Math 101 — Quiz 7
TUTOR HELP NOT OKAY!!

Name: Key

Directions: **Due Tuesday, Mar. 9th**

- You are not to work on this together or get help from mentors/tutors!!
- Show all work.
- Write your solutions on printer paper.
- Only use a calculator if the question specifically says to use one!
- Box off your solutions.
- No credit will be given for sloppy or illegible work.
- Do not write your solutions on this piece of paper.
- Staple your worked out solutions to this piece of paper and use this piece of paper as your top (or cover) sheet.

Exercises

1. **Section 3.3**

   Do problems 14, 17, 21, 30, 32, 37, 42, 46, 56, 58, 64, 65 and 74 on pages 207 – 208 in the textbook.

Solve

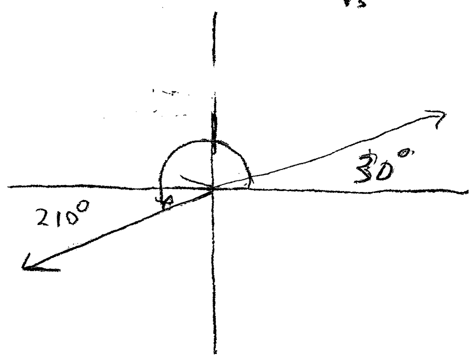
(14) $\tan^2\theta = \frac{1}{3}$

on the interval $[0, 2\pi)$

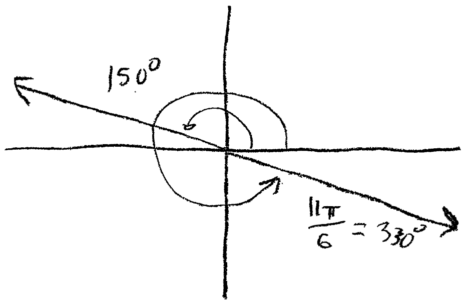
$\tan\theta = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$ by the sqrt theorem

Then $\tan\theta = \frac{1}{\sqrt{3}}$ or $\tan\theta = -\frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$



$\tan\theta = -\frac{1}{\sqrt{3}}$



So,

$\tan\theta = \frac{1}{\sqrt{3}}$

has 2 solutions

$\theta = 30°, 210°$

$\left(\frac{\pi}{6}, \frac{7\pi}{6}\right)$

and

$\tan\theta = -\frac{1}{\sqrt{3}}$

has 2 solns

$\theta = 150°, 330°$

$\left(\frac{5\pi}{6}, \frac{11\pi}{6}\right)$

$$\boxed{\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}}$$

(11) Solve $2\sin\theta + 3 = 2$ over $[0, 2\pi)$

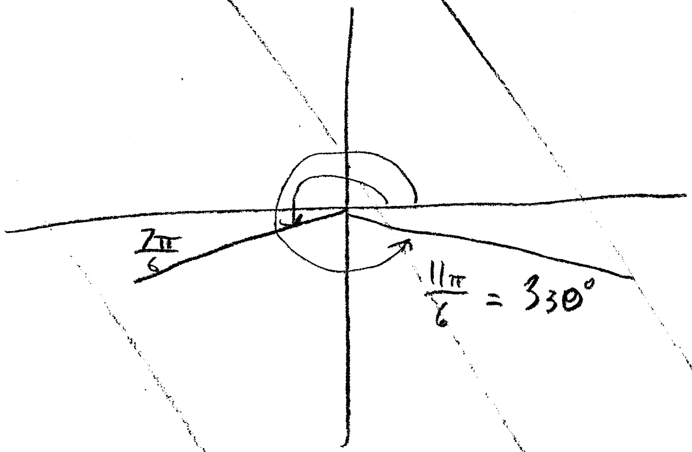
not on quiz 7

Solns in $[0, 2\pi)$ are $\frac{7\pi}{6}$ $\frac{11\pi}{6}$

$2\sin\theta = 2-3$

$2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$



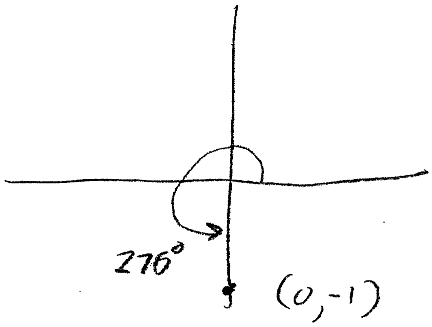
(17) $\sin(3\theta) = -1$

Let $x = 3\theta$. Then

$\sin 3\theta = -1$ is

equiv. to $\sin x = -1$,

(Now solve for x over $[0, 2\pi)$)



