56 total points

College Algebra - Test 1 **Name:** Key

1. (5 points) Suppose $g(x) = \begin{cases} 3x+8 & \text{if } x < -2 \\ \sqrt{4-x^2} & \text{if } -2 \le x < 0 \\ \sqrt{x} & \text{if } x \ge 0 \end{cases}$.

Evaluate the piecewise defined function at the values indicated below.

(a) $g(-3) = 3(-3)+8 = -9+8$ (a) -1

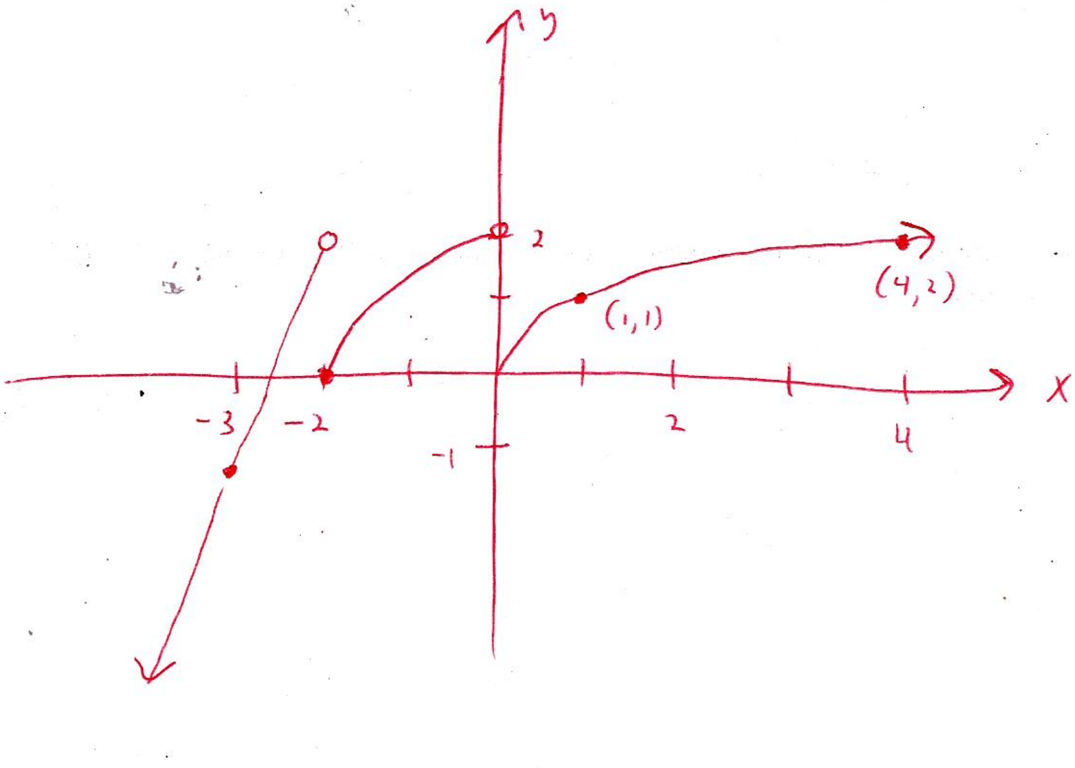
(b) $g(-2) = \sqrt{4-(-2)^2} = \sqrt{4-4} = 0$ (b) 0

(c) $g(-1) = \sqrt{4-(-1)^2} = \sqrt{4-1} = \sqrt{3}$ (c) $\sqrt{3}$

(d) $g(0) = \sqrt{0} = 0$ (d) 0

(e) $g(4) = \sqrt{4} = 2$ (e) 2

2. (4 points) Sketch the graph of the piecewise function defined above.

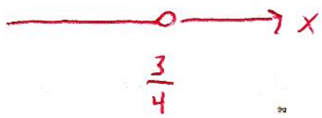


3. (5 points) Write the domain of $f(x) = \dfrac{1}{4x-3}$ using interval notation.

$4x - 3 = 0$

$4x = 3$

$x = \dfrac{3}{4}$



3. $\boxed{\left(-\infty, \tfrac{3}{4}\right) \cup \left(\tfrac{3}{4}, \infty\right)}$

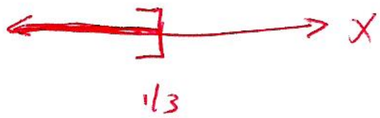
4. (5 points) Write the domain of $f(x) = \sqrt{1-3x}$ using interval notation.

$1 - 3x \geq 0$

$1 \geq 3x$

$3x \leq 1$

$x \leq 1/3$



4. $\boxed{(-\infty, 1/3]}$

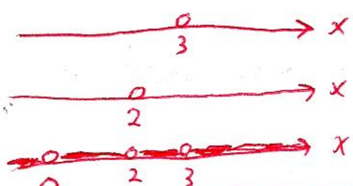
5. (5 points) Find f/g and its domain. $f(x) = \dfrac{2}{x-3}$ and $g(x) = \dfrac{x}{2-x}$

$f/g = \dfrac{2}{x-3} \div \dfrac{x}{2-x} = \dfrac{2}{x-3} \cdot \dfrac{2-x}{x} = \boxed{\dfrac{4-2x}{x^2-3x}}$

5. ________

$dom(f) = (-\infty, 3) \cup (3, \infty)$

$dom(g) = (-\infty, 2) \cup (2, \infty)$



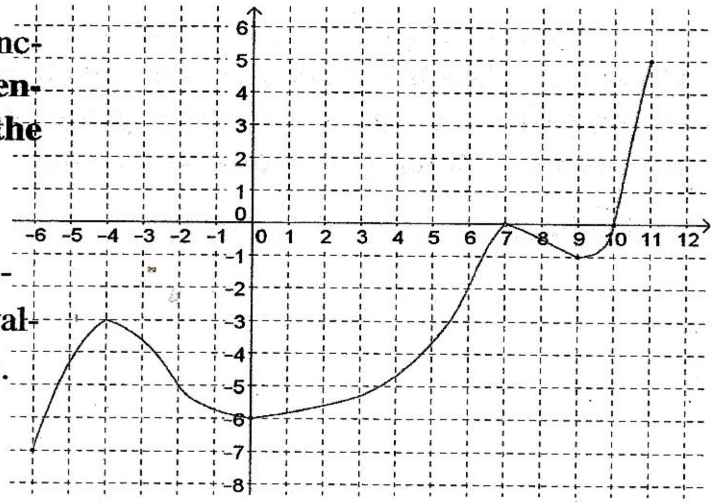
$\boxed{dom(f/g) = (-\infty, 0) \cup (0, 2) \cup (2, 3) \cup (3, \infty)}$

6. (5 points) Find the average rate of change of $f(x) = 5x^2 - 2x$ from $x_1 = 3$ to $x_2 = 4$

$\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{72 - 39}{4 - 3} = \dfrac{33}{1} = 33$

6. $\boxed{33}$

7. (12 points) The graph of a function f is given. **Assume the entire graph of f is shown in the figure.**



(a) Find all ***local*** and absolute maximum and minimum values of the function and the value of x at which each occurs.

local min $y = -6, -1$

local max $y = -3, 0$

abs max $y = 5$

abs min $y = -7$

(b) State the x intervals for which $f(x) > 0$.

