Ch. 2 Take-home part   Name: Key

Double check your solutions! Turn this in before class starts tomorrow morning. You are not allowed to work together or collaborate on this! You are encouraged to use a calculator on this part, but Show All of Your Work.

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1. (1 point) Use a calculator to find the exact value of $\cos^{-1}(0.23)$ in degrees. Round your answer to two decimal places

   1. $76.70°$

2. (1 point) Use a calculator to find the exact value of $\sin^{-1}(0.11)$ in degrees. Round your answer to two decimal places

   2. $6.32°$

3. (1 point) Use a calculator to find the exact value of $\tan^{-1}(0.57)$ in degrees. Round your answer to two decimal places

   3. $29.68°$

4. (1 point) Use a calculator to find the exact value of $\sec^{-1}(5.2)$ in degrees. Round your answer to two decimal places

   $= \cos^{-1}\left(\frac{1}{5.2}\right)$

   4. $78.91°$

5. (1 point) Use a calculator to find the exact value of $\csc^{-1}(3.23)$ in degrees. Round your answer to two decimal places

   5. $18.03°$

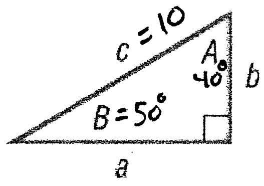
6. (1 point) Use a calculator to find the exact value of $\cot^{-1}(0.35)$ in degrees. Round your answer to two decimal places

   6. $70.71°$

7. (1 point) Use a calculator to find the exact value of $\cot^{-1}(-9.2)$ in degrees. Round your answer to two decimal places

   7. $173.80°$

8. (6 points) Use the given information to solve the right triangle. $c = 10$, and $A = 40°$. Find b, a and B



8. $\boxed{a = 6.4 \quad b = 7.7 \quad B = 50°}$

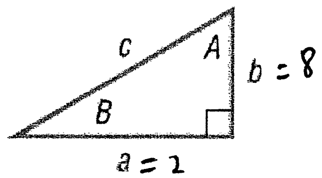
$$\sin 40° = \frac{a}{10}$$

$$10 \cdot \sin 40° = a \doteq 6.4$$

$$\sin 50° = \frac{b}{10}$$

$$10 \cdot \sin 50° = b \doteq 7.7$$

9. (6 points) Use the given information to solve the right triangle. $a = 2$, and $b = 8$. Find c, A and B.



9. $\boxed{A = 75.96° \quad B = 14.03° \quad c = 2\cdot\sqrt{17}}$

$$\tan B = \frac{8}{2}$$

$$B = \tan^{-1}(4) \doteq 75.96°$$

$$\tan A = \frac{2}{8}$$

$$A = \tan^{-1}\left(\frac{1}{4}\right) \doteq 14.03°$$

$$c^2 = 2^2 + 8^2$$

$$c^2 = 4 + 64$$

$$c^2 = 68$$

$$c = \sqrt{68} = \sqrt{4 \cdot 17} = \sqrt{4}\sqrt{17}$$

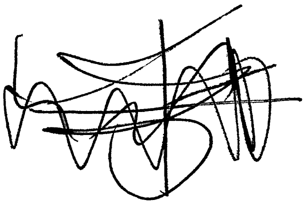
$$= 2\sqrt{17} \approx 8.25$$

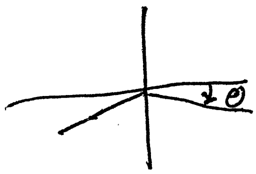
10. (4 points) Solve the equation on the interval $[0, 2\pi)$

$$\sin(\theta) = -0.24$$

10. $\left\{ 193.89^\circ,\ 346.89^\circ \right\}$

$\theta = \sin^{-1}(-0.24) \doteq -13.89^\circ$





Any angle coterminal to θ is also a soln. Add 360° to -13.89° to find the angle in $[0, 2\pi)$ that is coterminal. There is another angle in Q3 with the same reference angle as θ (13.89°).

$\theta_1 = -13.89^\circ + 360^\circ = \boxed{346.89^\circ}$

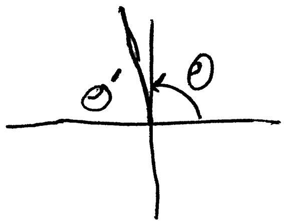
$\theta_2 = 180^\circ + 13.89^\circ = \boxed{193.89^\circ}$

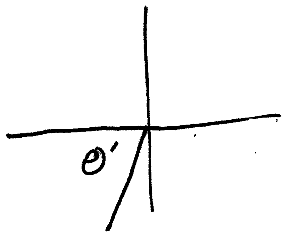
11. (4 points) Solve the equation on the interval $[0, 2\pi)$

$$\cos(\theta) = -0.31$$

11. $\left\{ 108.06^\circ,\ 251.94^\circ \right\}$

$\theta_1 = \cos^{-1}(-0.31) = 108.06^\circ$





The other solution has the same reference angle $\theta' = 180^\circ - 108.06^\circ = 71.94^\circ$

So, $\theta_2 = 180^\circ + 71.94^\circ = 251.94^\circ$

or $\theta_2 = 360^\circ - 108.06^\circ$

Test 2 in class part Name: Key

Double check your solutions! Use Algebraic Notation AND Show All of Your Work. No Assistance or Collaboration! You may not leave to use the restroom. No Calculator part.

