Math 101 — Quiz 9
TUTOR HELP NOT OKAY!!

Name: Key

Directions: **Due Monday, Mar. 23rd**

- You are not to work on this together or get help from mentors/tutors!!
- Show all work.
- Write your solutions on printer paper.
- Only use a calculator if the question specifically says to use one!
- Box off your solutions.
- No credit will be given for sloppy or illegible work.
- Do not write your solutions on this piece of paper.
- Staple your worked out solutions to this piece of paper and use this piece of paper as your top (or cover) sheet.

Exercises

1. **Section 3.5**

   Do problems 15, 16, 20, 32, 36, 62 and 78 on pages 227 – 228 in the textbook.

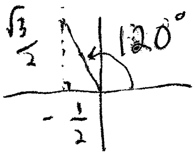
2. **Section 3.6**

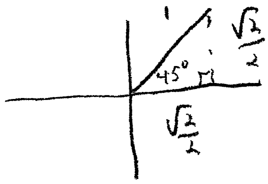
   Do problems 10, 12, 22, 24, 28, 50, 71 and 75 on pages 237 – 238 in the textbook.

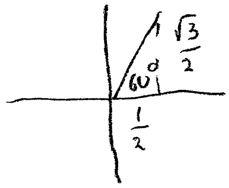
(15) $\cos(165°) = \cos(120°+45°)$

$$= \cos(120°)\cos 45° - \sin 120° \sin 45°$$

$$= -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{-\frac{1}{4}\left(\sqrt{2}+\sqrt{6}\right)}$$





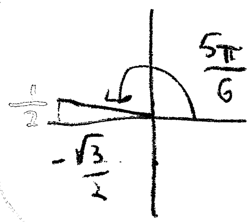


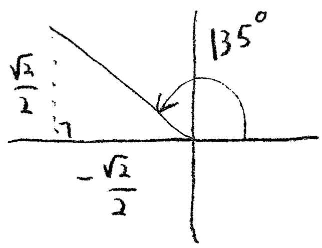
(16) $\sin(105°) = \sin(60°+45°)$

$$= \sin 60° \cos 45° + \sin 45° \cos 60°$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)}$$

(20) $\tan\frac{19\pi}{12} = \tan\left(\frac{10\pi}{12} + \frac{9\pi}{12}\right)$





$$= \tan\left(\frac{5\pi}{6} + \frac{3\pi}{4}\right)$$

$$= \frac{\tan\left(\frac{5\pi}{6}\right) + \tan\left(\frac{3\pi}{4}\right)}{1 - \tan\left(\frac{5\pi}{6}\right)\cdot\tan\left(\frac{3\pi}{4}\right)}$$

$$\tan\frac{5\pi}{6} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$\tan\frac{3\pi}{4} = -1$$

$$= \frac{-\frac{1}{\sqrt{3}} + -1}{1-\left(-\frac{1}{\sqrt{3}}\right)(-1)} = \frac{-1-\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}\cdot\frac{-\sqrt{3}}{-\sqrt{3}}$$

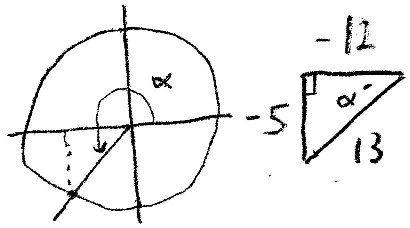
$$= \frac{\sqrt{3}+1}{-\sqrt{3}+1} = \frac{1+\sqrt{3}}{1-\sqrt{3}}\cdot\frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{1+2\sqrt{3}+3}{1-3} = \frac{4+2\sqrt{3}}{-2}$$

