Math 101 — Quiz 4
TUTOR HELP NOT OKAY!!

Name: Key

Directions: **Due Wednesday, Feb. 23th**

- You are not to work on this together or get help from mentors/tutors!!
- Show all work.
- Write your solutions on printer paper.
- Only use a calculator if the question specifically says to use one!
- Box off your solutions.
- No credit will be given for sloppy or illegible work.
- Do not write your solutions on this piece of paper.
- Staple your worked out solutions to this piece of paper and use this piece of paper as your top (or cover) sheet.

Exercises

1. **Section 3.1**
   Do problems 16, 20, 24, 36, 44, 48, 54, 56, 58, 68 and 70 on pages 192 – 193 in the textbook.

(16) Find the exact value of $\cos^{-1}(-1)$ without a calculator

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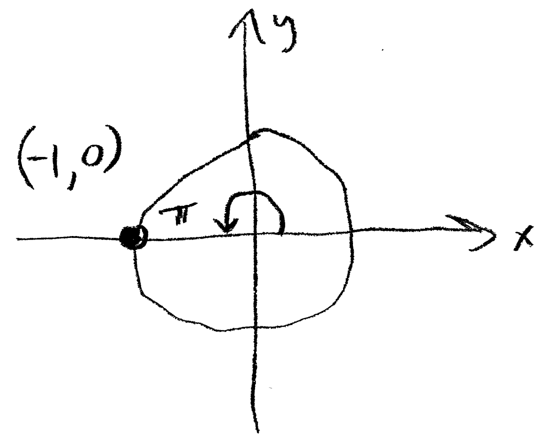
Soln/ We know inverse trig functions represent angles.
Let $\theta = \cos^{-1}(-1)$. Then, θ is an angle in the interval $[0,\pi]$. Also, we can take the cosine of both sides of $\theta = \cos^{-1}(-1)$ to get

$$\cos\theta = \cos\left(\cos^{-1}(-1)\right),$$ or equivalently,

that $\cos\theta = -1$, since $f\left(f^{-1}(x)\right) = x$ for all x in the domain of f^{-1}.

Therefore, the question asks us to find a specific angle θ in the interval $[0,\pi]$ whose cosine is -1.



$$\theta = \cos^{-1}(-1)$$
$$= \boxed{\pi}$$

(20) Find the exact value of $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ without a calculator

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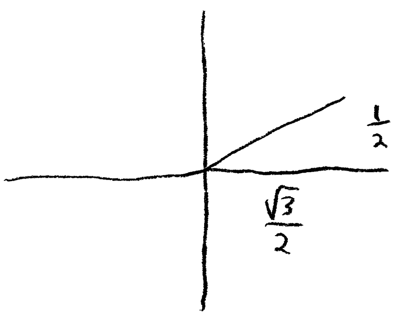
Soln/ Let $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$. Then θ is an angle in the range of inverse tangent, the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Moreover, $\tan\theta = \frac{\sqrt{3}}{3}$.

So, we need to find a common angle θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that has a tangent equal to $\frac{\sqrt{3}}{3}$. Since tangent is positive, we must be looking for a Q1 angle. Since the tangent has a $\sqrt{3}$ in it, we are looking at either the 30° or 60° angle. We calculate the tangent of both 30° and 60° to see that $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \boxed{30^\circ \text{ or } \frac{\pi}{6}}$



$$\tan 30^\circ = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \checkmark$$

(24) Find the exact value of $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ without a calculator

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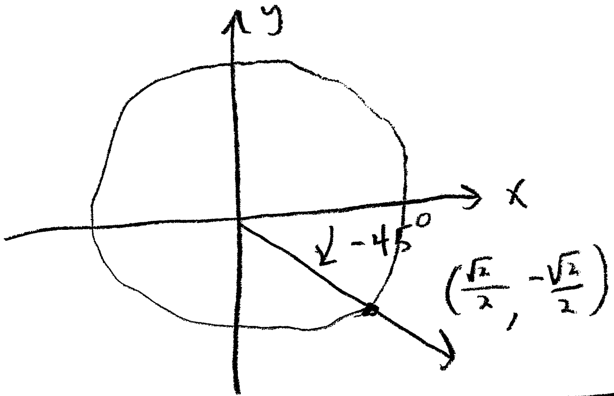
soln/ Let $\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, then $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

and $\sin\theta = -\frac{\sqrt{2}}{2}$.

