Linear Inequalities in One Variable

Professor Tim Busken

Department of Mathematics

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Learning Objectives:

- Solve a linear inequality in one variable and graph the solution set.
- Write solutions to inequalities using interval notation.
- Solve a compound inequality and graph the solution set.
- Solve application problems using inequalities.

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Linear Inequalities in One Variable

Definition

Replacing the equal sign in the general linear equation $a \cdot x + b = c$ by any of the symbols $<, \leq, >$ or $\geq$ gives a **linear inequality in one variable**.

For example, $2 \cdot x - 1 \leq 0$ and $3x + 5 > 8$ are two different linear inequalities in a single variable, x.

Solving Linear Inequalities

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For example, $\{x \mid x < -2\}$ is shorthand notation for the set of real numbers less than -2.

$\{x \mid x < -2\}$

x such that x is any real number less than -2

Addition Property for Inequalities

For any three algebraic expressions A, B and C,

$$\text{If} \quad A < B$$

$$\text{then} \quad A + C < B + C$$

In words: Adding the same quantity to both sides of an inequality will not change the solution set.

We can use the Addn. Prop. to write ***equivalent inequalities***.

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$$5x + 4 < 4x + 2$$

$$5x + 4 + (-4) < 4x + 2 + (-4)$$ Addition Prop. of Inequalities

$$5x + \left(4 + (-4)\right) < 4x + \left(2 + (-4)\right)$$ Associative Prop. of Addition

$$5x + 0 < 4x + (-2)$$ Additive Inverse & Closure Props.

$$5x < 4x - 2$$ Additive Identity & the Defn. of Subtraction

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$$5x + (-4x) < 4x - 2 + (-4x)$$ Addition Prop. of Inequalities

$$5x + (-4x) < 4x + (-4x) - 2$$ Commutative Prop. of Addn.

$$\Big(5x + (-4x)\Big) < \Big(4x + (-4x)\Big) - 2$$ Associative Prop. of Addn.

$$(5 - 4) \cdot x < 0 - 2$$ Distributive & Additive Inverse Props.

$$1 \cdot x < -2$$ Closure & Additive Identity Props.

$$x < -2$$ Multiplicative Identity Prop.

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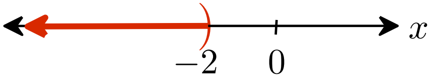
Graph: We can shade the number line to the left of -2 to give a graphical description of the solution set.



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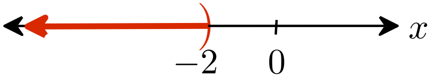

We use a left-opening parenthesis at -2 to indicate that -2 is not part of the solution set.

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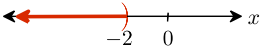

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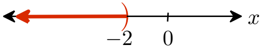
Set Notation	Line Graph	Interval Notation
$\{x \mid x < -2\}$		$(-\infty, -2)$

Multiplication Property of Inequalities

For any three algebraic expressions A, B and C, where $C \neq 0$,

$$\begin{array}{rll} \text{If} & A < B, & \\ \text{then} & C \cdot A < C \cdot B & \text{if } C \text{ is positive } (C > 0) \\ \text{or} & C \cdot A > C \cdot B & \text{if } C \text{ is negative } (C < 0) \end{array}$$

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If	$A < B$,	
then	$C \cdot A < C \cdot B$	if C is positive $(C > 0)$
or	$C \cdot A > C \cdot B$	if C is negative $(C < 0)$

In words: Multiplying both sides of an inequality by a positive quantity always produces an equivalent inequality. Multiplying both sides of an inequality by a negative number produces an equivalent inequality BUT it reverses the direction of the inequality symbol.