$(10, 11]$

(c) State the x intervals for which $f(x) < 0$.

$[-6, 7) \cup (7, 10)$

(d) Find the x intervals on which the function is ***increasing***.

$(-6, -4) \cup (0, 7) \cup (9, 11)$

(e) Find the x intervals on which the function is ***decreasing***.

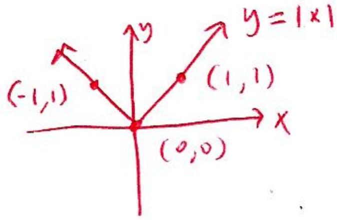
$(-4, 0) \cup (7, 9)$

(f) Find $f(-4)$. (f) -3

(g) Find $f(9)$. (g) -1

Directions: Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations. Label at least 3 points on your final graph.

8. (5 points) $h(x) = -3|x+3| + 4$ $= -3f(x+3)+4$ if $f(x) = |x|$

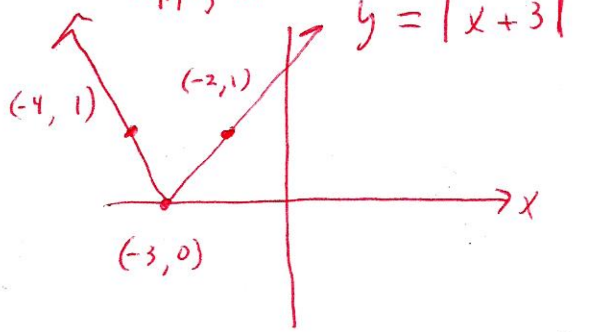


HS = -3

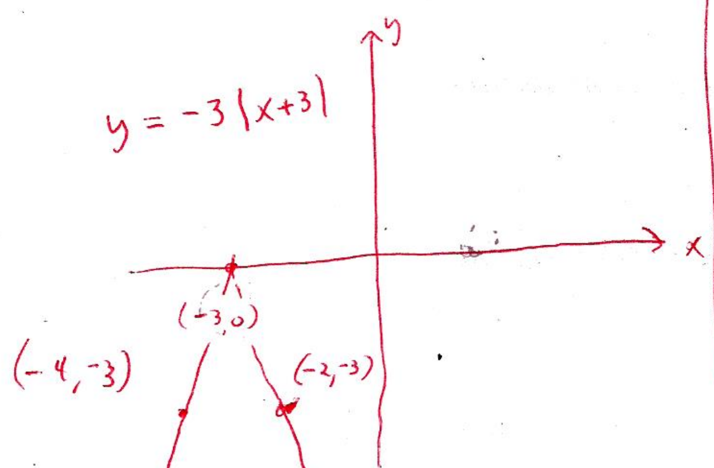
VS = 4

Refl/mag = -3

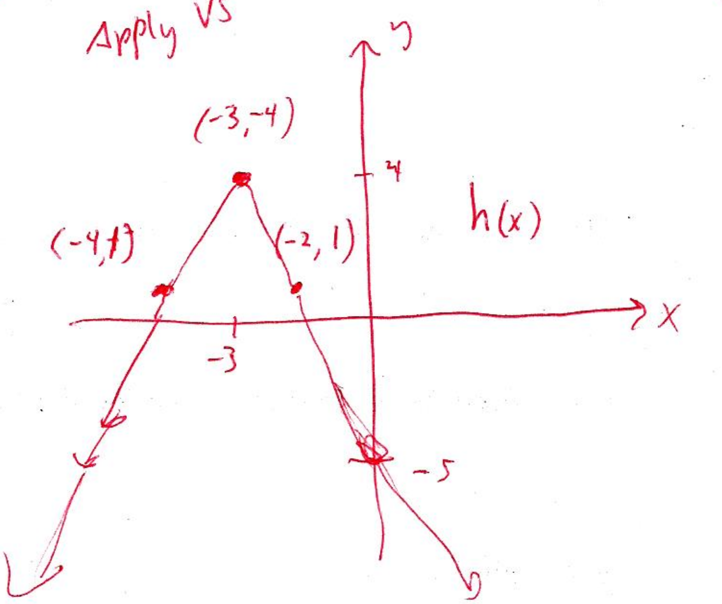
Apply HS



Apply Refl/mag



Apply VS



Find $f \circ g$ its domain.

9. (5 points) $f(x) = \sqrt{x+4}$ and $g(x) = x^2$.

$f \circ g = f(g(x)) = \sqrt{x^2+4}$

$\mathrm{dom}(f \circ g) = (-\infty, \infty)$

10. (5 points) Find the inverse function of $f(x) = \sqrt[3]{x+3}$

10. ________

$y = \sqrt[3]{x+3}$

$x = \sqrt[3]{y+3}$

$x^3 = y+3$

$y+3 = x^3$

$y = x^3 - 3$

$\boxed{f^{-1}(x) = x^3 - 3}$

76 total points

Math 110 – Exam 2 **Name:** Key

Directions: You are NOT allowed to use a calculator, computer, textbook or person as help on this Test. Show ALL of your work on ALL of the questions if you want full credit. Scratch paper is not allowed. Tutor help is not okay.

Use $f(x) = x^5 - x^4 - 5x^3 + x^2 + 8x + 4$ for questions 1 through 8.

1. (5 points) Find all the zeros of $f(x)$. What is the multiplicity of each root?

poss. rat. zeros $\{\pm 1, \pm 2, \pm 4\}$

Zeros $x = -1$ (mult 3)

1. $x = 2$ (mult 2)

-1	1	-1	-5	1	8	4
	↓	-1	2	3	-4	-4
	1	-2	-3	4	4	0

$\Rightarrow f(x) = (x+1)(x^4 - 2x^3 - 3x^2 + 4x + 4)$

2	1	-2	-3	4	4
	↓	2	0	-6	-4
	1	0	-3	-2	0

$\Rightarrow f(x) = (x-2)(x+1)(x^3 - 3x^2 - 2)$

2	1	0	-3	-2
	↓	2	4	2
	1	2	1	0

$\Rightarrow f(x) = (x-2)^2(x+1)(x^2 + 2x + 1)$

$= (x-2)^2(x+1)(x+1)(x+1)$

$= (x-2)^2(x+1)^3$

Key

2. (2 points) Write the complete factorization of $f(x)$ here.

2. $f(x) = (x-2)^2(x+1)^3$

3. (2 points) What is the domain of $f(x)$?