$x = \frac{3\pi}{2} + 2\pi k$

$\left(\frac{1}{3}\right) 3\theta = \frac{1}{3}\left(\frac{3\pi}{2} + 2\pi k\right)$

$\theta = \frac{\pi}{2} + \frac{2\pi k}{3}$

$= \frac{3\pi}{6} + \frac{4\pi k}{6} = \frac{3\pi + 4\pi k}{6}$

$= \frac{(3+4k)\pi}{6}$

When $k=0$, $\theta = \frac{\pi}{2}$

When $k=1$, $\theta = \frac{7\pi}{6}$

When $k=2$, $\theta = \frac{11\pi}{6}$

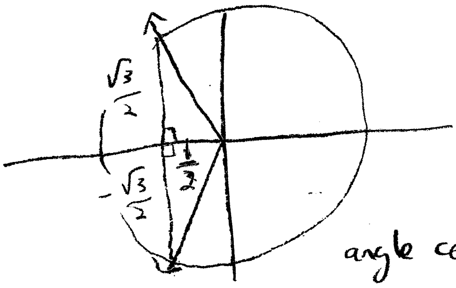
When $k=3$, $\theta = \frac{15\pi}{6} = \frac{5\pi}{2} \notin [0, 2\pi)$

$$\boxed{\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}}$$

21. $\sec\left(\frac{3\theta}{2}\right) = -2$

let $x = \frac{3\theta}{2}$ then solve $\sec(x) = -2$,

or $\cos(x) = -\frac{1}{2}$ cosine is negative in Q2 & Q3.



There are 2 solns for x over $[0, 2\pi)$, namely $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. Any angle coterminal to these 2 angles is also a soln. Solns are not separated by the same distance. All solns, x are then

$$x = \frac{2\pi}{3} + 2\pi k \quad \text{or} \quad x = \frac{4\pi}{3} + 2\pi k$$

$$x = \frac{2\pi}{3} + 2\pi k \quad \text{or} \quad x = \frac{4\pi}{3} + 2\pi k$$

$$\frac{3\theta}{2} = \frac{2\pi}{3} + 2\pi k \quad \text{or} \quad \frac{3\theta}{2} = \frac{4\pi}{3} + 2\pi k$$

$$\theta = \frac{2}{3}\left(\frac{2\pi}{3} + 2\pi k\right) \quad \text{or} \quad \theta = \frac{2}{3}\left(\frac{4\pi}{3} + 2\pi k\right)$$

$$\theta = \frac{4\pi}{9} + \frac{4\pi k}{3} \quad \text{or} \quad \theta = \frac{8\pi}{9} + \frac{4\pi k}{3}$$

$$\theta = \frac{4\pi + 12\pi k}{9} \quad \text{or} \quad \theta = \frac{8\pi + 12\pi k}{9}$$

$$\theta = (4 + 12k)\frac{\pi}{9} \quad \text{or} \quad \theta = (8 + 12k)\cdot\frac{\pi}{9}$$

$k=0$, $\theta = \frac{4\pi}{9}$	$k=0$, $\theta = \frac{8\pi}{9}$
$k=1$, $\theta = \frac{16\pi}{9}$	$k=1$, $\theta = \frac{14\pi}{9}$ not in $[0, 2\pi)$
$k=2$, $\theta = \frac{28\pi}{9}$ not in $[0, 2\pi)$	$\boxed{\left\{\frac{4\pi}{9}, \frac{8\pi}{9}, \frac{16\pi}{9}\right\}}$

