10.2 Rational Numbers as Exponents **Math 51 Professor Busken**

Objectives

- Use the definition of $a^{1/d}$
- Use the definition of $a^{n/d}$
- Use the definition of $a^{-n/d}$
- Simplify expressions with rational exponents
- Simplify radical expressions using rational using rational exponents

Definition 1.

- $a^{1/d} = \sqrt[d]{a}$
- $a^{n/d} = \sqrt[d]{a^n} = \left(\sqrt[d]{a}\right)^n$
- $a^{-n/d} = \frac{1}{\sqrt[d]{a^n}}$

Examples

- $64^{1/2} = \sqrt{64} = 8; \qquad 8^{1/3} = \sqrt[3]{8} = 2;$
- $27^{5/3} = \left(\sqrt[3]{27}\right)^5 = 3^5 = 3\cdot 3\cdot 3\cdot 3\cdot 3 = 243$
- $16^{-3/4} = \frac{1}{16^{3/4}} \frac{1}{\left(\sqrt[4]{16}\right)^3} = \frac{1}{2^3} = \frac{1}{8}$
- $\left(\sqrt[5]{2xy}\right)^3 = (7xy)^{3/5}$

Properties of integer exponents are true for rational exponents. An expression with rational exponents is simplified when :

1. no parenthesis appear,
2. no powers are raised to powers,
3. each base occurs once
4. and no negative or zero exponents appear.

$$\left(64x^{1/3}y^{-1/2}\right)^{1/3} = 64^{1/3}\left(x^{1/3}\right)^{1/3}\left(y^{-1/2}\right)^{1/3}$$
$$= 4x^{1/9}y^{-1/6}$$
$$= \boxed{\frac{4x^{1/9}}{y^{1/6}}}$$

Some radical expressions can be simplified using rational exponents.

1. Convert radical expressions to exponential expressions.
2. Use arithmetic and properties of exponents to simplify.
3. Convert back to radical notation.

Examples

$$\sqrt[6]{x^2} = x^{2/6} = x^{1/3} = \boxed{\sqrt[3]{x}}$$

$$\sqrt[5]{x^2}\cdot\sqrt[4]{x} = x^{2/5}\cdot x^{1/4}$$
$$= x^{2/5+1/4} = x^{8/20+5/20}$$
$$= x^{13/20} = \boxed{\sqrt[20]{x^{13}}}$$

Exercises

1. **Write each expression using radical notation and then simplify the result, if possible.**

a) $(-8)^{1/3}$ b) $(144)^{1/2}$ c) $-(144)^{1/2}$ d) $(-144)^{1/2}$

e) $\left(-144^{\frac{1}{2}}\right)$ f) $-(81)^{1/4}$ g) $(xyz)^{1/4}$ h) $(25x^6)^{1/2}$

2. **Write an equivalent expression using exponential notation.**

a) $\sqrt{x}$ b) $\sqrt[3]{y}$ c) $\sqrt[5]{b^3}$ d) $\sqrt{5a}$

e) $\left(\sqrt[6]{3x}\right)^5$ f) $\left(\sqrt[7]{2x^2y}\right)^3$

3. **Simplify as much as possible.**

a) $9^{3/2}$ b) $16^{3/4}$ c) $8^{-2/3}$ d) $\left(\frac{16}{81}\right)^{-3/4}$

4. **Write an equivalent expression with positive exponents and simplify, if possible.**

a) $16^{-1/4}$ b) $x^{-1/2}$ c) $\frac{1}{x^{-2/3}}$ d) $4x^3y^{-1/2}$

e) $(5xy)^{-4/5}$ f) $\left(\frac{1}{27}\right)^{-2/3}$

5. **Use laws of exponents and simplify, if possible. Do not use negative exponents in your answer.**

a) $\frac{x^{1/3}}{x^{2/3}}$ b) $\left(x^5\right)^{3/5}$ c) $\left(x^{2/5}\right)^{15}$ d) $x^{\frac{1}{3}} \cdot x^{\frac{5}{3}}$

e) $\frac{x^{-1/3}}{x^{-3/4}}$ f) $\left(5x^{1/7}y^{1/7}\right)^3$ g) $(9^{-4/3})^{3/4}$ h) $\left(x^{-1/4}y^{1/4}\right)^{1/4}$

6. **Use rational exponents to simplify. Write the answer in radical notation, if appropriate.**

a) $\sqrt[6]{x^2}$ b) $\sqrt{4x^2}$ c) $\sqrt[3]{(5x)^9}$ d) $\left(\sqrt[6]{7x}\right)^4$

e) $\sqrt[3]{-8x^6y^9}$ f) $\sqrt[4]{\frac{b^{12}}{16}}$ g) $\sqrt[5]{-32m^5}$

Answers: 1a) -2 1b) 12 1c) -12 1d) not a real number 1e) -12 1f) -3 1g) $\sqrt[4]{xyz}$ 1h) $|5x^3|$, 2a) $x^{1/2}$ 2b) $y^{1/3}$ 2c) $b^{3/5}$ 2d) $(5a)^{1/2}$ 2e) $(3x)^{5/6}$ 2f) $(2x^2y)^{3/7}$, 3a) 27 3b) 8 3c) 1/4 3d) 27/8 4a) 1/2 4b) $\frac{1}{x^{1/2}}$ 4c) $x^{2/3}$ 4d) $\frac{4x^3}{y^{1/2}}$ 4e) $\frac{1}{(5xy)^{4/5}}$ 4f) 9 5a) $\frac{1}{x^{1/3}}$ 5b) x^3, 5c) x^6, 5d) x^2, 5e) $x^{5/12}$, 5f) $125x^{3/7}y^{3/7}$, 5g) 1/9 5h) $\frac{y^{1/16}}{x^{1/16}}$ 6a) $|x^3|$ 6b) $|2x|$, 6c) $(5x)^3$, 6d) $\sqrt[3]{49x^2}$ 6e) $-2x^2y^3$, 6f) $b^3/2$, 6g) $-2m$