3. $\mathbb{R} = (-\infty, \infty)$

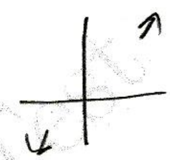
4. (2 points) Find the y-intercept of $f(x)$

$f(0) = 4$

4. $(0, 4)$

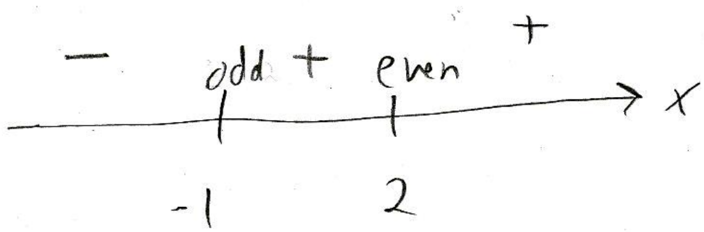
5. (2 points) Write an end behavior description for $f(x)$

$\left\{ \begin{array}{l} n = 5 \\ a_n = 1 > 0 \end{array} \right\} \Rightarrow$



5. $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

6. (2 points) Find the solution set to $f(x) > 0$

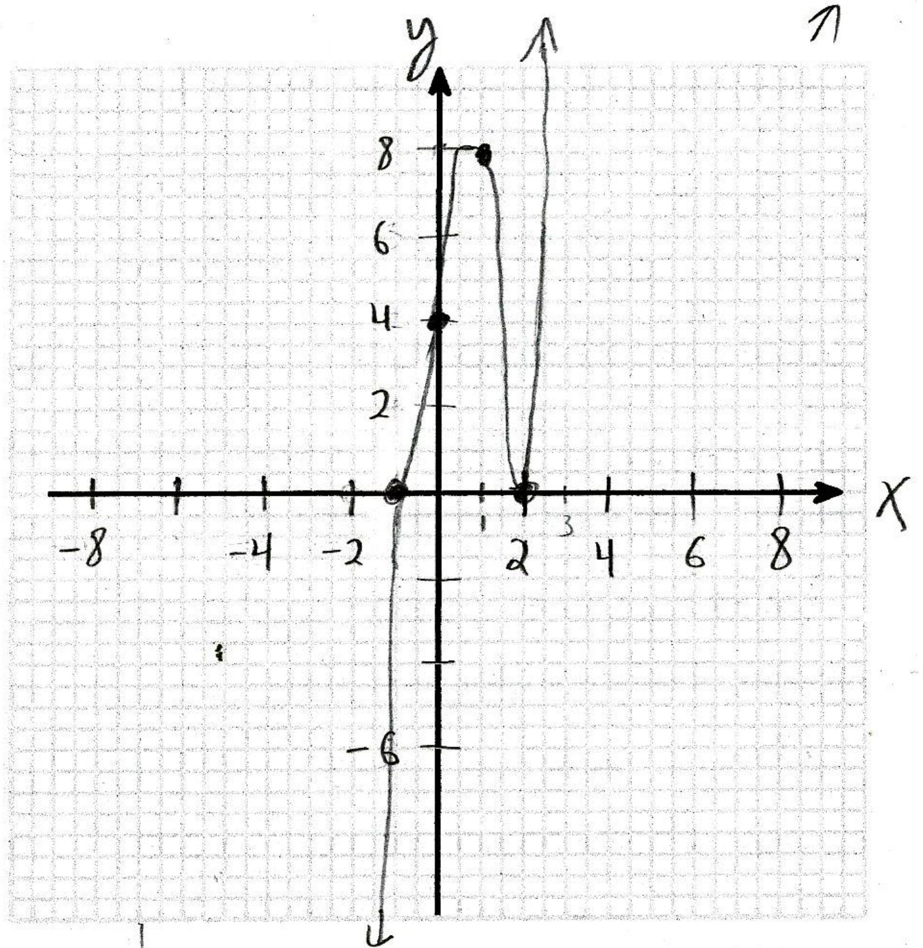


6. $(-1, 2) \cup (2, \infty)$

7. (2 points) Find the solution set to $f(x) < 0$

7. $(-\infty, -1)$

8. (3 points) Graph $f(x) = x^5 - x^4 - 5x^3 + x^2 + 8x + 4$



$$\begin{array}{r|rrrrrr} -3 & 1 & -1 & -5 & 1 & 8 & 4 \\ & \downarrow & -3 & 12 & -21 & 60 & -204 \\ \hline & 1 & -4 & 7 & -20 & 68 & -200 \end{array}$$

$$\begin{array}{r|rrrrrr} -2 & 1 & -1 & -5 & 1 & 8 & 4 \\ & \downarrow & -2 & 6 & -2 & 2 & -20 \\ \hline & 1 & -3 & 1 & -1 & 10 & -16 \end{array}$$

$$\begin{array}{r|rrrrrr} 1 & 1 & -1 & -5 & 1 & 8 & 4 \\ & \downarrow & 1 & 0 & -5 & -4 & 4 \\ \hline & 1 & 0 & -5 & -4 & 4 & 8 \end{array}$$

$$\begin{array}{r|rrrrrr} 3 & 1 & -1 & -5 & 1 & 8 & 4 \\ & \downarrow & 3 & 6 & 3 & 12 & 60 \\ \hline & 1 & 2 & 1 & 4 & 20 & 64 \end{array}$$

$$\begin{array}{r|rrrrrr} 4 & 1 & -1 & -5 & 1 & 8 & 4 \\ & \downarrow & 4 & 12 & 28 & 116 & 496 \\ \hline & 1 & 3 & 7 & 29 & 124 & 500 \end{array}$$

X	y
-3	-200
-2	-16
-1	0
0	4
1	8
2	0
3	64
4	500

For questions 9 through 16, use $f(x) = \dfrac{x-1}{x^3-4x}$

9. (2 points) Find the vertical asymptote(s) of $f(x)$. 9. $\boxed{\begin{matrix} x=0 \\ x=2 \\ x=-2 \end{matrix}}$

$x^3 - 4x = 0$

$x(x^2-4) = 0$

$x(x-2)(x+2) = 0$

$\Rightarrow \{x=0,\ x-2=0,\ x+2=0\}$

10. (2 points) Find the domain of $f(x)$ 10. ________

$\{x \mid x \neq \pm 2, 0\} \equiv (-\infty,-2) \cup (-2,0) \cup (0,2) \cup (2,\infty)$

11. (2 points) Find the x-intercept(s) of $f(x)$ 11. $\boxed{x=1}$

$x-1=0 \Rightarrow x=1$

12. (2 points) Find the y-intercept of $f(x)$ 12. ________

$f(0)$ is undefined. There is no y-int.