30. $4\sin\theta + 3\sqrt{3} = \sqrt{3}$

$$4\sin\theta + 3\sqrt{3} = 1\cdot\sqrt{3}$$

$$4\sin\theta = 1\cdot\sqrt{3} - 3\cdot\sqrt{3}$$

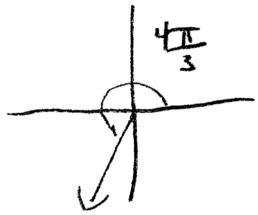
$$4\sin\theta = (1-3)\sqrt{3}$$

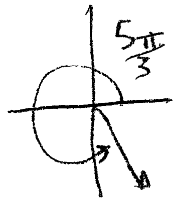
$$4\sin\theta = -2\sqrt{3}$$

$$\sin\theta = \frac{-2\sqrt{3}}{4}$$

$$\sin\theta = -\frac{\sqrt{3}}{2}$$

$\sin\theta < 0$ in Q3, Q4



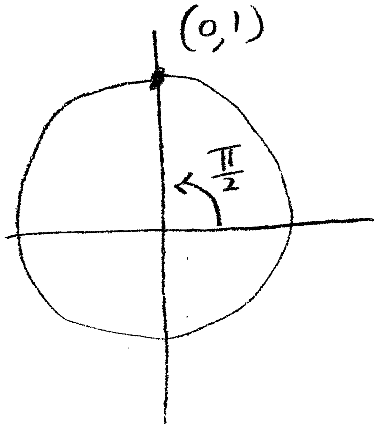


solns of $\sin\theta = -\frac{\sqrt{3}}{2}$ are $\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

or $\{240°, 300°\}$

(32) Solve $\sin\left(3\theta + \frac{\pi}{18}\right) = 1$ over $[0, 2\pi)$.

Let $X = 3\theta + \frac{\pi}{18}$, then $\sin\left(3\theta + \frac{\pi}{18}\right) = 1$ is equivalent to $\sin(x) = 1$. Then $\sin(x) = 1$ has solutions when $X = \frac{\pi}{2} + 2\pi k$ for $k \in \mathbb{Z}$.



Now backsubstitute $3\theta + \frac{\pi}{18}$ for X in the solution statement.

$\sin\left(3\theta + \frac{\pi}{18}\right) = 1$ has solutions

$$3\theta + \frac{\pi}{18} = \frac{\pi}{2} + 2\pi k, \text{ for } k \in \mathbb{Z}, \text{ or equivalently}$$

$$3\theta = -\frac{\pi}{18} + \frac{\pi}{2} + 2\pi k, \text{ or}$$

$$3\theta = -\frac{\pi}{18} + \frac{\pi}{2} \cdot \frac{9}{9} + \frac{2\pi k}{1} \cdot \frac{18}{18}, \text{ or}$$

$$3\theta = \frac{-1\pi + 9\pi + 36\pi k}{18}, \text{ or}$$

$$3\theta = \frac{8\pi + 36\pi k}{18}$$

$$3\theta = \frac{(4\pi + 18\pi k) \cdot 2}{18}$$ ← reduce. divide top and bottom by 2

$$3\theta = \frac{4\pi + 18\pi k}{9}$$

#32 continued

$$3\theta = \frac{(4 + 18k)\pi}{9}$$

$$3\theta = \frac{\pi}{9}(4 + 18k)$$

$$\frac{1}{3}\cdot 3\theta = \frac{1}{3}\cdot\frac{\pi}{9}(4+18k)$$

$$\theta = \frac{\pi}{27}(4+18k) \quad \text{or} \quad \theta = (4+18k)\cdot\frac{\pi}{27}$$

So, All the solns of the equation are in the set

$$\left\{\theta \mid \theta = (4+18k)\cdot\frac{\pi}{27},\ \text{where } k \in \mathbb{Z}\right\}.$$

To find solutions in the interval $[0, 2\pi)$, we let k take on integer values. Also, note that $\frac{54\pi}{27} = 2\pi$.

When $k = -1$, $\theta = \frac{-14\pi}{27}$ which is not in $[0, 2\pi)$.

When $k = 0$, $\theta = \frac{4\pi}{27}$

When $k = 1$, $\theta = \frac{22\pi}{27}$

When $k = 2$, $\theta = \frac{40\pi}{27}$

When $k = 3$, $\theta = \frac{58\pi}{27}$, which is greater than $\frac{54\pi}{27} = 2\pi$

The solns in the interval $[0, 2\pi)$ are $\left\{\frac{4\pi}{27}, \frac{22\pi}{27}, \frac{40\pi}{27}\right\}$

(37) $\tan\theta = -\frac{\sqrt{3}}{3}$

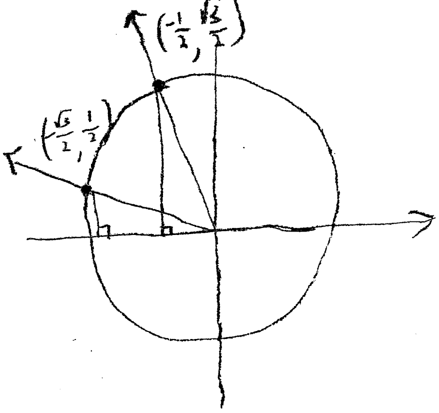
tangent is negative in Q2 and Q4.