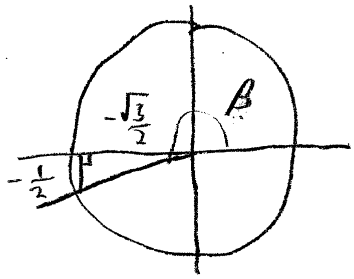
$$\boxed{-2+\sqrt{3}}$$

(32) $\sin\frac{\pi}{18}\cos\frac{5\pi}{18} + \cos\frac{\pi}{18}\sin\frac{5\pi}{18} = \sin\left(\frac{\pi}{18}+\frac{5\pi}{18}\right) = \sin\frac{6\pi}{18} = \sin\frac{\pi}{3} = \boxed{\frac{\sqrt{3}}{2}}$

(36) $\tan\alpha = \frac{5}{12}, \quad \pi < \alpha < \frac{3\pi}{2}$

$\sin\beta = -\frac{1}{2} \quad \pi < \beta < \frac{3\pi}{2}$





$\cos\alpha = -\frac{12}{13}$	$\cos\beta = -\frac{\sqrt{3}}{2}$
$\sin\alpha = \frac{-5}{13}$	$\sin\beta = -\frac{1}{2}$
$\tan\alpha = \frac{5}{12}$	$\tan\beta = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

(a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha = \frac{-5}{13}\cdot\frac{-\sqrt{3}}{2} + \frac{-1}{2}\cdot\frac{-12}{13} = \frac{5\sqrt{3}}{26} + \frac{12}{26}$

$= \boxed{\frac{1}{26}\left(12+5\sqrt{3}\right)}$

(b) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= -\frac{12}{13}\cdot\frac{-\sqrt{3}}{2} - \frac{-5}{13}\cdot\frac{-1}{2} = \frac{12\sqrt{3}}{26} - \frac{5}{26} = \boxed{\frac{1}{26}\left(12\sqrt{3}-5\right)}$

(c) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$= \frac{-5}{13}\cdot\frac{-\sqrt{3}}{2} - \frac{-1}{2}\cdot\frac{-12}{13} = \frac{5\sqrt{3}}{26} - \frac{12}{26} = \boxed{\frac{1}{26}\left(5\sqrt{3}-12\right)}$

(d) $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{\frac{5}{12}-\frac{\sqrt{3}}{3}}{1+\frac{5}{12}\cdot\frac{\sqrt{3}}{3}} \cdot \frac{36}{36}$

$= \frac{15-12\sqrt{3}}{36+5\sqrt{3}} \cdot \frac{36-5\sqrt{3}}{36-5\sqrt{3}} = \frac{540-507\sqrt{3}+180}{1296-75} = \frac{702-507\sqrt{3}}{1221} \begin{matrix}\div 3\\ \div 3\end{matrix}$

$= \boxed{\frac{240-169\sqrt{3}}{407}}$

(62) Prove $\dfrac{\cos(\alpha-\beta)}{\sin\alpha\cos\beta} = \cot\alpha + \tan\beta$

$$LHS = \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\cos\beta} = \frac{\cos\alpha\cancel{\cos\beta}}{\sin\alpha\cancel{\cos\beta}} - \frac{\cancel{\sin\alpha}\sin\beta}{\cancel{\sin\alpha}\cos\beta} =$$

$$= \frac{\cos\alpha}{\sin\alpha} - \frac{\sin\beta}{\cos\beta} = \cot\alpha + \tan\beta \quad ✓$$

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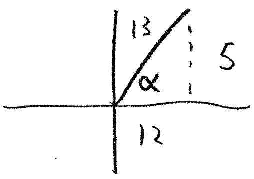
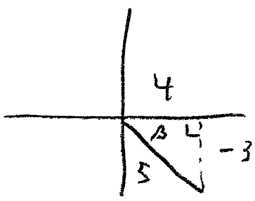
(78) $\cos\left[\tan^{-1}\left(\frac{5}{12}\right) - \sin^{-1}\left(-\frac{3}{5}\right)\right]$ Let $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$ and $\beta = \sin^{-1}\left(-\frac{3}{5}\right)$.

