Unit 1: History of Numbers

Learning Outcomes:

- Demonstrate an understanding of the evolution of our numeration system by relating counting, place value, and place holder concepts from the past to the present
- Recognize early numeration systems
- Write numbers in Egyptian, Babylonian, Greek and Roman
- Demonstrate a knowledge of our present system of place values and expanded notation

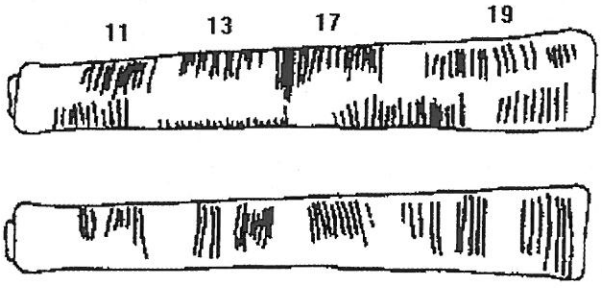
1.1 History of Numbers

In this unit, we will look at ancient civilizations and how they used numbers and operations. We will see how our present number system evolved from the numeration systems of many cultures. It is through an understanding of the past that we can appreciate the present.

In the beginning…

I. Tally marks using "grouping" were the first known form of counting. Larger grooves and crossed slashes were used to represent and record larger quantities. These marks can be found dating back to 3000 B.C. in Egypt and Babylonia.

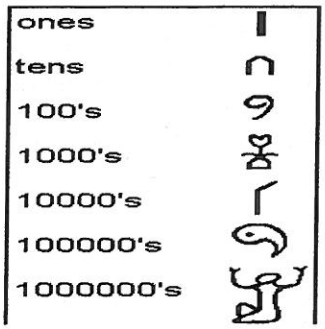
II. The **African** civilization has contributed to our present system of numbers in areas of measurement, (pyramids), irrigation systems, numeration ideas and geometry (patterns and modeling). Some of the oldest mathematical objects in the world were found in Africa. The Ishango Bone--dated between 23,000B.C. and 18,000 B.C.--was found in 1960 in Lake Edward, Zaire. Close inspection of the bone shows the notches represent our present day, prime numbers (see below).



An even older artifact is the Lebombo Bone, which resembles a calendar stick. The Lebombo Bone was found between South Africa and Swaziland and is the oldest mathematical object in the world dating to approximately 30,000 B.C.

III. The **Egyptians** used symbols painted in their pottery, cut numbers in stone and scrolled on papyrus to express counting. We have evidence that this system dates back to 3400 B.C.

The following chart depicts the Egyptian symbols for numbers:



These *hieroglyphic* symbols are a stroke, arch, rope, flower, finger, tadpole and an astonished man. These are pictorial symbols from everyday life. To make writing easier to read the repeated symbols can be grouped in two, three or four symbols arranged vertically.

The number 672 would be 𓏺𓏺𓏺𓏺𓏺𓏺𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓎆𓎆 or 𓏺𓏺𓏺𓍢𓍢𓍢𓍢𓍢𓎆𓎆
𓏺𓏺𓏺𓍢𓍢𓍢

- The Egyptian numeration system was *additive* and used *grouping by tens*
- The system was very cumbersome when writing large numbers and when operations were performed

Write the following numbers using the Egyptian numeration system:

a. 345

b. 42,320

c. 567,922

d. What are the advantages and disadvantages in using the Egyptian system? Explain.

IV. The **Greeks** used letters of their alphabet for their numbers. It was a *ciphered* numeration system that dates back to 3000 B.C.

The following chart depicts the Greek system for numbers:

1 = α	10 = ι	100 = ρ
2 = β	20 = κ	200 = σ
3 = γ	30 = λ	300 = τ
4 = δ	40 = μ	400 = υ
5 = ε	50 = ν	500 = φ
6 = ϛ (ϝ)	60 = ξ	600 = χ
7 = ζ	70 = ο	700 = ψ
8 = η	80 = π	800 = ω
9 = θ	90 = ϙ	900 = ϡ

EXAMPLE:

The number 672 would be χοβ .