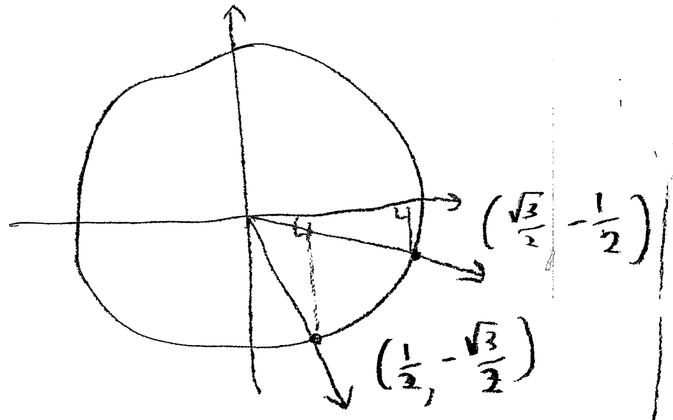
$\tan\theta = \frac{y}{x}$

$x<0$, $y>0$	$x>0$, $y>0$
$x<0$, $y<0$	$x>0$, $y<0$

So, solns in the interval will be in Q2 and Q4

Since "$\sqrt{3}$" appears in the tangent we are dealing with a multiple of a 30° or 60° type angle.





$\frac{2\pi}{3}$ or $\frac{5\pi}{6}$

$\frac{5\pi}{3}$ or $\frac{11\pi}{6}$

$\tan\frac{5\pi}{6} = -\frac{1}{2} \div \frac{\sqrt{3}}{2}$

$= -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

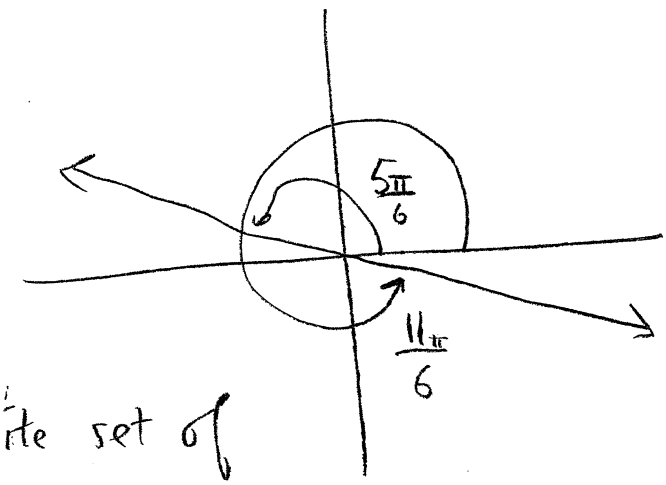
$\tan\frac{11\pi}{6} = -\frac{1}{2} \div \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{3}$

So solns in $[0, 2\pi)$ are $\left\{\frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

or $\{150°, 330°\}$

(37) Any angle coterminal to $\frac{5\pi}{6}$ or $\frac{11\pi}{6}$

is also a soln.



And solns are

separated by 180°

or π radians. The infinite set of

solutions is then $\{\theta \mid \theta = \frac{5\pi}{6} + k\pi$, where k is any integer$\}$

We can let K take on any 6 integer values to find 6 solutions.