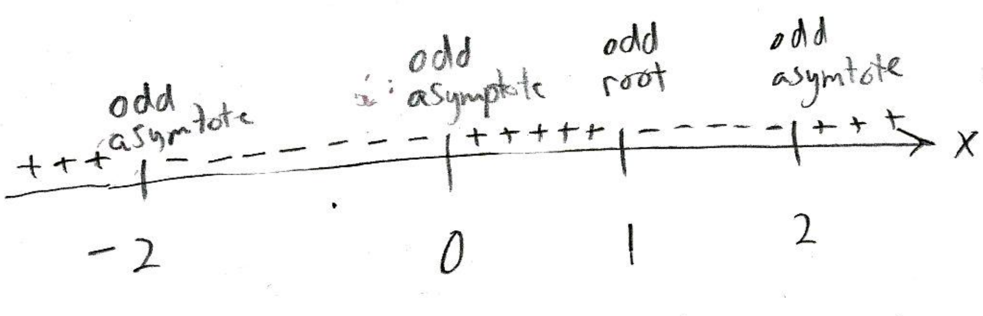
13. (2 points) Find the horizontal asymptote of $f(x)$ 13. $\boxed{y=0}$

$n=1$

$d=3$

$[n<d] \Rightarrow [y=0 \text{ is the HA}]$

14. (2 points) Find all x values for which $f(x) > 0$ 14. $\boxed{(-\infty,-2) \cup (0,1) \cup (2,\infty)}$



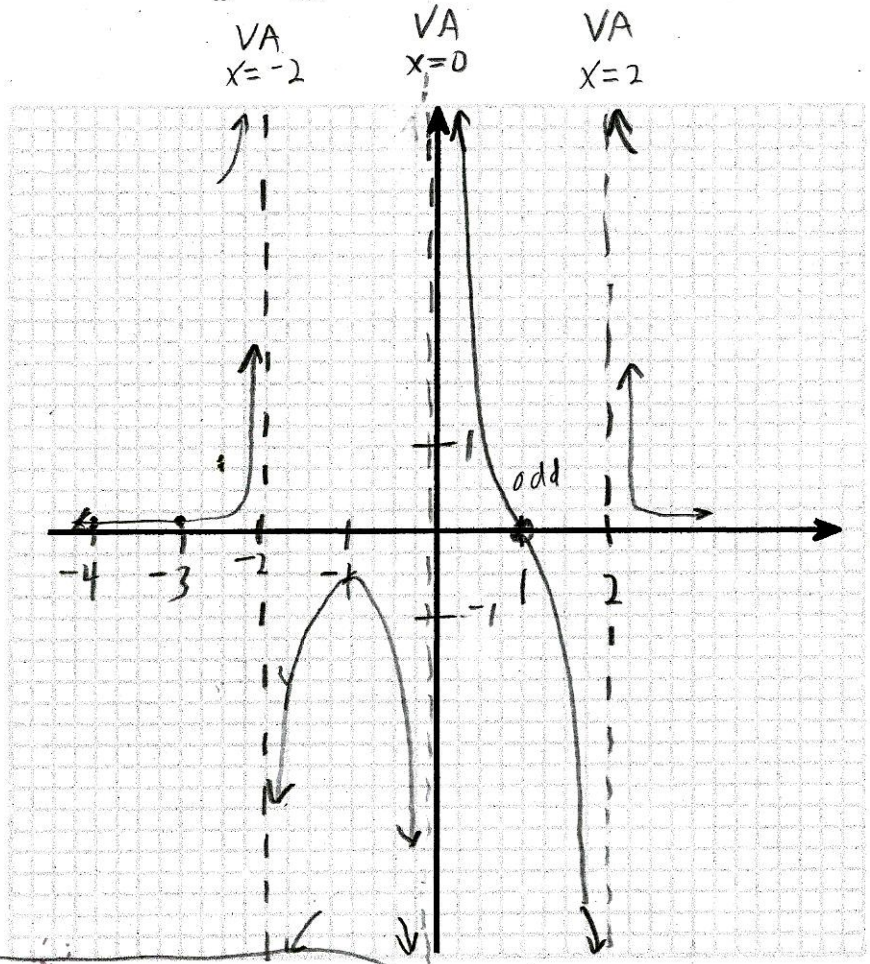
Test point

$f(-3) = \dfrac{-3-1}{(-3)^3-4(-3)} = \dfrac{-4}{-27+12} = \dfrac{-4}{-15} = \dfrac{4}{15}$

15. (2 points) Describe the behavior of the graph of f around its vertical asymptote(s). Use the arrow notation taught in class.

15. __________

16. (4 points) Graph $f(x) = \dfrac{x-1}{x^3-4x}$



VA's	
$x=-2$	$y \to \infty$ as $x \to -2^-$ and $y \to -\infty$ as $x \to -2^+$
$x=0$	$y \to -\infty$ as $x \to 0^-$ and $y \to \infty$ as $x \to 0^+$
$x=2$	$y \to -\infty$ as $x \to 2^-$ and $y \to \infty$ as $x \to 2^+$

Key

18. (4 points) Find the quotient and the remainder for $\dfrac{x^4 - 16}{x^2 + 3x + 1}$

$$
\begin{array}{r}
x^2 - 3x + 8 \\
x^2 + 3x + 1 \;\big)\; x^4 + 0x^3 + 0x^2 + 0x - 16 \\
-(x^4 + 3x^3 + 1x^2) \qquad\quad \\
\hline
-3x^3 - x^2 + 0x - 16 \\
-(-3x^3 - 9x^2 - 3x) \qquad \\
\hline
8x^2 + 3x - 16 \\
-(8x^2 + 24x + 8) \\
\hline
-21x - 24
\end{array}
$$

$Q = x^2 - 3x + 8$

$R = -21x - 24$

18. ______

Find a mathematical model that represents the statement. Then determine the value of the constant of proportionality, *k*.

19. (4 points) z varies jointly as x and y. It is known from experimental results that $z = 64$ when $x = 4$ and $y = 8$.

19. ______

$Z = Kxy$

$64 = K \cdot 4 \cdot 8$

$\dfrac{64 = 32K}{32}$

$K = 2$

so $Z = 2xy$

Find a mathematical model for the verbal statement.

20. (2 points) y varies inversely as the square of x.

$y = \frac{k}{x^2}$

20. ______

21. (4 points) Find a polynomial with real coefficients that has zeros at $x = 2$ and $x = 3 - 2i$. Write the polynomial in descending order (leaving your polynomial in factored form doesn't constitute a full credit answer).

21. ______

$(x-2)(x-3+2i)(x-3-2i)$

$= (x-2)\big((x-3)+2i\big)\big((x-3)-2i\big)$

$= (x-2)\big((x-3)^2 - 4i^2\big)$

$= (x-2)(x^2-6x+9+4)$

$= (x-2)(x^2-6x+13)$

$= x(x^2-6x+13) - 2(x^2-6x+13)$

$= x^3 - 6x^2 + 13x - 2x^2 + 12x - 26$

$= \boxed{x^3 - 8x^2 + 25x - 26}$

Key

Use $g(x) = 3x^2 + 2x - 7$ to answer questions 22 through 28.