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12. (6 points) Show that f and g are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$. Assume $f(x) = \dfrac{2x+3}{x+2}$ and $g(x) = \dfrac{3-2x}{x-2}$.

$$f(g(x)) = \frac{2\cdot\left(\frac{3-2x}{x-2}\right)+3}{\frac{3-2x}{x-2}+2}$$

$$= \frac{\frac{6-4x}{x-2}+\frac{3(x-2)}{x-2}}{\frac{3-2x}{x-2}+\frac{2(x-2)}{x-2}}$$

$$= \frac{6-4x+3x-6}{3-2x+2x-4}$$

$$= \frac{-x}{-1}$$

$$= x \checkmark$$

$$g(f(x)) = \frac{3-2\cdot\left(\frac{2x+3}{x+2}\right)}{\frac{2x+3}{x+2}-2}$$

$$= \frac{3+\frac{-4x-6}{x+2}}{\frac{2x+3}{x+2}-\frac{2(x+2)}{x+2}}$$

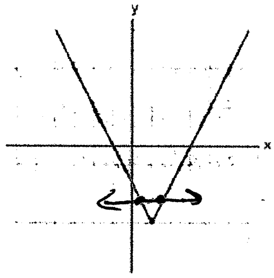
$$= \frac{\frac{3(x+2)}{x+2}+\frac{-4x-6}{x+2}}{\frac{2x+3-2x-4}{x+2}}$$

$$= \frac{3x+6+-4x-6}{-1}$$

$$= \frac{-x}{-1} = x \checkmark$$

Key

13. (5 points) The graph of a function f is given below. Use the horizontal line test to determine whether f is one-to-one. Is the function one-to-one??



13. NO

14. (3 points) Find the exact value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

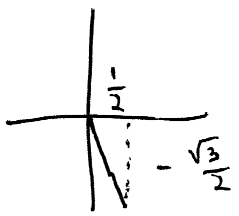
$\theta = \frac{5\pi}{3}$ is not in the range of inverse sine, which is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so we cant use $f^{-1}(f(x)) = x$ here.

14. $-\frac{\pi}{3}$

Instead,

$$\sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$= -\frac{\pi}{3}$$



15. (3 points) Find the exact value of $\tan\left(\tan^{-1}(0)\right)$

Since 0 is in the domain of $y = \tan^{-1} x$ and 0 is in the range of $y = \tan x$, we can use $f(f^{-1}(x)) = x$

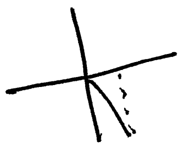
15. 0

17. (3 points) Find the exact value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

17. $\boxed{-\frac{\pi}{3}}$

Let $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ then

$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\theta = -\frac{\sqrt{3}}{2}$



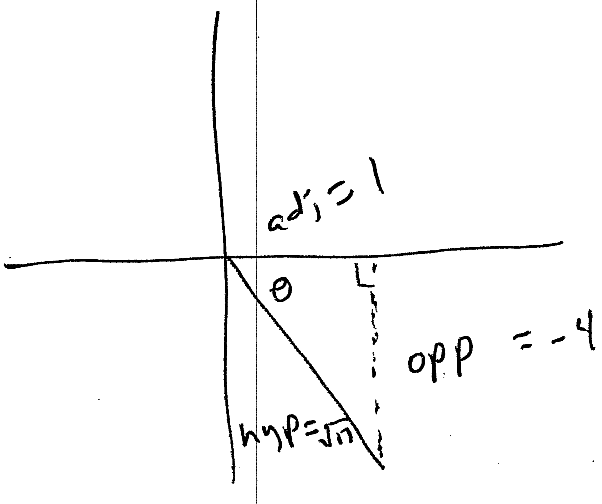
18. (4 points) Find the exact value of $\cos\left[\tan^{-1}(-4)\right]$

18. $\boxed{\frac{\sqrt{17}}{17}}$

Let $\theta = \tan^{-1}(-4)$

then $\tan\theta = -4$ and $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan\theta = \frac{opp}{adj} = \frac{-4}{1}$