Then, $\theta = \frac{5\pi}{6} + k\pi \cdot \frac{6}{6} = \frac{5\pi}{6} + \frac{6k\pi}{6} = \frac{5\pi + 6k\pi}{6}$, or

$$\theta = \frac{(5+6k)\cdot\pi}{6} \quad \text{or} \quad \boxed{\theta = (5+6k)\cdot\frac{\pi}{6}}$$

When $k=0$, $\theta = \frac{5\pi}{6}$;

When $k=1$, $\theta = \frac{11\pi}{6}$

When $k=2$, $\theta = \frac{17\pi}{6}$

$k=3$, $\theta = \frac{23\pi}{6}$

$k=4$, $\theta = \frac{29\pi}{6}$

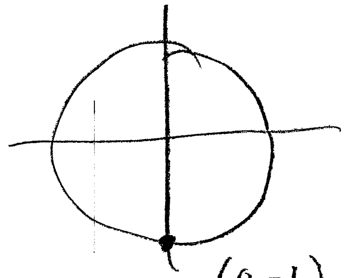
$k=5$, $\theta = \frac{35\pi}{6}$

6 solns

$$\left\{\frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}, \frac{23\pi}{6}, \frac{29\pi}{6}, \frac{35\pi}{6}\right\}$$

42) $\sin(2\theta) = -1$

Let $x = 2\theta$, then solve $\sin(x) = -1$.



$$x = \frac{3\pi}{2} + 2\pi k, \quad k \in \mathbb{Z}$$

$$2\theta = \frac{3\pi}{2} + 2\pi k$$

$$\frac{1}{2} \cdot 2\theta = \frac{1}{2}\left(\frac{3\pi}{2} + 2\pi k\right)$$

$$\theta = \frac{3\pi}{4} + \pi k$$

$$\theta = \frac{3\pi}{4} + \frac{4\pi k}{4}$$

$$\theta = (3+4k) \cdot \frac{\pi}{4}$$

When $k=0$, $\theta = \frac{3\pi}{4}$

When $k=1$, $\theta = \frac{7\pi}{4}$

When $k=2$, $\theta = \frac{11\pi}{4}$

When $k=3$, $\theta = \frac{15\pi}{4}$

$k=4$, $\theta = \frac{19\pi}{4}$

$k=5$, $\theta = 23\pi$

All solns:

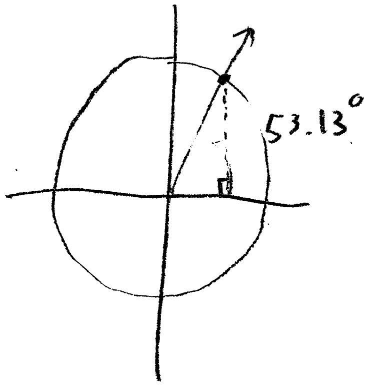
$$\left\{\theta \mid \theta = (3+4k)\cdot\frac{\pi}{4}, k \in \mathbb{Z}\right\}$$

6 solns ↓

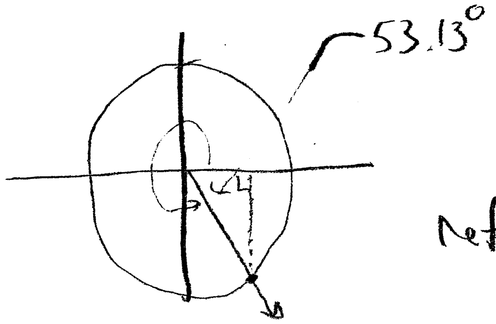
$$\left\{\frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}, \frac{19\pi}{4}, \frac{23\pi}{4}\right\}$$

(46) Solve $\cos\theta = 0.6$ over $[0, 2\pi)$ with a calculator

$\theta = \cos^{-1}(0.6) \doteq 53.13^\circ$



But cosine is also positive in Q4



Both angles have the same reference angle.

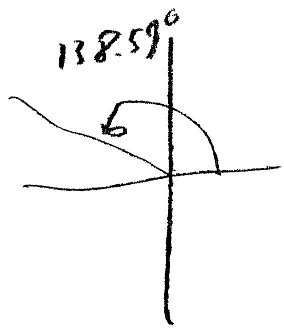
The other soln over $[0, 2\pi)$ is then $360^\circ - 53.13^\circ$
$= 306.86^\circ$

