Math 101 — Quiz 6 **Name:** Key
TUTOR HELP NOT OKAY!!

Directions: **Due Monday, Mar. 9th**

- You are not to work on this together or get help from mentors/tutors!!
- Show all work.
- Write your solutions on printer paper.
- Only use a calculator if the question specifically says to use one!
- Box off your solutions.
- No credit will be given for sloppy or illegible work.
- Do not write your solutions on this piece of paper.
- Staple your worked out solutions to this piece of paper and use this piece of paper as your top (or cover) sheet.

Exercises

(0,1)

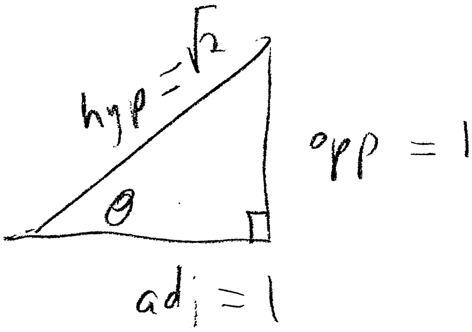
1. **Section 3.2**

   Do problems 15, 24, 32, 43, 44, 53, 54, 58, 64 and 66 on pages 198 – 199 in the textbook.

(15) $\csc\left(\tan^{-1}(1)\right)$

Let $\theta = \tan^{-1}(1)$, then $\tan\theta = 1$ and $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

~~Also~~



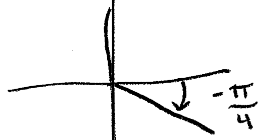
$c^2 = 1^2 + 1^2$

$c^2 = 2$

$c = \sqrt{2}$

Then $\csc\left(\tan^{-1}(1)\right) = \csc\theta = \frac{\text{hyp}}{\text{opp}} = \boxed{\sqrt{2}}$

(24) $\cos^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right]$



$\tan\left(-\frac{\pi}{4}\right) = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -$

Since $\tan\left(-\frac{\pi}{4}\right) = -1$, then $\cos^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right) = \cos^{-1}(-1)$.

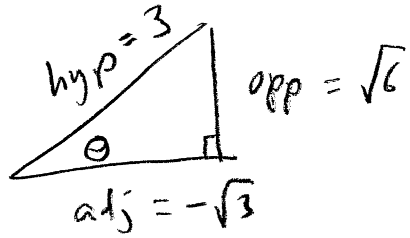
Let $\theta = \cos^{-1}(-1)$ then $\cos\theta = -1$ and $\theta \in [0, \pi]$.

So, $\boxed{\theta = \pi}$

(32) $\cot\left[\cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right]$

Let $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ then $\cos\theta = -\frac{\sqrt{3}}{3}$

and $\theta \in [0, \pi]$. Also $\cos\theta = \frac{adj}{hyp}$



$$3^2 = (-\sqrt{3})^2 + b^2$$
$$9 = 3 + b^2$$
$$b^2 = 6$$
$$b = \sqrt{6}$$

Then $\cot\left[\cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right]$

$$= \cot(\theta)$$

$$= \frac{adj}{opp} = \frac{-\sqrt{3}}{\sqrt{6}} = -\sqrt{\frac{3}{6}}$$

$$= -\sqrt{\frac{1}{2}} = -\frac{\sqrt{1}}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{-\frac{\sqrt{2}}{2}}$$

(43) Find the exact value of

$\cot^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

Let $\theta = \cot^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ then $\theta \in (0, \pi)$, the range of the inverse cotangent. Recall that

$$\cot^{-1}(x) = \begin{cases} \tan^{-1}\left(\frac{1}{x}\right) + 180^\circ, & \text{if } x<0 \quad \left(\text{if } \cot^{-1}(x) \in Q2\right) \\ \text{undefined if } x & \text{if } x = 0 \\ \tan^{-1}\left(\frac{1}{x}\right), & \text{if } x>0 \quad \left(\text{If } \cot^{-1}(x) \in Q1\right) \end{cases}$$

Since $-\frac{\sqrt{3}}{3}$ is less than zero, we know our angle θ is a Q2 angle. To find this angle we first find the angle $\tan^{-1}\left(\frac{1}{-\frac{\sqrt{3}}{3}}\right)$ then add 180° to it.