Then $\tan\alpha = \frac{5}{12}$ (with α in Q1) and $\sin\beta = \frac{-3}{5}$ (with β in Q4)





Moreover,

$$\cos\left[\tan^{-1}\left(\frac{5}{12}\right) - \sin^{-1}\left(-\frac{3}{5}\right)\right] = \cos(\alpha-\beta)$$

$$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$= \frac{12}{13}\cdot\frac{4}{5} + \frac{5}{13}\cdot\frac{-3}{5}$$

$$= \frac{48}{65} + \frac{-15}{65} = \boxed{\frac{33}{65}}$$

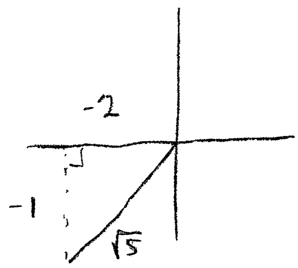
$$\begin{array}{r} 48 \\ -15 \\ \hline 33 \end{array}$$

(10) $\tan\theta = \frac{1}{2}, \pi < \theta < \frac{3\pi}{2}$

(a) $\sin 2\theta = 2\sin\theta\cos\theta$

$= \frac{2}{1}\cdot\left(-\frac{1}{\sqrt5}\right)\left(-\frac{2}{\sqrt5}\right)$

$= \boxed{\frac{4}{5}}$



$2^2 + 1^2 = c^2$

$5 = c^2$

$\sqrt5 = c$

(b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{2}{\sqrt5}\right)^2 - \left(\frac{-1}{\sqrt5}\right)^2 = \frac{4}{5} - \frac{1}{5} = \boxed{\frac{3}{5}}$

(c) Multiply $\pi < \theta < \frac{3\pi}{2}$ by $\frac{1}{2}$ to get an estimate on $\frac{1}{2}\theta$.

Then, $\frac{\pi}{2} < \frac{1}{2}\theta < \frac{3\pi}{4}$ or $\frac{1}{2}\theta$ is between $90°$ & $135°$.

This places $\frac{1}{2}\theta$ in Q2. Then,

$\sin\frac{1}{2}\theta = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1 - \frac{-2}{\sqrt5}}{2}} = \sqrt{\frac{1}{2}\left(1+\frac{2}{\sqrt5}\right)} = \sqrt{\frac{1}{2} + \frac{1}{\sqrt5}}$

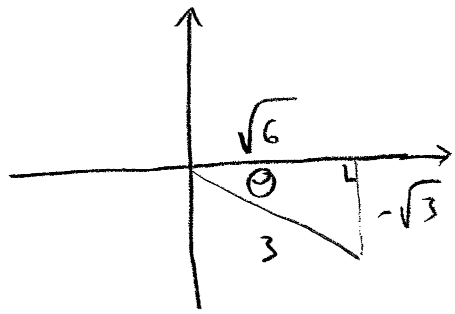
$= \sqrt{\frac{1}{2} + \frac{1}{\sqrt5}\cdot\frac{\sqrt5}{\sqrt5}} = \sqrt{\frac{1}{2} + \frac{\sqrt5}{5}} = \sqrt{\frac{5}{10} + \frac{2\sqrt5}{10}} = \boxed{\sqrt{\frac{5+2\sqrt5}{10}}}$

(d) $\cos\frac{1}{2}\theta = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1}{2}\left(1-\frac{2}{\sqrt5}\right)} = -\sqrt{\frac{1}{2} - \frac{1}{\sqrt5}}$

$= -\sqrt{\frac{1}{2} - \frac{1}{\sqrt5}\cdot\frac{\sqrt5}{\sqrt5}} = -\sqrt{\frac{1}{2} - \frac{\sqrt5}{5}} = -\sqrt{\frac{5}{10} - \frac{2\sqrt5}{10}} = \boxed{-\sqrt{\frac{5-2\sqrt5}{10}}}$

(12) $\sin\theta = -\frac{\sqrt{3}}{3}$, $\frac{3\pi}{2} < \theta < 2\pi$

θ is in Q4



$a^2 + (-\sqrt{3})^2 = 3^2$

$a^2 + 3 = 9$

$a^2 = 6$

$a = \sqrt{6}$

$\sin\theta = -\frac{\sqrt{3}}{3}$, $\cos\theta = \frac{\sqrt{6}}{3}$

Also, multiplying $\frac{3\pi}{2} < \theta < 2\pi$ by $\frac{1}{2}$ gives $\frac{3\pi}{4} < \frac{\theta}{2} < \pi$,

or that $135° < \frac{1}{2}\theta < 180°$. This places $\frac{1}{2}\theta$ in Q2.