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Solution:

$-2x - 3 \leq 3$	
$-2x - 3 + 3 < 3 + 3$	Addition Prop. of Inequalities
$-2x \leq 6$	Additive Inverse & Identity Props
$\left(-\frac{1}{2}\right) \cdot (-2x) \geq \left(-\frac{1}{2}\right) \cdot 6$	Multiplication Prop. of Inequalities
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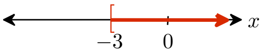

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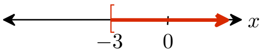
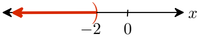
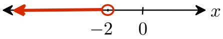


Interval Notation
$[-3, \infty)$

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$x < -2$	$(-\infty, -2)$		

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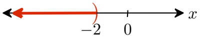
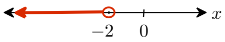
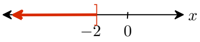
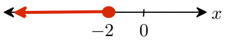
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		

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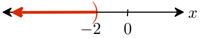
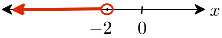
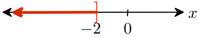
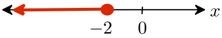
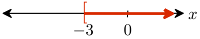
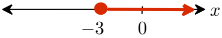
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		
$x \geq -3$	$[-3, \infty)$		

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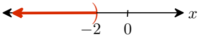
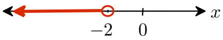
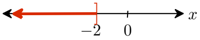
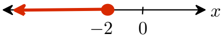
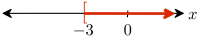
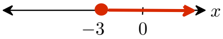
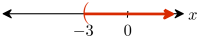
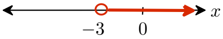
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		
$x \geq -3$	$[-3, \infty)$		
$x > -3$	$(-3, \infty)$		

Classroom Example: Solve the following inequality.

- $3(2x + 5) \leq -3x$

Classroom Examples: Take the next five minutes to work these 6 problems. Graph the solution set to the given inequality, then write the solution set using interval notation.

- $x \leq -6$
- $x > 5$
- $x \geq -1$
- $x > 10$

Classroom Examples: Solve each inequality. Graph the solution set, then write the solution set using interval notation.

- $2x - 1 \leq -6$
- $-3x < 2x - 6$

Definition

A compound inequality is two or more simple inequalities {sets} joined by the terms 'and' or 'or' .

For Example, the set $\left\{ x \;\middle|\; 3x - 6 \le -3 \quad \text{or} \quad 3x - 6 \ge 3 \right\}$ is a compound inequality.

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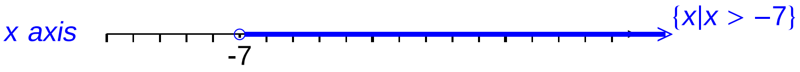

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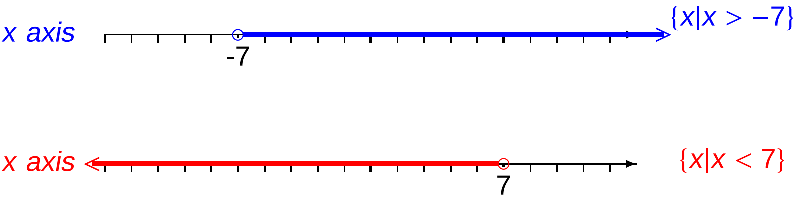

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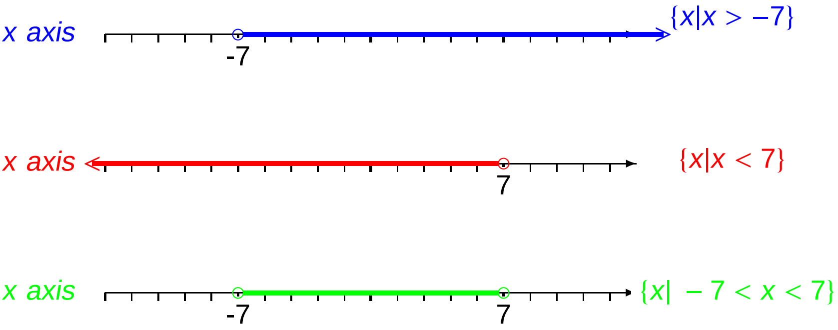

Linear Inequalities in One Variable

Classroom Examples: Solve the following compound inequalities. Graph the solution set on a number line, then write the solution set using interval notation.

- $-7 \leq 2x + 1 \leq 7$
- $3x - 6 \leq -3$ or $3x - 6 \geq 3$

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
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Theorem

Suppose a and b are any real numbers with the restriction that $b > 0$. Then the equation $|a| = b$ is equivalent to $a = b$ or $a = -b$.

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Classroom Examples: Solve the following absolute value equations.

- $|x| = 4$
- $|3x - 6| = 9$
- $|4x - 3| + 2 = 3$