Write the following numbers using the Greek numeration system:

a. 345

b. 949

c. 888

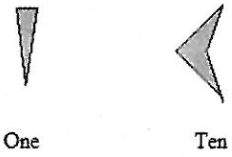
d. Convert to our number system: Φμβ

e. What are the advantages and disadvantages in using the Greek system? Explain.

- The Greeks were the first to use a comma to write large numbers
  Example: ,**β** = 2000 or (2 • 1000)

V. The **Babylonian** numeration system is one of the oldest. It dates back to 3000 B.C. Babylonians began using tally marks, which evolved into a wedge shaped symbol called a *cuneiform*. Many of their clay tablets are still around today. The Babylonians used a base 60 system instead of the base 10 system that we use today. It is believed to be related to the concept of time – 24 hours in a day, 60 minutes in an hour and 60 seconds in a minute. There are only two symbols, a modernized wedge pointed down, ▼, for one and a modernized wedge pointing to the left for ten, **<** .

The following represent numbers in the Babylonian system:



One Ten

EXAMPLES:

- The number 42 would be **<<<<▼▼**
- The number 672 would be **<▼ <▼▼**

$$(11 \cdot 60) + (12 \cdot 1)$$

672 = (11 groups of 60) + (12 groups of 1)

- In approximately 3400 B.C. a space was used to indicate a place holder (zero was only an idea; the Babylonians knew they needed something to distinguish between place values)
- A space was used to show breaks between place values

Write the following numbers using the Babylonian numeration system:

a. 48

b. 67

c. 132

d. 3702

e. What are the advantages and disadvantages in using the Babylonian system? Explain.

The Babylonians introduced:
- Place values
- Positional system
- Space for a place holder

VI. The **Roman Numerals** go back to 500 B.C. Today, Roman numerals can be seen on clocks, chapters of a book and outlines. This book uses Roman numerals to show topic changes.

The following chart depicts the Roman system for numbers:

Roman Numeral	I	V	X	L	C	D	M
Number	1	5	10	50	100	500	1000

EXAMPLE:

The number 672 would be DCLXXII

600 + 70 + 2

↓ ↓ ↓

DC + LXX + II

- Within the Roman system, when more than three symbols are required, the concept of subtraction is used

EXAMPLE:

The number 493 would be CDXCIII

400 + 90 + 3

↓ ↓ ↓

CD + XC + III

↓ ↓

(500-100) (100 -10)

Write the following numbers using the Roman numeration system:

a. 777

b. 24

c. 1098

d. 479

e. Convert XL to our present system.

f. Convert MDCXII to our present system.

g. Convert MCMLXIV to our present system.

h. Write your age with Roman numerals.

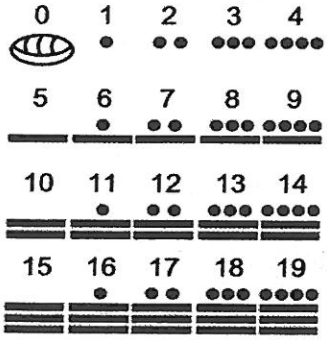
i. Write your birth year with Roman numerals.

j. What are the advantages and disadvantages in using the Roman system? Explain.

The Roman system included:
- Grouping
- Additivity
- Subtraction

VII. The **Mayan** civilization number contributions date back to 300 A.D. The Mayans used a base 20 system. It is thought that the base of 20 was used because 20 was the number of fingers and toes.

The following chart depicts the Mayan system for numbers:



- The Mayans were the first to use a symbol for zero
- The was used as placeholder
- Interesting note: It is believed that our present day calendar was invented by the Mayans

VIII. "*Much ado about nothing*"... William Shakespeare

Some facts about *zero:*

- First used as a placeholder by the Mayans, approximately 4th century
- Used as a number in the 7th century in India


Complete the following:

a. The sum of zero and a number is ...

b. Zero subtracted from a number is ...

c. Zero multiplied by a number is ...

d. Zero divided by a number is ...

e. A non-zero number divided by zero is ...

f. Zero divided by zero is ...