$$\boxed{\{53.13^\circ, 306.86^\circ\}}$$

Solve

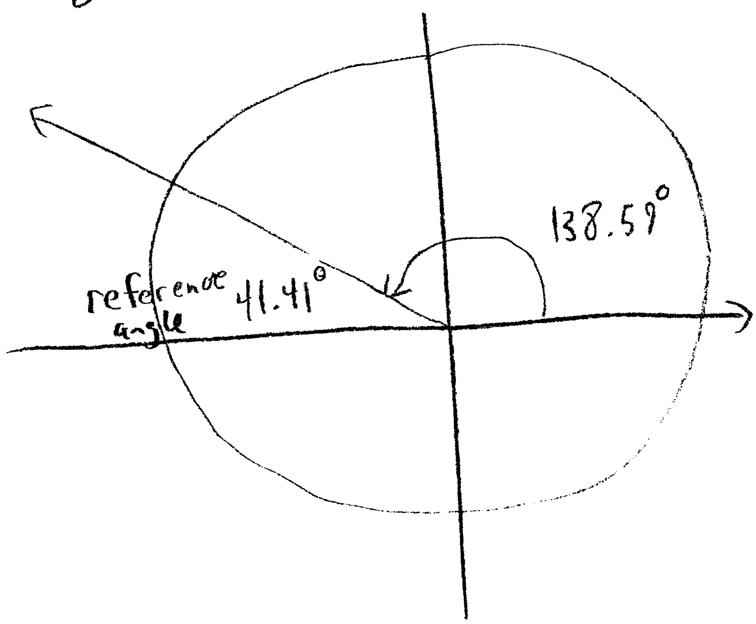
56) $4\cos\theta + 3 = 0$ over $[0, 2\pi)$ with a calculator

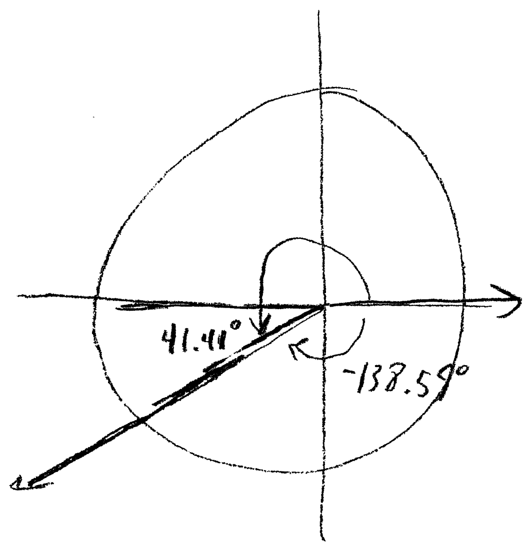
$$\cos\theta = -\frac{3}{4}$$



$$\theta = \cos^{-1}(-3/4) = 138.59°$$

But cosine is negative in Q3 too, so there is another soln in $[0, 2\pi)$. Both solns will have the same reference angle.





So the other soln is $180° + 41.41°$ or $360° - 138.59°$

Two solns in $[0, 2\pi)$

Approximated to 2 decimal places

$$\boxed{\{138.59°, 221.41°\}}$$

58. Solve

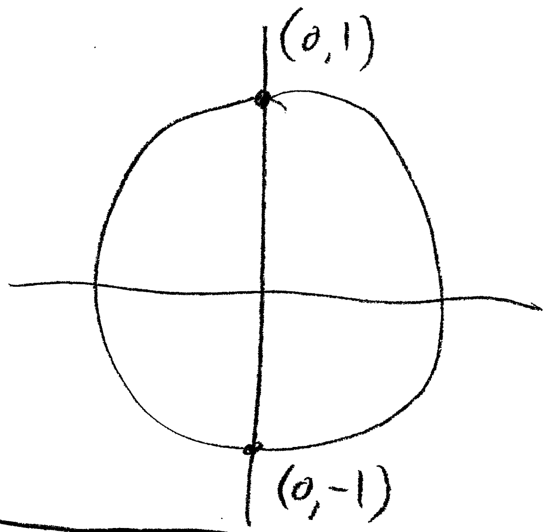
$\sin^2\theta - 1 = 0$ over $[0, 2\pi)$

$\sin^2\theta = 1$

$(\sin\theta)^2 = 1$

$\sin\theta = \pm\sqrt{1}$

$\sin\theta = -1$ or $\sin\theta = 1$



2 solns in $[0, 2\pi)$ $\boxed{\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\} \text{ or } \left\{90^\circ, 270^\circ\right\}}$

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64. Solve $\cos^2\theta - \sin^2\theta + \sin\theta = 0$ over $[0, 2\pi)$

Substitute $1 - \sin^2\theta$ in place of $\cos^2\theta$, then an equivalent equation is

$1 - \sin^2\theta - \sin^2\theta + \sin\theta = 0$ Let $x = \sin\theta$, then an equiv eqn is

