $\frac{5}{4}\sin 95^\circ \doteq 1.25$ and $y = \sin^{-1}(x)$ is not defined for x values outside $[-1, 1]$, there is no triangle

30



SSS

LAW of Cosines

(5 points)

$c^2 = a^2 + b^2 - 2ab\cos C$ (solve for C)

$$2ab\cos C = a^2 + b^2 - c^2$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$C = \cos^{-1}\left(\frac{a^2+b^2-c^2}{2ab}\right) = \cos^{-1}\left(\frac{4^2+3^2-6^2}{2(4)(3)}\right) = \cos^{-1}\left(\frac{-11}{24}\right) = 117.3^\circ$$

Then,

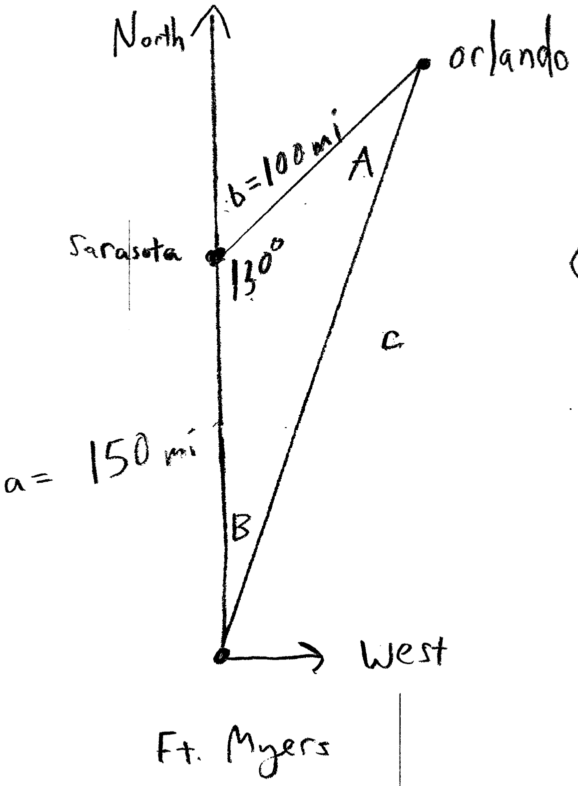
$$\left(\frac{\sin 117.3^\circ}{6} = \frac{\sin A}{4}\right) \Rightarrow \left(A = \sin^{-1}\left[\frac{2}{3}\sin 117.3^\circ\right] \doteq 36.3^\circ\right)$$

And

$$B = 180^\circ - 117.3^\circ - 36.3^\circ = 26.4^\circ$$

$$\boxed{\begin{aligned} A &= 36.3^\circ \\ B &= 26.4^\circ \\ C &= 117.3^\circ \end{aligned}}$$

44



SAS

5 points

(a) $c^2 = a^2+b^2-2ab\cos C$

$$c = \sqrt{a^2+b^2-2ab\cos C}$$

$$= \sqrt{150^2+100^2-2(150)(100)\cos 130^\circ}$$

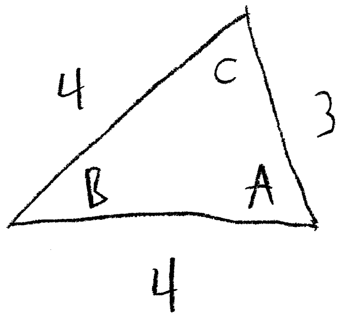
$$\doteq \boxed{227.56 \text{ miles}}$$

(b) $\left[\frac{\sin B}{100\text{ mi}} = \frac{\sin 130^\circ}{227.56}\right] \Rightarrow \left[B = \sin^{-1}\left(\frac{100\sin 130^\circ}{227.56}\right) \doteq 19.7^\circ\right]$

$$\boxed{B = 19.7^\circ}$$

$$\boxed{\text{Bearing } N\ 19.7^\circ E}$$

12



(5 points)

$$\left(\begin{array}{c}\text{Semi}\\ \text{perimeter}\end{array}\right) = s = \frac{1}{2}(a+b+c) = \frac{1}{2}(4+4+3) = \frac{1}{2}(11) = 5.5$$

$$s = 5.5$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{5.5(5.5-4)(5.5-4)(5.5-3)}$$

$$= \sqrt{5.5(1.5)(1.5)(2.5)}$$

$$\doteq \boxed{5.56 \text{ unit squared}}$$