Then, $\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$= \boxed{-45^\circ \text{ or } -\frac{\pi}{4}}$



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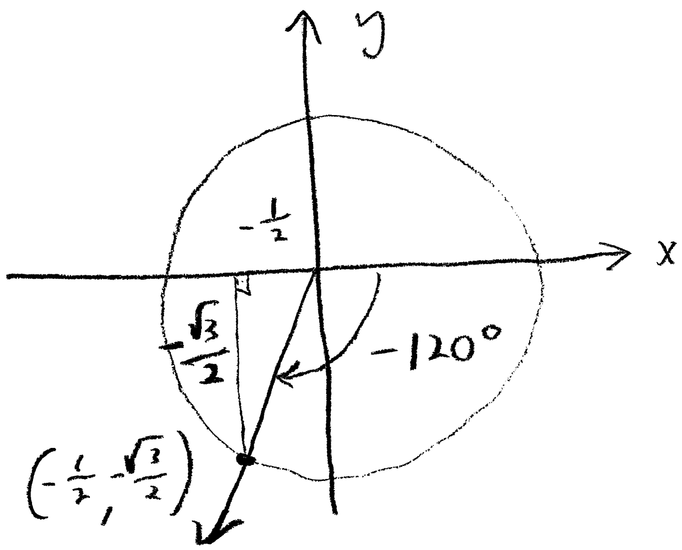
(36) Use a calculator to find the exact value of $\sin^{-1}\frac{\sqrt{3}}{5}$

$\boxed{\approx 20.3^\circ \text{ or } 0.35 \text{ radians}}$

(44) Find the exact value of $\tan^{-1}\left(\tan\left(-\frac{2\pi}{3}\right)\right)$

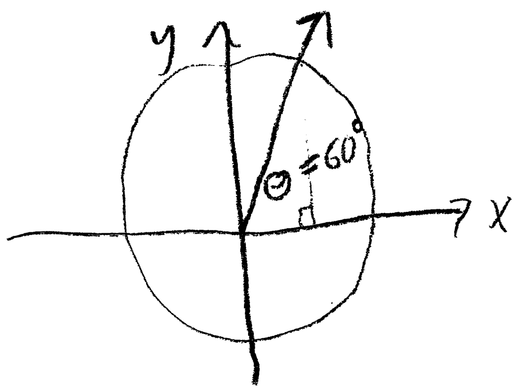
Soln/ The input angle is $-\frac{2\pi}{3}$ or -120°



This angle is outside of the range of inverse tangent, so $\tan^{-1}\left(\tan\left(-\frac{2\pi}{3}\right)\right) \neq -\frac{2\pi}{3}$.

Since $\tan\left(-\frac{2\pi}{3}\right) = \sqrt{3}$, $\tan^{-1}\left(\tan\left(-\frac{2\pi}{3}\right)\right) = \tan^{-1}(\sqrt{3})$.

Then, let $\theta = \tan^{-1}(\sqrt{3})$. It follows that $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.



Since tangent is positive here, θ must be in Q1. Since $\tan\theta = \sqrt{3}$, we need the 30° or 60° angle.

But the answer is $\boxed{\theta = 60^\circ \text{ or } \frac{\pi}{3}}$ since $\tan\frac{\pi}{3} = \sqrt{3}$.

(48) Find the exact value of $\tan\left(\tan^{-1}(-2)\right)$

without a calculator

Soln/ the innermost function is inverse tangent.

The domain of inverse tangent is $\mathbb{R}$, so the input value of -2 will not make $\tan^{-1}(-2)$ "not defined."

Therefore, $\tan\left(\tan^{-1}(-2)\right) = \boxed{-2.}$

This is justified by the property below

* Suppose f has an inverse. Then,
  For all x in the domain of f^{-1}

$$f\left(f^{-1}(x)\right) = x.$$

Let $f(x) = \tan(x)$ then $f^{-1}(x) = \tan^{-1}(x)$ and $\text{dom}(f^{-1}) = \mathbb{R}$.

Since -2 is in the set of real numbers, the equation

$f\left(f^{-1}(x)\right) = x$ is equivalent to

$\tan\left(\tan^{-1}(-2)\right) = -2$ if $x = -2$.

(54) Suppose $f(x) = 2\tan x - 3$; $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(A) Find $f^{-1}(x)$

(B) Find $\text{rng}(f)$

(c) Find $\text{dom}(f^{-1})$

(d) Find $\text{rng}(f^{-1})$

no calculators

(A)