EXAMPLES:

$8/0 =$	$0 \div 5 =$	$2\overline{)0}$
$0/8 =$	$0 \div 0 =$	$0\overline{)2}$
$0/0 =$	$5 \div 0 =$	$0\overline{)0}$

IX. Our present system of numbers is called the **Hindu-Arabic** number system. This is a base 10 number system and combines concepts of numbers that developed in many nations over 1000s of years. The ten symbols (1, 2, 3, 4, 5, 6, 7, 8, 9, 0) have their origin in India around 100 A.D. By approximately 800 A.D., these number symbols had evolved to what closely resembles their modern form and were adopted by Arab mathematicians, scientists, merchants and traders.

An Italian mathematician by the name of Leonardo of Pisa (later known as Fibonacci) learned the Hindu-Arabic numbers while visiting northern Africa as a young man around 1185 A.D. He discovered how much easier any calculation was with Hindu-Arabic numbers compared to Roman numerals. Fibonacci wrote a book in 1202 A.D. to introduce the Hindu-Arabic numbers to people of Italy. He used many real-life examples for people who would need to use these numbers in business.

Fibonacci's book was a huge success. Over the next few hundred years, all of Europe began to replace Roman numerals with Hindu-Arabic numerals. By about 1400 A.D., with the invention of the printing press, the number symbols became more standardized and began to look essentially the same as the symbols we use today. By about 1500 A.D., the Hindu-Arabic numbers had almost completely replaced Roman numerals through all of Europe. After the 1500s, with Hindu-Arabic numbers, mathematics began to develop quickly through Europe, leading to better mathematical tools for business and science. From that point on, Hindu-Arabic numerals became the most common number system used throughout the world.

Brahmi		—	=	≡	+	५	६	७	९	?
Hindu	०	१	२	३	४	५	६	७	८	९
Arabic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Medieval	o	I	2	3	४	ς	6	∧	8	9
Modern	0	1	2	3	4	5	6	7	8	9

From Apices to Modern Digits

X century	I	ट	ट	४	५	६	७	४	९
XI century	ı	ट	ट	४	५	६	७	४	९
XII century	ı	ट	४	४	५	६	७	४	९
Modern	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*

Charts courtesy of Archimedes' Laboratory.

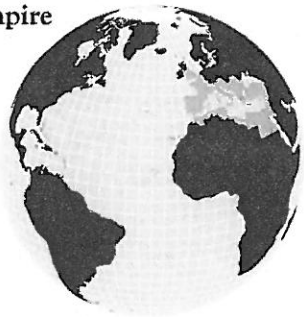
X. Where in the world are these ancient civilizations?

Egyptian Extent
1500 B.C.

Greek Hearth
& Colonies,
700 B.C.

Babylonia
1750 B.C.

Roman Empire
15 A.D.



Luba & Lunda
Empires
1600 A.D.

Mayan Extent
500 B.C.

Assignment 1.1 Name:_____

Use your notes to identify the civilization:

1. Groupings of ten with symbols from everyday life .....
2. A comma before the symbol multiplies the value by a thousand .....
3. A base 60 place value system .....
4. A zero as a place holder .....
5. Subtraction concept decreased the number of symbols needed .....
6. Oldest mathematical artifacts in existence .....
7. A base 10 place value system with a place holder .....

Write the following numbers in the indicated numeration system:

	Egyptian	Greek	Babylonian	Roman
1204				
95				
60				
3833				
Your age				

Respond to the following questions:

8. Since zero means "nothing," you should not have to write it and only leave a space. Do you agree? Why/why not?

9. Think about the number 676 in Roman, Babylonian and Egyptian numeration systems.

   a. Which system would require the greatest number of symbols?

   b. Which system would require the least number of symbols?

   c. Do your answers hold true for most numbers?

10. State some of the benefits in using the Hindu-Arabic numeration system.

Supplemental Exercises

Unit 1
Section 1.1

Write the following numbers in the Egyptian numeration system.

1. 4,301

2. 106

3. 31,055

4. 28

5. 999

Write the following numbers in the Greek numeration system.

6. 203

7. 59

8. 6,850

9. 998

10. 1,001

Write the following numbers in the Babylonian numeration system.

11. 39

12. 93

13. 141

14. 2,460

15. 7,572

Write the following numbers in the Roman numeration system.

16. 38

17. 449

18. 999

19. 1,046

20. 2,905