Let $\theta' = \tan^{-1}\left(\frac{1}{-\frac{\sqrt{3}}{3}}\right) = \tan^{-1}\left(-\frac{3}{\sqrt{3}}\right) = \tan^{-1}\left(-\frac{3}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}\right)$

$= \tan^{-1}\left(-\frac{3\sqrt{3}}{3}\right) = \tan^{-1}\left(-\sqrt{3}\right)$.

That is, let $\theta' = \tan^{-1}\left(-\sqrt{3}\right)$. Then $\theta' \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

and $\tan\theta' = -\sqrt{3}$.

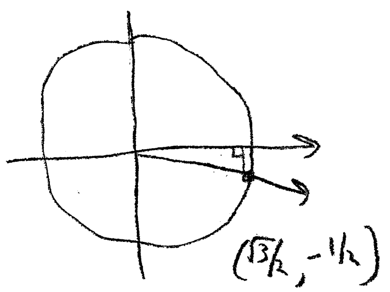
43 (continued)

Since $\theta' \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, θ' is a Q1 or negative angle in Q4.

But tangent is positive in Q1, and $\tan\theta' = -\sqrt{3} < 0$, so θ' is a negative Q4 angle. Moreover, θ' is either $-30°$ or $-60°$ since we are told $\tan\theta'$ has a "$\sqrt{3}$" in it.

Check both:

$$\tan(-30°) = \frac{y}{x} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$$



and $\tan(-60°) = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

so $\theta' = -60°$.

Then θ, our original angle in Q2 is

$$\theta = \theta' + 180° = -60° + 180° = \boxed{120° \text{ or } \frac{2\pi}{3}}$$

Find the exact value of

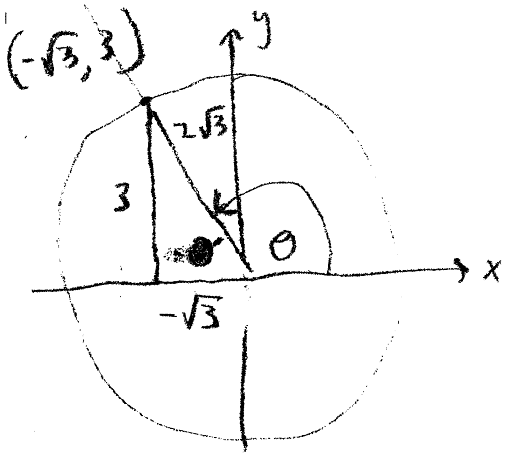
(43) $\cot^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

Alternate Soln

Let $\theta = \cot^{-1}\left(-\frac{\sqrt{3}}{3}\right)$, then $\theta \in (0, \pi)$

and $\cot\theta = -\frac{\sqrt{3}}{3}$. Since $\cot\theta < 0$, $\theta \in Q2$.

Since $y = \cot^{-1}(x)$ and $y = \cos^{-1}(x)$ have the same range, we proceed to find the cosine of θ.



$$c^2 = (-\sqrt{3})^2 + 3^2$$

$$c^2 = 3 + 9$$

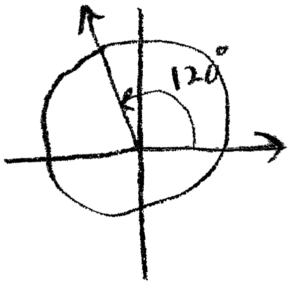
$$c^2 = 12$$

$$c = \sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$$

$$\cos\theta = \frac{-\sqrt{3}}{2\sqrt{3}} = -\frac{1}{2}.$$

So, $\cos\theta = -\frac{1}{2}$ and

$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$ and $\theta \in [0, \pi]$ and $\theta \in Q2$.



These conditions imply

$$\boxed{\theta = 120^\circ \text{ or } \frac{2\pi}{3}}$$

44. $\csc^{-1}\left(-\frac{2\sqrt{3}}{3}\right) = \sin^{-1}\left(-\frac{3}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}\right)$

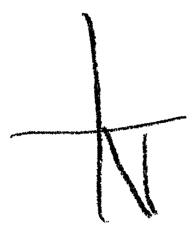
$$= \sin^{-1}\left(-\frac{\cancel{3}\sqrt{3}}{2\cdot \cancel{3}}\right)$$

$$= \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right).$$

Let $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ then $\sin\theta = -\frac{\sqrt{3}}{2}$

and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Since $\sin\theta < 0$, $\theta \in Q4$.

