Quiz 11 Key

29 E   35 D

38. $r = 1 + \sin\theta$

The graph will be a cardioid. Check for symmetry:

Polar axis: Replace θ by $-\theta$. The result is $r = 1 + \sin(-\theta) = 1 - \sin\theta$. The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

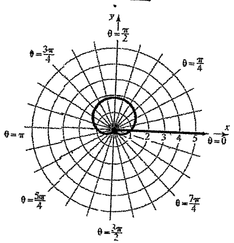
$$\begin{aligned} r &= 1 + \sin(\pi - \theta) \\ &= 1 + [\sin(\pi)\cos\theta - \cos(\pi)\sin\theta] \\ &= 1 + (0 + \sin\theta) \\ &= 1 + \sin\theta \end{aligned}$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

The pole: Replace r by $-r$. $-r = 1 + \sin\theta$. The test fails.

Due to symmetry with respect to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$r = 1 + \sin\theta$
$-\frac{\pi}{2}$	0
$-\frac{\pi}{3}$	$1 - \frac{\sqrt{3}}{2} \approx 0.1$
$-\frac{\pi}{6}$	$\frac{1}{2}$
0	1
$\frac{\pi}{6}$	$\frac{3}{2}$
$\frac{\pi}{3}$	$1 + \frac{\sqrt{3}}{2} \approx 1.9$
$\frac{\pi}{2}$	2



42. $r = 2 - \cos\theta$

The graph will be a limaçon without an inner loop. Check for symmetry:

Polar axis: Replace θ by $-\theta$. The result is $r = 2 - \cos(-\theta) = 2 - \cos\theta$. The graph is symmetric with respect to the polar axis.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

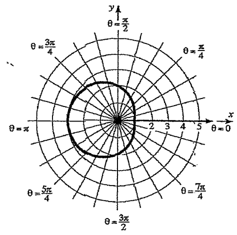
$$\begin{aligned} r &= 2 - \cos(\pi - \theta) \\ &= 2 - [\cos(\pi)\cos\theta + \sin(\pi)\sin\theta] \\ &= 2 - (-\cos\theta + 0) \\ &= 2 + \cos\theta \end{aligned}$$

The test fails.

The pole: Replace r by $-r$. $-r = 2 - \cos\theta$. The test fails.

Due to symmetry with respect to the polar axis, assign values to θ from 0 to π.

θ	$r = 2 - \cos\theta$
0	1
$\frac{\pi}{6}$	$2 - \frac{\sqrt{3}}{2} \approx 1.1$
$\frac{\pi}{3}$	$\frac{3}{2}$
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	$\frac{5}{2}$
$\frac{5\pi}{6}$	$2 + \frac{\sqrt{3}}{2} \approx 2.9$
π	3



46. $r = 1 - 2\sin\theta$

The graph will be a limaçon with an inner loop. Check for symmetry:

Polar axis: Replace θ by $-\theta$. The result is $r = 1 - 2\sin(-\theta) = 1 + 2\sin\theta$. The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

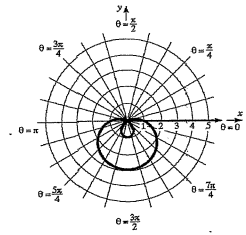
$$\begin{aligned} r &= 1 - 2\sin(\pi - \theta) \\ &= 1 - 2[\sin(\pi)\cos\theta - \cos(\pi)\sin\theta] \\ &= 1 - 2(0 + \sin\theta) \\ &= 1 - 2\sin\theta \end{aligned}$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

The pole: Replace r by $-r$. $-r = 1 - 2\sin\theta$. The test fails.

Due to symmetry with respect to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$r = 1 - 2\sin\theta$
$-\frac{\pi}{2}$	3
$-\frac{\pi}{3}$	$1 + \sqrt{3} \approx 2.7$
$-\frac{\pi}{6}$	2
0	1
$\frac{\pi}{6}$	0
$\frac{\pi}{3}$	$1 - \sqrt{3} \approx -0.7$
$\frac{\pi}{2}$	-1



50. $r = 2\sin(3\theta)$

The graph will be a rose with three petals. Check for symmetry:

Polar axis: Replace θ by $-\theta$.