(a) $\sin 2\theta = 2\sin\theta\cos\theta = \frac{2}{1}\cdot\frac{-\sqrt{3}}{3}\cdot\frac{\sqrt{6}}{3} = \frac{-2\sqrt{18}}{9} = \frac{-2\cdot 3\sqrt{2}}{9} = \boxed{\frac{-2\sqrt{2}}{3}}$

(b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{\sqrt{6}}{3}\right)^2 - \left(\frac{-\sqrt{3}}{3}\right)^2 = \frac{6}{9} - \frac{3}{9} = \frac{3}{9} = \boxed{\frac{1}{3}}$

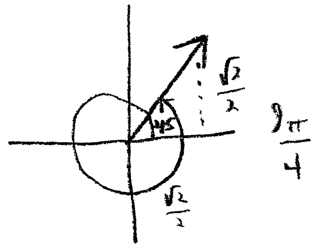
(c) $\sin\frac{\theta}{2} = \sqrt{\frac{1}{2}(1-\cos\theta)} = \sqrt{\frac{1}{2}\left(1-\frac{\sqrt{6}}{3}\right)} = \sqrt{\frac{1}{2} - \frac{\sqrt{6}}{6}} = \sqrt{\frac{3}{6} - \frac{\sqrt{6}}{6}} = \boxed{\sqrt{\frac{3-\sqrt{6}}{6}}}$

(d) $\cos\frac{\theta}{2} = -\sqrt{\frac{1}{2}(1+\cos\theta)} = -\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{6}}{3}\right)} = -\sqrt{\frac{1}{2} + \frac{\sqrt{6}}{6}} = -\sqrt{\frac{3}{6} + \frac{\sqrt{6}}{6}}$

$= \boxed{-\sqrt{\frac{3+\sqrt{6}}{6}}}$

(22) $\tan \frac{9\pi}{8} = \tan\left(\frac{1}{2} \cdot \frac{9\pi}{4}\right)$

$= \tan\left(\frac{1}{2}\alpha\right)$ with $\alpha = \frac{9\pi}{4}$



$$= \frac{1-\cos\alpha}{\sin\alpha} = \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} \cdot \frac{2}{2} = \frac{2-\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} = \frac{2}{\sqrt{2}} - 1 = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} - 1 = \frac{2\sqrt{2}}{2} - 1$$

$$= \boxed{\sqrt{2} - 1}$$

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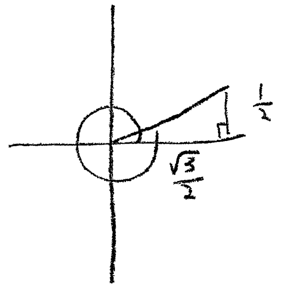
(24) $\sin 195° = \sin\left(\frac{1}{2} \cdot 390°\right)$

$= \sin \frac{1}{2}\alpha$ with $\alpha = 390°$



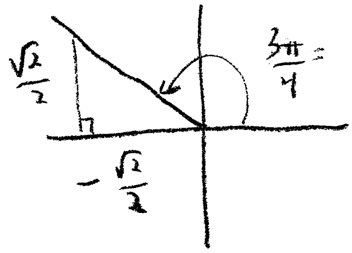
$$= -\sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1}{2}\left(1-\frac{\sqrt{3}}{2}\right)}$$

$$= -\sqrt{\frac{1}{2} - \frac{\sqrt{3}}{4}} = \sqrt{\frac{2}{4} - \frac{\sqrt{3}}{4}}$$

$$= -\sqrt{\frac{2-\sqrt{3}}{4}} = -\frac{\sqrt{2-\sqrt{3}}}{\sqrt{4}} = -\frac{\sqrt{2-\sqrt{3}}}{2} = \boxed{\frac{-1}{2}\sqrt{2-\sqrt{3}}}$$