$\boxed{\theta = -60^\circ}$ Since $\sin(-60^\circ) = -\frac{\sqrt{3}}{2}$



(53) $\csc^{-1}\left(-\frac{3}{2}\right) = \sin^{-1}\left(-\frac{2}{3}\right) \doteq \boxed{-41.81^\circ \text{ or } -0.73 \text{ radians}}$

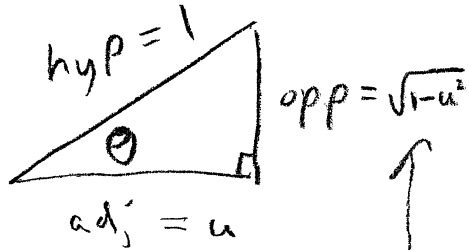
(54) $\sec^{-1}\left(-\frac{4}{3}\right) = \cos^{-1}\left(-\frac{3}{4}\right) \doteq \boxed{138.59^\circ \text{ or } 2.42 \text{ radians}}$

---

(58) $\sin(\cos^{-1} u)$

Let $\theta = \cos^{-1}(u)$, then $\theta \in [0, \pi]$

and $\cos\theta = u = \frac{u}{1} = \frac{adj}{hyp}$



$c^2 = a^2 + b^2$

$1^2 = u^2 + b^2$

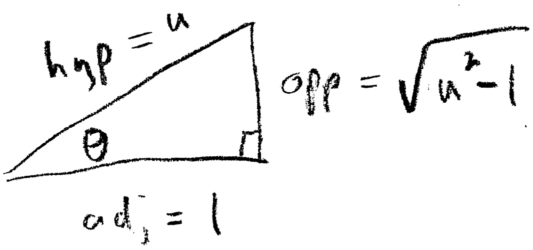
$b^2 = 1 - u^2$

$b = \sqrt{1-u^2}$

Then $\sin(\cos^{-1} u) = \sin\theta = \frac{opp}{hyp} = \boxed{\sqrt{1-u^2}}$

(64) $\cos(\sec^{-1} u)$

Let $\theta = \sec^{-1}(u)$ then $\sec\theta = \frac{u}{1} = \frac{hyp}{adj}$



$c^2 = a^2 + b^2$

$u^2 = 1^2 + b^2$

$b^2 = u^2 - 1$

$b = \sqrt{u^2-1}$

Then $\cos(\sec^{-1} u) = \cos\theta = \frac{adj}{hyp} = \boxed{\frac{1}{u}}$

Alternate soln path

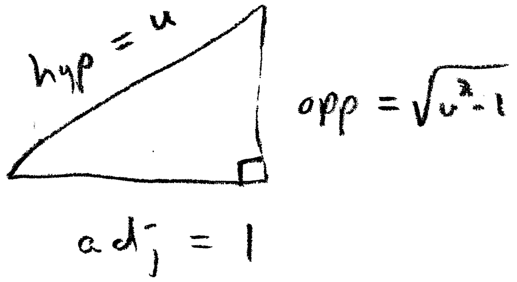
$$\cos(\sec^{-1} u) = \cos\left(\cos^{-1}\left(\frac{1}{u}\right)\right)$$

$$= \boxed{\frac{1}{u}}$$

(66) $\tan\left(\sec^{-1}(u)\right)$

Let $\theta = \sec^{-1}(u) = \cos^{-1}\left(\frac{1}{u}\right)$.

Then, $\cos\theta = \frac{1}{u} = \frac{adj}{hyp}$ and $\theta \in [0,\pi]$



$c^2 = a^2 + b^2$

$u^2 = 1^2 + b^2$

$b^2 = u^2 - 1$

$b = \sqrt{u^2-1}$

Then,

$$\tan\left(\sec^{-1}u\right) = \tan\theta$$

$$= \frac{opp}{adj}$$

$$= \boxed{\sqrt{u^2-1}}$$