$y = 2\tan(x) - 3$

$x = 2\tan(y) - 3$

$x + 3 = 2\tan(y)$

$\frac{x+3}{2} = \tan(y)$

$\frac{1}{2}x + \frac{3}{2} = \tan(y)$

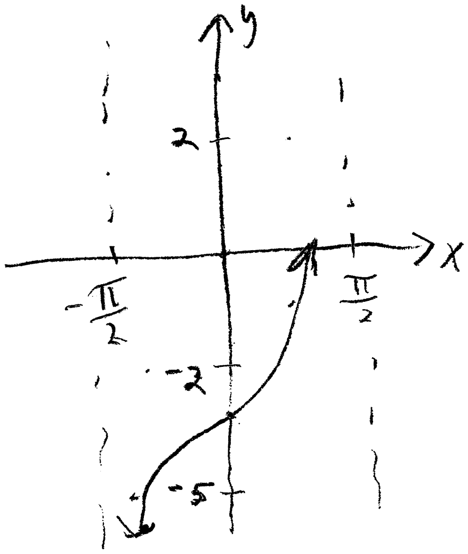
$\tan(y) = \frac{1}{2}x + \frac{3}{2}$

$\tan^{-1}(\tan(y)) = \tan^{-1}\left(\frac{1}{2}x + \frac{3}{2}\right)$

$y = \tan^{-1}\left(\frac{1}{2}x + \frac{3}{2}\right)$

$\boxed{f^{-1}(x) = \tan^{-1}\left(\frac{1}{2}x + \frac{3}{2}\right)}$

(B) We can get the range of f from f's graph



$\text{rng}(f) = \mathbb{R}$ or $(-\infty, \infty)$

(C) Then, since $\text{rng}(f) = \text{dom}(f^{-1})$,

$\text{dom}(f^{-1}) = \mathbb{R}$

(D) Since $\text{dom } f = \text{rng}(f^{-1})$,

$\text{rng } f^{-1} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(A) $f^{-1}(x) = \tan^{-1}\left(\frac{1}{2}x + \frac{3}{2}\right)$

(B) $\text{rng}(f) = \mathbb{R}$

(C) $\text{dom}(f^{-1}) = \mathbb{R}$

(D) $\text{rng}(f^{-1}) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(56) Suppose $f(x) = 3\sin(2x);\ -\frac{\pi}{4} \le x \le \frac{\pi}{4}$

(a) Find $f^{-1}(x)$

(b) Find $\text{rng}(f)$

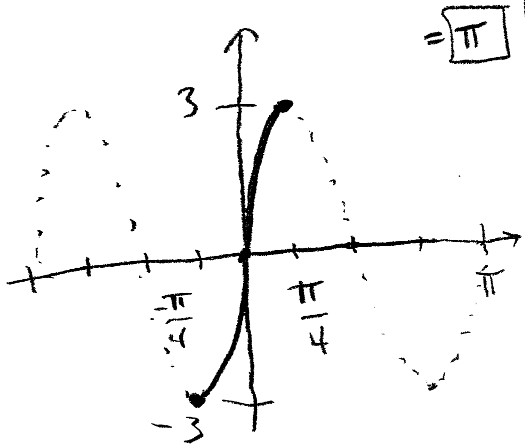
(c) Find $\text{dom}(f^{-1})$

(d) Find $\text{rng}(f^{-1})$

(a)
$$y = 3\sin(2x)$$
$$x = 3\sin(2y)$$
$$\frac{x}{3} = \sin(2y)$$
$$\sin(2y) = \frac{1}{3}x$$
$$\sin^{-1}\left(\sin(2y)\right) = \sin^{-1}\left(\frac{1}{3}x\right)$$
$$2y = \sin^{-1}\left(\frac{1}{3}x\right)$$
$$\frac{1}{2}\cdot 2y = \frac{1}{2}\sin^{-1}\left(\frac{1}{3}x\right)$$
$$y = \frac{1}{2}\sin^{-1}\left(\frac{1}{3}x\right)$$
$$\boxed{f^{-1}(x) = \frac{1}{2}\sin^{-1}\left(\frac{1}{3}x\right)}$$

(b) graph f

$A = 3$, $pd = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \boxed{\pi}$



$\text{rng}(f) = [-3, 3]$

(c) $\text{dom}(f^{-1}) = \text{rng}(f) = [-3, 3]$

(d) $\text{rng}(f^{-1}) = \text{dom } f = \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(a) $f^{-1}(x) = \frac{1}{2}\sin^{-1}\left(\frac{1}{3}x\right)$

(b) $\text{rng } f = [-3, 3]$

(c) $\text{dom } f^{-1} = [-3, 3]$

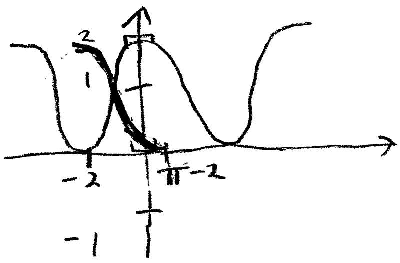
(d) $\text{rng } f = \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(58) Suppose $f(x) = \cos(x+2)+1$; $-2 \le x \le \pi - 2$