$1 - x^2 - x^2 + x = 0$, or

$1 - 2x^2 + x = 0$, or

$-2x^2 + x + 1 = 0$, or multiplying by -1

$2x^2 - x - 1 = 0$

(64) Continued

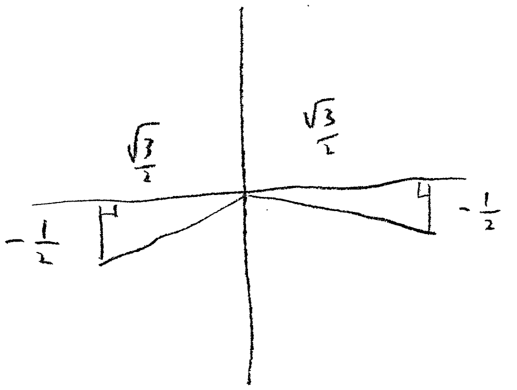
So, $2x^2 - x - 1 = 0$ is equiv to

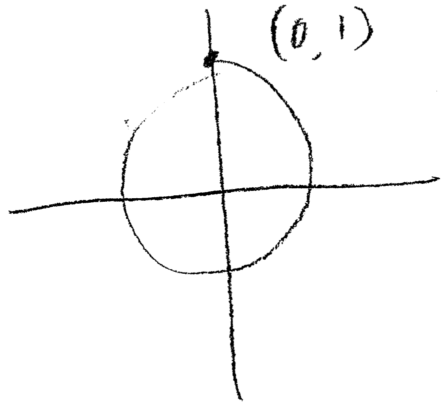
$(2x+1)(x-1) = 0$. The zero product property

implies $2x+1 = 0$ or $x - 1 = 0$, or equivalently

that $x = -\frac{1}{2}$ or $x = 1$, or that

$\sin\theta = -\frac{1}{2}$ or $\sin\theta = 1$





$\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ OR $\left\{\frac{\pi}{2}\right\}$

3 solns over $[0, 2\pi)$ $\boxed{\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\} \text{ or } \{90°, 210°, 330°\}}$

65

Solve

$\sin^2\theta = 6(\cos(-\theta)+1)$ over $[0, 2\pi)$

$\sin^2\theta = 6\cos\theta + 6$ since $a(b+c) = ab+ac$ and $\cos(-\theta) = \cos\theta$

$1-\cos^2\theta = 6\cos\theta + 6$ Replace $\sin^2\theta$ with $1-\cos^2\theta$

Now let $x = \cos\theta$, then an equiv eqn is

$1 - x^2 = 6x + 6$, or

$x^2 + 6x + 5 = 0$, or

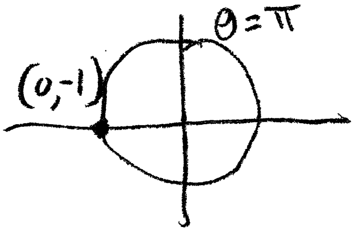
$(x+1)(x+5) = 0$, or

$x+1 = 0$ or $x+5 = 0$

by the zero product property

$x = -1$ or $x = -5$

$\cos\theta = -1$ or $\cos\theta = -5$



but $\cos\theta \neq -5$ since $-1 \leq \cos\theta \leq 1$

So, the only soln over $[0, 2\pi)$ is

$\boxed{\{\pi\}}$

74 solve $2\cos^2\theta - 7\cos\theta - 4 = 0$ over $[0, 2\pi)$

Let $x = \cos\theta$, then $2x^2 - 7x - 4 = 0$, or

$(2x+1)(x-4) = 0$, or

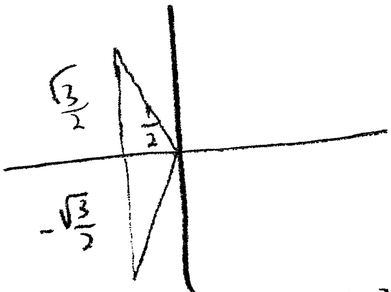
$2x+1=0$ or $x-4=0$

$2x = -1$ or $x = 4$

$x = -\frac{1}{2}$ or $x = 4$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = 4$

no soln since $-1 \le \cos\theta \le 1$



$$\boxed{\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\} \text{ or } \{120^\circ, 240^\circ\}}$$