22. (4 points) Use the quadratic formula to find the zeros of

$$g(x) = 3x^2 + 2x - 7$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{4-4(3)(-7)}}{6}$$

$$= \frac{-2 \pm \sqrt{4+84}}{6} = \frac{-2 \pm \sqrt{88}}{6}$$

$$= -\frac{2}{6} \pm \frac{\sqrt{88}}{6} = -\frac{1}{3} \pm \frac{2\sqrt{22}}{6} = \boxed{-\frac{1}{3} \pm \frac{\sqrt{22}}{3}}$$

23. (3 points) Estimate the value of each root without a calculator.

$$\begin{array}{r} 9.4 \\ \times\ 9.4 \\ \hline 376 \\ 8460 \\ \hline 88.36 \end{array}$$

So $\sqrt{88} \approx 9.4$

and $\frac{\sqrt{88}}{6} \approx \frac{9.4}{6} = 1.56$

$$\begin{array}{r} 1.56 \\ 6\overline{)9.4} \\ 6 \\ \hline 34 \\ 30 \\ \hline 40 \end{array}$$

So $-\frac{1}{3} \pm \frac{\sqrt{88}}{6}$

$\approx -0.33 \pm 1.57$

$= \{-1.9, 1.24\}$

$$\begin{array}{r} 1.57 \\ -\ .33 \\ \hline 1.24 \end{array}$$

$$\begin{array}{r} 1.57 \\ .33 \\ \hline 1.90 \end{array}$$

$b=\frac{2}{3}$; $\frac{b}{2}=\frac{1}{2}b=\frac{1}{2}\cdot\frac{2}{3}=\frac{1}{3}$; $\left(\frac{b}{2}\right)^2=\frac{1}{9}$

24. (4 points) Express the quadratic function $g(x) = 3x^2+2x-7$ in standard (vertex) form.

$$= 3\left(x^2+\frac{2}{3}x\right)-7 = 3\left(x^2+\frac{2}{3}x+\frac{1}{9}\right)-7-3\cdot\frac{1}{9}$$

$$= 3\left(x+\frac{1}{3}\right)^2 + -7 + -1/3 = 3\left(x+1/3\right)^2 + \frac{-21}{3} + \frac{-1}{3}$$

$$\boxed{g(x) = 3\left(x+1/3\right)^2 - \frac{22}{3}}$$

Key

25. (1 point) Find the vertex of $g(x) = 3x^2 + 2x - 7$. Does f open up or down?

opens up

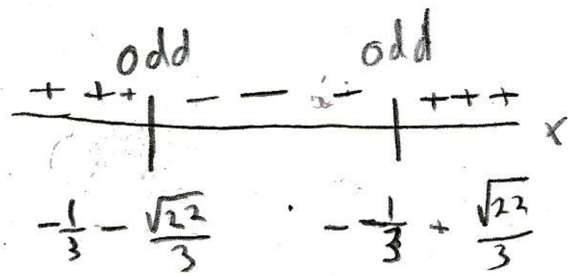
25. $\left(-1/3, -\frac{22}{3}\right)$

26. (2 points) What is the range of g?

26. $\left[-\frac{22}{3}, \infty\right)$

27. (3 points) For what x values is the graph of g below the x axis?

27. $\left(-\frac{1}{3}-\frac{\sqrt{22}}{3}, -\frac{1}{3}+\frac{\sqrt{22}}{3}\right)$



$f(0) = -7$

28. (2 points) Find the solutions to the inequality $g(x) > 0$.

28. ________

$$\left(-\infty, -\frac{1}{3}-\frac{\sqrt{22}}{3}\right) \cup \left(-\frac{1}{3}+\frac{\sqrt{22}}{3}, \infty\right)$$

total 73 pts

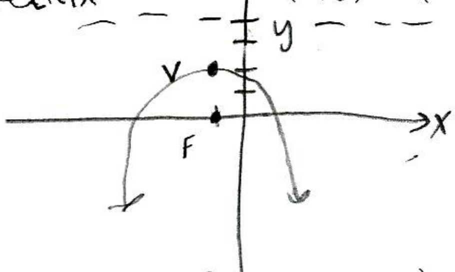
Math 110 Test 3 Name: Key

No Calculators or Computing Devices on this section. Once you turn this section in, you may NOT have it back! Use Algebraic Notation AND Show All of Your Work.

---

1. (5 points) **Find the standard form of the equation of the parabola with the given characteristic(s).**

directrix Vertex: $(x, y) = (-1, 2)$; focus: $(-1, 0)$



1. ________

$x^2 + 2x + 1 = -16y + 32$

$x^2 + 2x - 31 = -16y$

$\boxed{y = -\frac{1}{16}(x^2 + 2x - 31)}$

$(x-h)^2 = 4p(y-k)$

$(x+1)^2 = 4(-4)(y-2)$

$\boxed{(x+1)^2 = -16(y-2)}$

2. (5 points) **Write the equation of a circle in standard form, and then find its center and radius.**

$$x^2 + y^2 - 4y = 0$$

$x^2 + (y^2 - 4y + \underline{4}) = 0 + \underline{4}$

2. ________

$x^2 + (y-2)^2 = 4$

$(x-0)^2 + (y-2) = 4$

Center $(x, y) = (0, 2)$

radius $r = \sqrt{4} = 2$

3. (6 points) **Identify the conic by writing its equation in standard form, then sketch its graph. Be sure to label foci, asymptotes and vertices on your graph if that is appropriate.**

$$y^2 - x^2 + 4y = 0$$

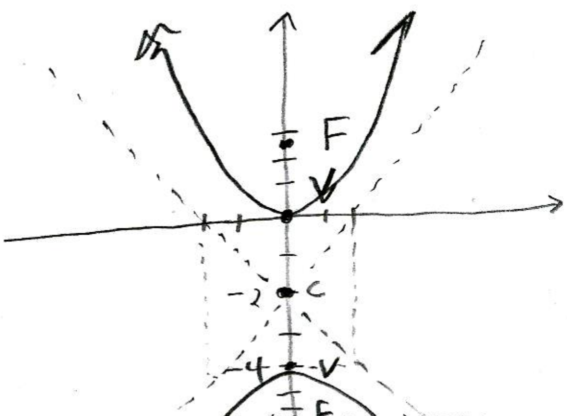
3. ________

$(y^2 + 4y + \underline{4}) - (x-0)^2 = 0 + \underline{4}$

$(y+2)^2 - (x-0)^2 = 4$

$\dfrac{(y+2)^2}{4} - \dfrac{(x-0)^2}{4} = 1$

Std form of a hyperbola Centered at $(0,-2)$ with $a = b = 2$, Opens in y-direction



Foci

$c^2 = a^2 + b^2$

$c^2 = 4 + 4$

$c^2 = 8$

$c = \sqrt{8} = 2\sqrt{2} \approx (2)\cdot(1.4) = 2.8$

$$\boxed{\begin{array}{l} \text{Center } (0,-2) \\ \text{Vertices } (0,0),\ (0,-4) \\ \text{Foci } (0, 2+2\sqrt{2}) \\ \quad (0,-4-2\sqrt{2}) \end{array}}$$

4. (5 points) This is a ***Matching question*** associated with the theory on graphical translations of functions. Suppose $f(x) = 3^x$. Relative to the graph of $f(x)$ the graphs of the following functions have been changed in what way?