$b^2 + a^2 = c^2$

$1^2 + 4^2 = c^2$

$17 = c^2$

$c = \sqrt{17}$

$hyp = \sqrt{17}$

$\cos\theta = \frac{adj}{hyp}$

$= \frac{1}{\sqrt{17}}$

$= \frac{1}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}}$

$= \frac{\sqrt{17}}{17}$

16. Assume $f(x) = -2\cos(4x)$ for domain $0 \le x \le \frac{\pi}{4}$ for parts a through d below.

(a) (2 points) Find the inverse function, f^{-1} of the function f defined above.

$$\boxed{f^{-1}(x) = \frac{1}{4}\cos^{-1}\left(-\frac{1}{2}x\right)}$$

(a)

$y = -2\cos(4x)$

$x = -2\cos(4y)$

$\frac{x}{-2} = \cos(4y)$ divide by -2

$-\frac{1}{2}x = \cos(4y)$

$\cos^{-1}\left(-\frac{1}{2}x\right) = \cos^{-1}(\cos(4y))$

$4y = \cos^{-1}\left(-\frac{1}{2}x\right)$

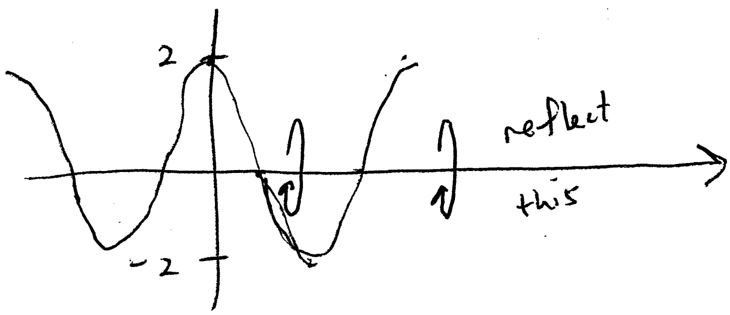
$\frac{1}{4}\cdot 4y = \frac{1}{4}\cos^{-1}\left(-\frac{1}{2}x\right)$

$y = \frac{1}{4}\cos^{-1}\left(-\frac{1}{2}x\right)$

(b) (2 points) Find the range of f.

(b) $[-2, 2]$

$y = -2\cos 4x$



(c) (1 point) Find the domain of f^{-1}. $[-2, 2]$

(c) $\cancel{[0, \frac{\pi}{4}]}$

(d) (1 point) Find the range of f^{-1}. $[0, \frac{\pi}{4}]$

(d) $\cancel{[-2, 2]}$

21. (6 points) Solve the equation on the interval $[0, 2\pi)$

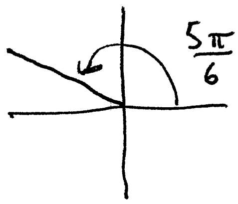
$$\cos(3\theta) = \frac{-\sqrt{3}}{2}$$

$\left\{ \frac{7\pi}{18}, \frac{5\pi}{18}, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18} \right\}$

21.

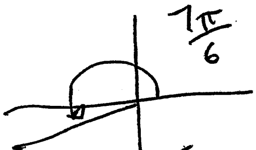
Let $x = 3\theta$, then solve $\cos x = -\frac{\sqrt{3}}{2}$.

cosine is negative in Q2 & Q3



We know $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$ and $\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

Solns are not separated by an equal distance.



$\left\{ x \mid x = \frac{5\pi}{6} + 2k\pi \right\} \cup \left\{ x \mid x = \frac{7\pi}{6} + 2k\pi \right\}$

Now resubstitute 3θ in place of x.

Then $3\theta = \frac{5\pi}{6} + 2\pi k$ or $3\theta = \frac{7\pi}{6} + 2k\pi$ are solns.

$\frac{1}{3} \cdot 3\theta = \frac{1}{3}\left(\frac{5\pi}{6} + 2\pi k\right)$ or $\frac{1}{3} \cdot 3\theta = \frac{1}{3}\left(\frac{7\pi}{6} + 2\pi k\right)$

$\theta = \frac{5\pi}{18} + \frac{2\pi k}{3}$ or $\theta = \frac{7\pi}{18} + \frac{2\pi k}{3}$

$\theta = \frac{5\pi}{18} + \frac{12\pi k}{18}$ or $\theta = \frac{7\pi}{18} + \frac{12\pi k}{18}$

note $\frac{36\pi}{18} = 2\pi$

$\theta = (5 + 12k)\frac{\pi}{18}$ or $\theta = (7 + 12k)\frac{\pi}{18}$

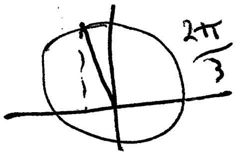
$k = 0,\ \theta = \frac{5\pi}{18}$	$k = 0,\ \theta = \frac{7\pi}{18}$
$k = 1,\ \theta = \frac{17\pi}{18}$	$k = 1,\ \theta = \frac{19\pi}{18}$
$k = 2,\ \theta = \frac{29\pi}{18}$	$k = 2,\ \theta = \frac{31\pi}{18}$
$k = 3,\ \theta = \frac{41\pi}{18} > 2\pi$	$k = 3\ \ \theta = \frac{42\pi}{18} > 2\pi$

