

Objectives

- Use the Product Rule to Multiply Radical Expressions
- Use factoring and the product rule to simplify radicals
- Multiply radicals then simplify

A number that is the square of a rational number is called a **perfect square number**. A number is a **perfect cube** if it is the cube of a rational number. An expression is called a **perfect square expression** whenever you can write it as some other expression squared. An expression is called a perfect cube expression whenever you can write it as another expression cubed. An expression is called a **perfect fourth-powered expression** when it can be written as some other expression raised to the fourth power. And, in general, an expression is called a **perfect n^{th} -powered expression** when it can be written as some other expression raised to the n^{th} power. In this section, we use the multiplication property for radicals to factor out the largest perfect n^{th} -powered expression from underneath the radical symbol.

Multiplication Property of Radicals: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$

Exercises

1. Use the product property to simplify.

a) $\sqrt{5} \cdot \sqrt{6}$ b) $\sqrt{x+4} \cdot \sqrt{x-4}$ c) $\sqrt[3]{7} \cdot \sqrt[3]{3}$ d) $\sqrt[5]{3x} \cdot \sqrt[5]{4x^2}$

2. Simplify. Assume all variables in a radicand represent positive real numbers and no radicands involve negative quantities raised to even powers.

a) $\sqrt{90}$ b) $\sqrt{200}$ c) $\sqrt{75x^5}$ d) $\sqrt{128x^3}$
 e) $\sqrt{x^6y^9z^8}$ f) $\sqrt[3]{16x^{10}y^{15}}$ g) $\sqrt[5]{32x^9y^7z^{10}}$ h) $\sqrt[5]{96x^8y^6}$

3. Multiply and simplify, if possible. Assume all variables in a radicand represent positive real numbers and no radicands involve negative quantities raised to even powers.

a) $\sqrt{3} \cdot \sqrt{4}$ b) $\sqrt{18} \cdot \sqrt{50}$ c) $5\sqrt[3]{2} \cdot 10\sqrt[3]{16}$ d) $\sqrt[3]{5x^2} \cdot \sqrt[3]{50x^2}$
 e) $\sqrt[4]{8x^2y^4} \cdot \sqrt[4]{4x^5y^2}$ f) $\sqrt[3]{(x+3y)^5} \cdot \sqrt[3]{(x+3y)^4}$

Answers: 1a) $\sqrt{30}$ 1b) $\sqrt{x^2-16}$ 1c) $\sqrt[3]{21}$ 1d) $\sqrt[5]{12x^3}$ 2a) $3\sqrt{10}$ 2b) $10\sqrt{2}$
 2c) $5x^2\sqrt{3x}$ 2d) $8x\sqrt{2x}$ 2e) $x^3y^4z^4\sqrt{y}$
 2f) $2x^3y^5\sqrt[3]{2x}$, 2g) $2xyz^2\sqrt[5]{x^4y^2}$, 2h) $2xy\sqrt[5]{3x^3y}$, 3a) $2\sqrt{3}$ 3b) 30
 3c) $100\sqrt[3]{4}$ 3d) $5x\sqrt[3]{2x}$ 3e) $2xy\sqrt[4]{2x^3y^2}$ 3f) $(x+3y)^3$

Notes:

- A radical of index n is simplified when its radicand has no factors other than 1 that are perfect n^{th} powered expressions.
- Perfect n^{th} powered expressions have exponents that are divisible by n .