__b__ $g(x) = -3^x$ a.) shifted 5 units right

__c__ $g(x) = 3^{(x+5)}$ b.) reflected about the x axis

__e__ $g(x) = 3^x + 5$ c.) shifted 5 units left

__a__ $g(x) = 3^{(x-5)}$ d.) shifted 5 units down

__d__ $g(x) = 3^x - 5$ e.) shifted 5 units vertically up

5. (4 points) **Use the One-to-One Property to solve the equation for x.**

$$3^{2x-3} = \frac{1}{81}$$

Soln

$3^{2x-3} = 3^{-4}$

$2x - 3 = -4$

$2x = -4 + 3$

$2x = -1$

$\boxed{x = -1/2}$

Side work

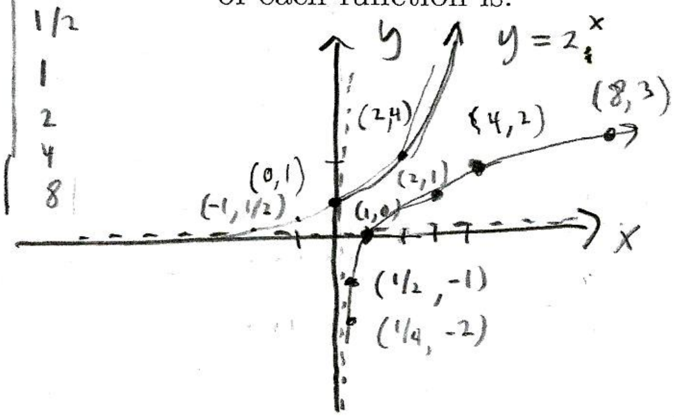
$1/81$

$= 1/3^4 = 3^{-4}$

5. $\boxed{x = -1/2}$

x	$y = 2^x$
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4
3	8

6. (5 points) Sketch the graphs of $f(x) = 2^x$ and $g(x) = \log_2(x)$ in the same coordinate system, and then tell me what the domain and range of each function is.



$y = 2^x$

domain $x \in \mathbb{R}$ or $(-\infty, \infty)$

range $y > 0$ or $(0, \infty)$

$y = \log_2(x)$

domain $x > 0$ or $(0, \infty)$

range $y \in \mathbb{R}$ or $(-\infty, \infty)$

7. (1 point) What number is $\log_7(49)$ equal to?

7. $\boxed{2}$

8. (1 point) What number is $\log_5(\sqrt{5})$ equal to?

$\log_5(5^{1/2})$

8. $\boxed{1/2}$

9. (1 point) What number is $\ln(e^3)$ equal to?

9. $\boxed{3}$

10. (1 point) What number is $\ln(e^3)$ equal to?

10. $\boxed{3}$

11. (1 point) What equation represents the vertical asymptote for

$$f(x) = \log_2(x)$$

11. $\boxed{X = 0}$

12. (5 points) Condense the following expression to the logarithm of a single quantity.

$$\frac{1}{3}\left[\log_8 y + 2\log_8(y+4)\right] - \log_8(y-1)$$

$$= \frac{1}{3}\left[\log_8 y + \log_8 (y+4)^2\right] - \log_8 (y-1)$$

$$= \frac{1}{3}\left[\log_8\left(y(y+4)^2\right)\right] - \log_8(y-1)$$

$$= \frac{1}{3}\log_8\left(y(y+4)^2\right) - \log_8(y-1)$$

$$= \log_8\left(y(y+4)^2\right)^{1/3} - \log_8(y-1) = \boxed{\log_8\left[\frac{\left(y(y+4)^2\right)^{1/3}}{y-1}\right] \text{ or } \log_8 \frac{\sqrt[3]{y(y+4)}}{y-1}}$$

13. (4 points) Solve the equation.

$$\log_7(x-3) + 1 = 2$$

$\log_7(x-3) = 2 - 1$

$\log_7(x-3) = 1$

$7^1 = (x-3)$

$7 = x - 3$

$7 + 3 = x$

$x = 10$

Check

$\log_7(10-3) + 1 = 2$

$\log_7 7 + 1 = 2$

$1 + 1 = 2$ ✓

13. $\boxed{X = 10}$

$0 = 3x^2+2x-8$

$= 3x^2+6x-4x-8 = 3x(x+2)-4(x+2) = (3x-4)(x+2)$

14. (5 points) Use the method of substitution to solve the system

$$\begin{cases} 3x^2+4x-y=7 \\ 2x-y=-1 \end{cases}$$

$$= \begin{Bmatrix} 3x^2+4x-y=7 \\ y=2x+1 \end{Bmatrix} = \begin{Bmatrix} 3x^2+4x-(2x+1)=7 \\ y=2x+1 \end{Bmatrix} = \begin{Bmatrix} 3x^2+2x-8=0 \\ y=2x+1 \end{Bmatrix}$$

$$= \begin{Bmatrix} (3x-4)(x+2)=0 \\ y=2x+1 \end{Bmatrix} = \begin{Bmatrix} 3x-4=0 \\ x+2=0 \\ y=2x+1 \end{Bmatrix} = \begin{Bmatrix} x=4/3 \\ x=-2 \\ y=2x+1 \end{Bmatrix}$$

System has 2 solutions $(x,y) = (4/3,\ 2\cdot\frac{4}{3}+1) = \boxed{(\frac{4}{3}, \frac{11}{3})}$

and $(x,y) = (-2,\ 2(-2)+1) = \boxed{(-2,-3)}$

15. (5 points) Solve the system $\begin{cases} x-2y=3 \\ -2x+4y=1 \end{cases}$

mult eqn 1 by 2

$$= \begin{Bmatrix} 2x-4y=6 \\ -2x+4y=1 \end{Bmatrix}$$

Replace eqn 2 with the sum of eqns 1 + 2

15. ________

$$= \begin{Bmatrix} 2x-4y=6 \\ 0=5 \end{Bmatrix}$$

A false statement has been derived, so the system is inconsistent, and there is no solution

16. (5 points) Solve the system $\begin{cases} 2x - y = 1 \\ 4x - 2y = 2 \end{cases}$

Replace eqn 2 with the sum of
-2 times eqn 1 plus eqn 2

16. ________

$$\begin{array}{lr} -2 \text{ eq } 1 & -4x + 2y = -2 \\ + \quad \text{eq } 2 & 4x - 2y = 2 \\ \hline & 0 = 0 \end{array}$$