$r = 2\sin[3(-\theta)] = 2\sin(-3\theta) = -2\sin(3\theta)$. The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

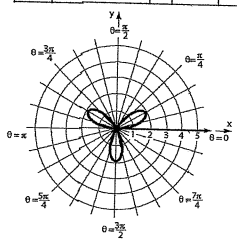
$$\begin{aligned} r &= 2\sin[3(\pi - \theta)] \\ &= 2\sin(3\pi - 3\theta) \\ &= 2[\sin(3\pi)\cos(3\theta) - \cos(3\pi)\sin(3\theta)] \\ &= 2[0 + \sin(3\theta)] \\ &= 2\sin(3\theta) \end{aligned}$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

The pole: Replace r by $-r$. $-r = 2\sin(3\theta)$. The test fails.

Due to symmetry with respect to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$r = 2\sin(3\theta)$	θ	$r = 2\sin(3\theta)$
$-\frac{\pi}{2}$	2	$\frac{\pi}{6}$	2
$-\frac{\pi}{3}$	0	$\frac{\pi}{4}$	$\sqrt{2} \approx 1.4$
$-\frac{\pi}{4}$	$-\sqrt{2} \approx -1.4$	$\frac{\pi}{3}$	0
$-\frac{\pi}{6}$	-2	$\frac{\pi}{2}$	-2
0	0		



54. $r^2 = \sin(2\theta)$

The graph will be a lemniscate. Check for symmetry:

Polar axis: Replace θ by $-\theta$.

$r^2 = \sin(2(-\theta)) = \sin(-2\theta) = -\sin(2\theta)$. The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$\begin{aligned} r^2 &= \sin[2(\pi - \theta)] \\ &= \sin(2\pi - 2\theta) \\ &= \sin(2\pi)\cos 2\theta - \cos(2\pi)\sin(2\theta) \\ &= 0 - \sin(2\theta) \\ &= -\sin(2\theta) \end{aligned}$$

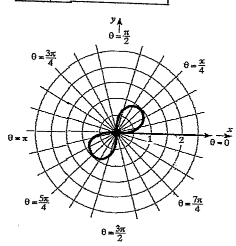
The test fails.

The pole: Replace r by $-r$.

$$\begin{aligned} (-r)^2 &= \sin(2\theta) \\ r^2 &= \sin(2\theta) \end{aligned}$$

The graph is symmetric with respect to the pole. Due to symmetry, assign values to θ from 0 to π.

θ	$r = \pm\sqrt{\sin(2\theta)}$
0	0
$\frac{\pi}{6}$	$\pm\sqrt{\frac{\sqrt{3}}{2}}$
$\frac{\pi}{3}$	$\pm\sqrt{\frac{\sqrt{3}}{2}}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	undefined
$\frac{5\pi}{6}$	undefined
π	0



58. $r = 3 + \cos\theta$

The graph will be a limaçon without an inner loop. Check for symmetry:

Polar axis: Replace θ by $-\theta$. The result is $r = 3 + \cos(-\theta) = 3 + \cos\theta$. The graph is symmetric with respect to the polar axis.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$\begin{aligned} r &= 3 + \cos(\pi - \theta) \\ &= 3 + [\cos(\pi)\cos\theta + \sin(\pi)\sin\theta] \\ &= 3 + (-\cos\theta + 0) \\ &= 3 - \cos\theta \end{aligned}$$

The test fails.

The pole: Replace r by $-r$. $-r = 3 + \cos\theta$. The test fails.

Due to symmetry, assign values to θ from 0 to π.

θ	$r = 3 + \cos\theta$
0	4
$\frac{\pi}{6}$	$3 + \frac{\sqrt{3}}{2} \approx 3.9$
$\frac{\pi}{3}$	$\frac{7}{2}$
$\frac{\pi}{2}$	3
$\frac{2\pi}{3}$	$\frac{5}{2}$
$\frac{5\pi}{6}$	$3 - \frac{\sqrt{3}}{2} \approx 2.1$
π	2