(28) $\cos\left(-\frac{3\pi}{8}\right) = \cos\frac{3\pi}{8}$ since $\cos(-\theta) = \cos\theta$

$$= \cos\left(\frac{1}{2}\cdot\frac{3\pi}{4}\right)$$



$$= \cos\tfrac{1}{2}\alpha \text{ with } \alpha = \frac{3\pi}{4}$$

$$= -\sqrt{\tfrac{1}{2}(1+\cos\alpha)}$$

$$= -\sqrt{\tfrac{1}{2}\left(1-\tfrac{\sqrt{2}}{2}\right)} = -\sqrt{\tfrac{1}{2}-\tfrac{\sqrt{2}}{4}}$$

$$= -\sqrt{\tfrac{2}{4}-\tfrac{\sqrt{2}}{4}} = -\sqrt{\frac{2-\sqrt{2}}{4}}$$

$$= -\frac{\sqrt{2-\sqrt{2}}}{\sqrt{4}} = -\frac{\sqrt{2-\sqrt{2}}}{2} = \boxed{-\frac{1}{2}\sqrt{2-\sqrt{2}}}$$

(50) $\cot 2\theta = \frac{1}{2}(\cot\theta - \tan\theta)$

$$\text{LHS} = \frac{1}{\tan 2\theta} = \frac{1}{\frac{2\tan\theta}{1-\tan^2\theta}} = \frac{1-\tan^2\theta}{2\tan\theta} = \frac{1}{2}\left(\frac{1-\tan^2\theta}{\tan\theta}\right)$$

$$= \frac{1}{2}\left[\frac{1}{\tan\theta} - \frac{\tan^2\theta}{\tan\theta}\right] = \frac{1}{2}(\cot\theta - \tan\theta)$$

✓

71 Solve $\cos 2\theta = \cos\theta$ over $[0, 2\pi)$

$2\cos^2\theta - 1 = \cos\theta$

$2\cos^2\theta - \cos\theta - 1 = 0$ Let $x = \cos\theta$

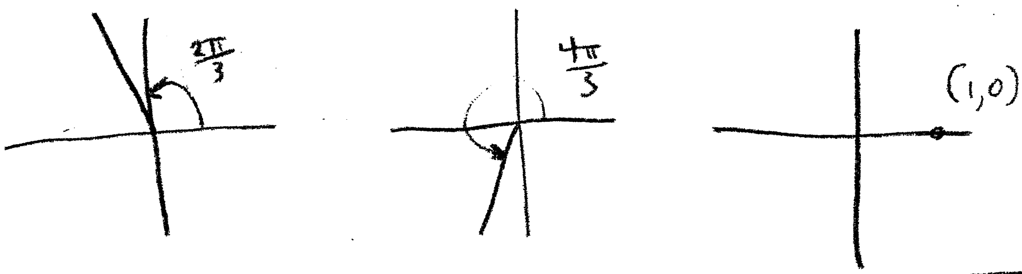
$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$\Rightarrow 2x+1 = 0$ or $x - 1 = 0$

$\Rightarrow x = -\frac{1}{2}$ or $x = 1$

$\Rightarrow \cos\theta = -\frac{1}{2}$ or $\cos\theta = 1$



$$\boxed{\text{Solns } \theta \in \left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\} = \left\{0^\circ, 120^\circ, 240^\circ\right\}}$$

(75) Solve $3 - \sin\theta = \cos 2\theta$ over $[0, 2\pi)$

$3 - \sin\theta = 1 - 2\sin^2\theta$, or

$2\sin^2\theta - \sin\theta + 2 = 0$ Let $x = \sin\theta$

$2x^2 - x + 2 = 0$

The quadratic is not factorable, and the quadratic formula produces complex solns. We know that $x = \sin\theta$ can only take on real numbered values, therefore the eqn has no solns.