So, $\begin{Bmatrix} 2x - y = 1 \\ 4x - 2y = 2 \end{Bmatrix} = \begin{Bmatrix} 2x - y = 1 \\ 0 = 0 \end{Bmatrix} = \{ 2x - y = 1 \}$

$= \{ y = -2x + 1 \}$ there is an inf numb of solns

Let $x = t$, where t is any real number

then the inf solns are

$$\{ (x, y) = (t, -2t + 1), \ t \in \mathbb{R} \}$$

or simply $\{ (t, -2t+1) \text{ for } t \in \mathbb{R} \}$

18. (5 points) Solve algebraically. Approximate the result to three decimal places.

$$4^{x-5} + 21 = 30$$

$4^{x-5} = 30 - 21$

$4^{x-5} = 9$

$\ln 4^{x-5} = \ln 9$

$(x-5) \ln 4 = \ln 9$

$\frac{\ln 4 (x-5)}{\ln 4} = \frac{\ln 9}{\ln 4}$

$x - 5 = \frac{\ln 9}{\ln 4}$

$\boxed{x = 5 + \frac{\ln 9}{\ln 4}}$

≈ 6.585

19. (5 points) Solve algebraically. Approximate the result to three decimal places.

$$\ln(4x) - \ln(2) = 8$$

$\ln \frac{4x}{2} = 8$

$\ln 2x = 8$

$e^{\ln 2x} = e^8$

$2x = e^8$

$\boxed{x = \frac{1}{2} e^8}$ ≈ 1490.479

Test 4 Name: Key

No Calculators or Computing Devices allowed! Use Algebraic Notation AND Show All of Your Work.

---

1. (6 points) Use Gaussian elimination to find the complete solution of the system, or show that no solution exists.

$-\text{eqn}①+\text{eqn}② \to \text{eqn}②$

$-\text{eqn}①+\text{eqn}③ \to \text{eqn}③$

$$\begin{cases} x+y+z=0 \\ x-y+z=0 \\ x-y-z=0 \end{cases}$$

1. $\boxed{(x,y,z) = (0,0,0)}$

$$= \begin{cases} x+y+z=0 \\ -2y=0 \\ -2y-2z=0 \end{cases} = \begin{cases} x+y+z=0 \\ \frac{-2y}{-2} = \frac{0}{-2} \\ \frac{-2y}{-2} + \frac{-2z}{-2} = \frac{0}{-2} \end{cases}$$

$$= \begin{cases} x+y+z=0 \\ y=0 \\ y+z=0 \end{cases}$$

$$= \begin{cases} x+0+z=0 \\ y=0 \\ 0+z=0 \end{cases} = \begin{cases} x+z=0 \\ y=0 \\ z=0 \end{cases}$$

$$= \begin{cases} x+0=0 \\ y=0 \\ z=0 \end{cases} = \boxed{\begin{cases} x=0 \\ y=0 \\ z=0 \end{cases}}$$

2. (6 points) Use Matrices and Elementary Row Operations to find the complete solution of the system, or show that no solution exists.

$$\begin{cases} x - 2y - 3z = -1 \\ 2x + y + z = 6 \\ x + 3y - 2z = 13 \end{cases}$$

2. $(x, y, z) = (2, 3, -1)$

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & -1 \\ 2 & 1 & 1 & 6 \\ 1 & 3 & -2 & 13 \end{array}\right] \begin{array}{l} \\ -2R_1 + R_2 \to R_2 \\ -R_1 + R_3 \to R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & -3 & -1 \\ 0 & 5 & 7 & 8 \\ 0 & 5 & 1 & 14 \end{array}\right] \begin{array}{l} \\ -R_2 + R_3 \to R_3 \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & -3 & -1 \\ 0 & 5 & 7 & 8 \\ 0 & 0 & -6 & 6 \end{array}\right] \begin{array}{l} \\ \frac{1}{5}R_2 \to R_3 \\ -\frac{1}{6}R_3 \to R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & -3 & -1 \\ 0 & 1 & 7/5 & 8/5 \\ 0 & 0 & 1 & -1 \end{array}\right] \begin{array}{l} \\ \\ -\frac{7}{5}R_3 + R_2 \to R_2 \end{array}$$

System is in REF

$$= \left[\begin{array}{ccc|c} 1 & -2 & -3 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right] = \begin{cases} x - 2y - 3z = -1 \\ y = 3 \\ z = 1 \end{cases}$$

$$= \begin{cases} x - 2y - 3z = -1 \\ y = 1/5 \\ z = -1 \end{cases}$$

$$= \begin{cases} x - 2\cdot(3) - 3(-1) = -1 \\ y = 3 \\ z = -1 \end{cases}$$

$$= \begin{cases} x - 6 + 3 = -1 \\ y = 3 \\ z = -1 \end{cases}$$

$$= \begin{cases} x = 2 \\ y = 3 \\ z = -1 \end{cases}$$

Check ① $2 - 6 + 3 = -1$ ✓

② $4 + 3 + -1 = 6$ ✓

③ $2 + 9 + 2 = 13$ ✓

3. (a) (2 points) Write a matrix equation equivalent to the following system.

$$\begin{cases} 2x + 3y = 2 \\ x - 2y = 8 \end{cases} \qquad \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$$

(a) ____________

$A\vec{x} = \vec{b}$

(b) (4 points) Find the inverse of the coefficient matrix, ~~and use it to~~ solve the system.

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-4-3}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}$$

(b) $\boxed{(x,y) = (4,-2)}$

$$\vec{x} = A^{-1}\vec{b} = -\frac{1}{7}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 8 \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -2(2) + -3(8) \\ -1(2) + 2(8) \end{bmatrix}$$

$2\times 2 \quad 2\times 1$

$$= -\frac{1}{7}\begin{bmatrix} -4 - 24 \\ -2 + 16 \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -28 \\ 14 \end{bmatrix}$$

$$= \begin{bmatrix} -28/-7 \\ 14/-7 \end{bmatrix}$$

4. (5 points) Solve $\begin{cases} 2x + 5y = 16 \\ 3x - 7y = 24 \end{cases}$ using Cramer's Rule.

4. $(x,y) = (8,0)$

$$A = \begin{bmatrix} 2 & 5 \\ 3 & -7 \end{bmatrix}, \quad \det(A) = -14 - 15 = -29$$

$16 \times 7 = 112$; $24 \times 5 = 120$

$$D_x = \det\left(\begin{bmatrix} 16 & 5 \\ 24 & -7 \end{bmatrix}\right) = 16(-7) - 24(5)$$

$$= -112 - 120 = -232$$

$232 \div 29 = 8$; $232 - 232 = 0$

$$D_y = \det\left(\begin{bmatrix} 2 & 16 \\ 3 & 24 \end{bmatrix}\right) = 48 - 48 = 0$$

$$x = \frac{D_x}{\det(A)} = \frac{-232}{-29} = 8 \quad ; \quad y = \frac{D_y}{\det(A)} = \frac{0}{-29} = 0$$

5. Let $A = \begin{bmatrix} 1 & -5 \\ -3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 7 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 & 1 \\ -2 & 7 & 2 \end{bmatrix}$

Carry out the indicated operation, or explain, using complete sentences, why it cannot be performed.