19. (3 points) Find the exact value of $\sec^{-1}(-2)$

$\sec^{-1}(-2) = \cos^{-1}\left(-\frac{1}{2}\right)$

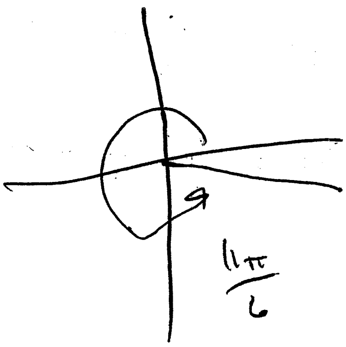
19. $\frac{2\pi}{3}$ or $120°$

Let $\theta = \cos^{-1}(-\frac{1}{2})$, then $\theta \in [0,\pi]$ and $\cos\theta = -\frac{1}{2}$.



20. (4 points) Find the exact value of $\cos^{-1}\left[\sin\left(\frac{11\pi}{6}\right)\right]$

20. $\frac{2\pi}{3}$ or $120°$



$\sin\frac{11\pi}{6} = -\frac{1}{2}$,

So, $\cos^{-1}\left(\sin\frac{11\pi}{6}\right) = \cos^{-1}\left(-\frac{1}{2}\right)$

$= 120°$

or

$\frac{2\pi}{3}$

22. (6 points) Solve the equation on the interval $[0, 2\pi)$

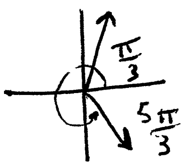
$$\cos\left(\frac{\theta}{3} - \frac{\pi}{4}\right) = \frac{1}{2}$$

22. $\frac{7\pi}{4}$

Let $x = \frac{\theta}{3} - \frac{\pi}{4}$. Then solve $\cos(x) = \frac{1}{2}$.

The solns are $x = \frac{\pi}{3} + 2\pi k$ or $x = \frac{5\pi}{3} + 2\pi k$.

Resubstitute then,



$$\frac{\theta}{3} - \frac{\pi}{4} = \frac{\pi}{3} + 2\pi k \quad \text{or} \quad \frac{\theta}{3} - \frac{\pi}{4} = \frac{5\pi}{3} + 2\pi k$$

(add $\frac{\pi}{4}$)

$$\frac{\theta}{3} = \frac{\pi}{4} + \frac{\pi}{3} + 2\pi k \quad \text{or} \quad \frac{\theta}{3} = \frac{\pi}{4} + \frac{5\pi}{3} + 2\pi k$$

(multiply by 3)

$$\frac{3}{1} \cdot \frac{\theta}{3} = \frac{3}{1} \cdot \left(\frac{\pi}{4} + \frac{\pi}{3} + 2\pi k\right) \quad \text{or} \quad \frac{3}{1} \cdot \frac{\theta}{3} = \frac{3}{1}\left(\frac{\pi}{4} + \frac{5\pi}{3} + 2\pi k\right)$$

$$\theta = \frac{3\pi}{4} + \pi + 6\pi k \quad \text{or} \quad \theta = \frac{3\pi}{4} + 5\pi + 6\pi k$$

$$\theta = \frac{3\pi}{4} + \frac{4\pi}{4} + \frac{24\pi k}{4} \quad \text{or} \quad \theta = \frac{3\pi}{4} + \frac{20\pi}{4} + \frac{24\pi k}{4}$$

$$\theta = \frac{7\pi + 24\pi k}{4} \quad \text{or} \quad \theta = \frac{23\pi + 24\pi k}{4}$$

$$\theta = (7 + 24k)\frac{\pi}{4} \quad \text{or} \quad \theta = (23 + 24k)\frac{\pi}{4}$$

↑ no solns in $[0, 2\pi)$

When $k = 0$, $\theta = \frac{7\pi}{4}$

23. (6 points) Solve the equation on the interval $[0, 2\pi)$

$$2\sin^2(\theta) - \sin(\theta) - 1 = 0$$

23. $\left\{ \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

or $\{90°, 210°, 330°\}$

Let $x = \sin\theta$ then

$2x^2 - x - 1 = 0$, or

$(2x+1)(x-1) = 0$, or

$\Rightarrow$ $2x+1 = 0$ or $x - 1 = 0$ by the zero product property

$\Rightarrow$ $x = -\frac{1}{2}$ or $x = 1$

$\Rightarrow$ $\sin\theta = -\frac{1}{2}$ or $\sin\theta = 1$

$\Rightarrow$ $\theta \in \left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$ or $\theta \in \left\{ \frac{\pi}{2} \right\}$