Ⓐ Find $f^{-1}(x)$
Ⓑ Find $\text{rng } f$
Ⓒ Find $\text{dom } f^{-1}$
Ⓓ Find $\text{rng } f^{-1}$

Ⓐ
$$y = \cos(x+2)+1$$
$$x = \cos(y+2)+1$$
$$x-1 = \cos(y+2)$$
$$\cos(y+2) = x-1$$
$$\cos^{-1}(\cos(y+2)) = \cos^{-1}(x-1)$$
$$y+2 = \cos^{-1}(x-1)$$
$$y = \cos^{-1}(x-1)-2$$
$$\boxed{f^{-1}(x) = \cos^{-1}(x-1)-2}$$

Ⓑ graph f



From the graph, we see that

$$\text{rng } f = [0,2]$$

Ⓒ By the property of inverse functions

$$\text{dom}(f^{-1}) = \text{rng}(f) = [0,2]$$

Ⓓ By the property of inverse functions

$$\text{rng}(f^{-1}) = \text{dom}(f) = [-2, \pi-2]$$

Ⓐ $f^{-1}(x) = \cos^{-1}(x-1)-2$
Ⓑ $\text{rng}(f) = [0,2]$
Ⓒ $\text{dom}(f^{-1}) = [0,2]$
Ⓓ $\text{rng}(f^{-1}) = [-2, \pi-2]$

Solve for x ; no calculator

(68) $5\sin^{-1}(x) - 2\pi = 2\sin^{-1}(x) - 3\pi$

Soln/ add 2π to both sides.

Subtract $2\sin^{-1}(x)$ from both sides

$$5\sin^{-1}(x) - 2\sin^{-1}(x) = 2\pi - 3\pi$$

$$(5-2)\cdot \sin^{-1}(x) = -\pi$$

$$3\cdot \sin^{-1}(x) = -\pi$$

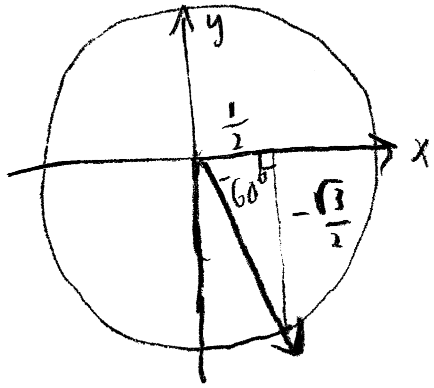
$$\frac{1}{3}\cdot 3\cdot \sin^{-1}(x) = \left(\frac{1}{3}\right)\cdot(-\pi)$$

$$\sin^{-1}(x) = \frac{-\pi}{3}$$

$$\sin\left(\sin^{-1}(x)\right) = \sin\left(\frac{-\pi}{3}\right)$$

$$x = \sin\left(-\frac{\pi}{3}\right)$$

$$\boxed{x = -\frac{\sqrt{3}}{2}}$$



(70) Calculator Problem

Use $D = 24\left[1 - \frac{\cos^{-1}(\tan(i) \cdot \tan(\theta))}{\pi}\right]$ hours

with angles i and θ expressed in radians to calculate the number of daylight hours in New York, i.e., assuming $\theta = 45^\circ 45'$

Use $\theta = 40^\circ + 45' \cdot \frac{1^\circ}{60'} = 40^\circ + \left(\frac{45}{60}\right)^\circ = 40.75^\circ \cdot \frac{\pi}{180} = \frac{40.75\pi}{180}$

(a) $D = 24 \cdot \left[1 - \frac{\cos^{-1}\left(\tan\left(\frac{23.5\pi}{180}\right) \cdot \tan\left(\frac{40.75\pi}{180}\right)\right)}{\pi}\right]$

$\boxed{\approx 14.93 \text{ hrs} \text{ or } 14 \text{ hrs, } 56 \text{ min}}$ and $(0.93 \text{ hr}) \cdot \frac{60 \text{ min}}{1 \text{ hr}} \approx 56 \text{ min}$

(b) $D = 24\left[1 - \frac{\cos^{-1}\left(\tan(0) \cdot \tan\left(\frac{40.75\pi}{180}\right)\right)}{\pi}\right] \approx \boxed{12 \text{ hrs}}$

(c) $D = 24\left[1 - \frac{\cos^{-1}\left(\tan\left(\frac{22.8\pi}{180}\right) \tan\left(\frac{40.75\pi}{180}\right)\right)}{\pi}\right] \approx \boxed{14.83 \text{ hrs or } 14 \text{ hrs, } 50 \text{ min}}$

$i = 22^\circ + 48' \cdot \frac{1^\circ}{60'} = 22.8^\circ$

$0.83 \text{ hr}\left(\frac{60 \text{ min}}{1 \text{ hr}}\right) \approx 50 \text{ min}$