(a) (4 points) $2A + B$

$$= 2\begin{bmatrix} 1 & -5 \\ -3 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 7 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -10 \\ -6 & 14 \end{bmatrix} + \begin{bmatrix} 2 & 7 \\ 1 & 0 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 4 & -3 \\ -5 & 14 \end{bmatrix}}$$

(b) (4 points) BC

$$= \begin{bmatrix} 2 & 7 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 1 \\ -2 & 7 & 2 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \end{bmatrix} = \boxed{\begin{bmatrix} -12 & 55 & 16 \\ 1 & 3 & 1 \end{bmatrix}}$$

2×2 $\quad 2 \times 3$ $\quad 2 \times 3$

$\alpha_{11} = 2(1) + 7(-2) = -12$

$\alpha_{12} = 2(3) + 7(7) = 55$

$\alpha_{13} = 2(1) + 7(2) = 16$

$\alpha_{21} = 1(1) + 0(-2) = 1$

$\alpha_{22} = 1(3) + 0(7) = 3$

$\alpha_{23} = 1(1) + 0(2) = 1$

(c) (2 points) C^{-1} doesn't exist since inverses are only for square matrices

(d) (2 points) $\det(C)$ doesn't exist since determinants are for square matrices.

6. (6 points) Find the partial fraction decomposition of $\frac{x-2}{x^2-6x+5}$.

6. ________

$$\frac{x-2}{(x-5)(x-1)} = \frac{A}{x-5} + \frac{B}{x-1}$$

$$\boxed{x-2 = A(x-1) + B(x-5) \quad \text{general eqn}}$$

let $x=1$, then	let $x=5$, then
$-1 = -4B$, or	$3 = 4A$, or
$B = 1/4$	$A = 3/4$

Then,

$$\boxed{\frac{x-2}{x^2-6x+5} = \frac{3}{4(x-5)} + \frac{1}{4(x-1)}}$$

7. Only one of the following two matrices has an inverse.

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \qquad A = \begin{bmatrix} 3 & 1 & -2 \\ 4 & 2 & 5 \\ -6 & 3 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 5 & -5 \\ 1 & -6 & 6 \end{bmatrix}$$

(a) (5 points) Find the determinant of each matrix. (a) $\boxed{\begin{array}{l} \det(A) = -125 \\ \det(B) = 0 \end{array}}$

$$\det(A) = a_{11} \cdot C_{11} + a_{12} \cdot C_{12} + a_{13} \cdot C_{13}$$

$$= 3 \cdot C_{11} + 1 \cdot C_{12} + -2 \cdot C_{13}$$

$$= 3 \cdot \begin{vmatrix} 2 & 5 \\ 3 & -1 \end{vmatrix} - \begin{vmatrix} 4 & 5 \\ -6 & -1 \end{vmatrix} - 2 \begin{vmatrix} 4 & 2 \\ -6 & 3 \end{vmatrix}$$

$$= 3(-2 - 15) - (-4 - -30) - 2(12 - -12)$$

$$= 3(-17) - (-4 + 30) - 2(12 + 12) = -51 - 26 - 48$$

$$= \boxed{-125}$$

$$\begin{array}{r} 51 \\ 26 \\ +48 \\ \hline 125 \end{array}$$

$$\det(B) = 3 \begin{vmatrix} 5 & -5 \\ -6 & 6 \end{vmatrix} + 0 \begin{vmatrix} 2 & -5 \\ 1 & 6 \end{vmatrix} + 0 \begin{vmatrix} 2 & 5 \\ 1 & -6 \end{vmatrix}$$

$$= 3(30 - 30) + 0 + 0$$

$$= 3(0) = \boxed{0}$$

(b) (1 point) Use the determinants from part (a) to identify which matrix has an inverse.

B doesn't have an inverse since its determinant is zero.

A has an inverse (b) ______

8. (6 points) Sketch the graph (and label the vertices, or boundary intersections) of the solution set of ordered pairs of the system.

$$\begin{Bmatrix} 3x + y < 3 \\ 4 - y < 2x \end{Bmatrix} = \begin{Bmatrix} y < -3x + 3 \\ y > -2x + 4 \end{Bmatrix}$$

Find intersection pt.

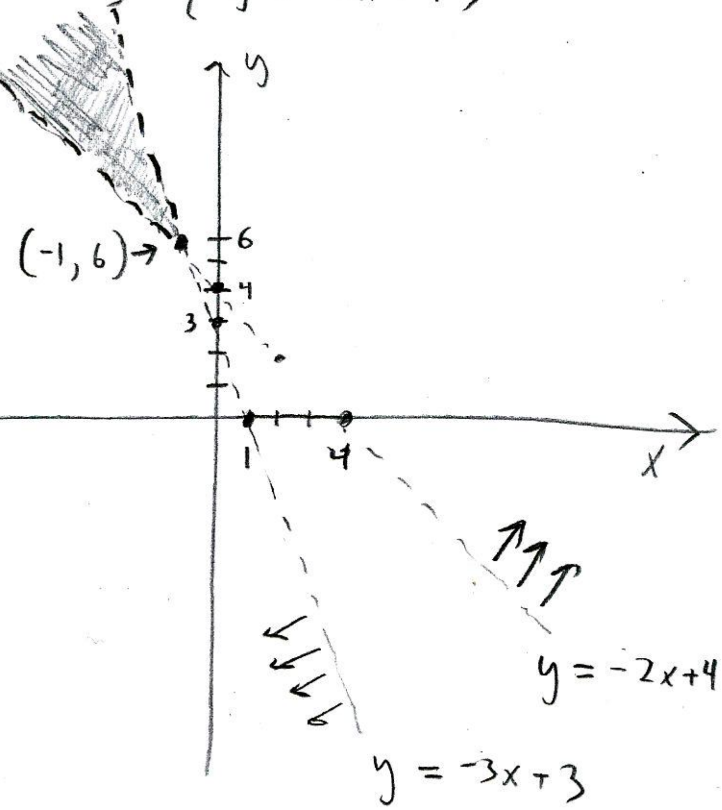
Solve

$$\begin{Bmatrix} y = -3x+3 \\ y = -2x+4 \end{Bmatrix}$$

$$= \begin{Bmatrix} y = -3x+3 \\ -3x+3 = -2x+4 \end{Bmatrix}$$

$$= \begin{Bmatrix} y = -3x+3 \\ -x = 1 \end{Bmatrix}$$

$$= \begin{Bmatrix} x = -1 \\ y = -3(-1)+3 \end{Bmatrix} = \begin{Bmatrix} x = -1 \\ y = 6 \end{Bmatrix}$$



Test pt (0,0)