

10.5 Adding, Subtracting, and Multiplying Radical Expressions

Learning Objectives:

1. Add or subtract radical expressions.
2. Multiply radical expressions.

Examples:

1. Add or subtract as indicated. Assume that all variables represent positive real numbers.

a) $\sqrt{63} - \sqrt{7}$

b) $-3\sqrt{200} - 5\sqrt{8} + 9\sqrt{98}$

c) $\sqrt{300x^3} - x\sqrt{12x}$

d) $\sqrt[3]{8x} - \sqrt[3]{27x}$

e) $7\sqrt[3]{x^3y^{13}} + 5xy\sqrt[3]{8y^{10}}$

f) $\frac{2\sqrt{2}}{3} + \frac{3\sqrt{2}}{5}$

g) $\frac{2x\sqrt{11}}{5} + \sqrt{\frac{11x^2}{100}}$

h) $10\sqrt[4]{x^7} - 2x\sqrt[4]{x^3}$

i) $\sqrt{\frac{20}{x^2}} + \sqrt{\frac{5}{4x^2}}$

2. Multiply. Then simplify if possible. Assume that all variables represent positive real numbers.

a) $\sqrt{6}(\sqrt{5} + \sqrt{7})$

b) $\sqrt{7}(\sqrt{11} + \sqrt{7})$

c) $(\sqrt{7} - \sqrt{2})^2$

d) $\sqrt{2x}(\sqrt{2} - \sqrt{x})$

e) $(6\sqrt{y} + z)(3\sqrt{y} - 1)$

f) $(\sqrt[3]{x} + 5)(\sqrt[3]{x} + 2)$

g) $(5\sqrt{3} + 9)(6\sqrt{3} - 4)$

h) $(\sqrt{x-4} + 3)^2$

i) $(\sqrt[3]{x} + 7)(\sqrt[3]{x} - 7\sqrt{x} + 2)$

Answers: 1a) $2\sqrt{7}$; b) $23\sqrt{2}$; c) $8x\sqrt{3x}$; d) $-\sqrt[3]{x}$; e) $17xy^4\sqrt[3]{y}$; f) $\frac{19\sqrt{2}}{15}$; g) $\frac{x\sqrt{11}}{2}$; h) $8x^4\sqrt{x^3}$; i) $\frac{5\sqrt{5}}{2x}$;
2a) $\sqrt{30} + \sqrt{42}$; b) $\sqrt{77} + 7$; c) $9 - 2\sqrt{14}$; d) $2\sqrt{x} - x\sqrt{2}$; e) $18y + (3z - 6)\sqrt{y} - z$; f) $\sqrt[3]{x^2} + 7\sqrt[3]{x} + 10$;
g) $54 + 34\sqrt{3}$; h) $x + 5 + 6\sqrt{x-4}$; i) $\sqrt[3]{x^2} - 7\sqrt[6]{x^5} + 9\sqrt[3]{x} - 49\sqrt{x} + 14$

Objectives

- Rationalize denominators or numerators that have one term
- Terms with different indices

Exercises

4. **Rationalize each denominator. Assume all variables represent positive numbers.**

a) $\frac{6}{2-\sqrt{7}}$ b) $\frac{\sqrt{5} + \sqrt{2x}}{\sqrt{5} - \sqrt{2x}}$ c) $\frac{3\sqrt{2} + 5\sqrt{3}}{5\sqrt{3} - 4\sqrt{2}}$

5. **Rationalize each numerator. Assume all variables represent positive numbers.**

a) $\frac{2-\sqrt{11}}{6}$ b) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ c) $\frac{\sqrt{8}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

6. **Perform the indicated operation. Assume all variables represent positive numbers.**

a) $\sqrt[3]{x} \sqrt[6]{x}$ b) $\sqrt{xy^3} \sqrt[3]{x^2y}$ c) $\sqrt[4]{xy^2} \left(\sqrt[3]{xy} - \sqrt[5]{x^2y^3} \right)$

Answers: 4a) $-2(2 + \sqrt{7})$ 4b) $\frac{5+2\sqrt{10x+2x}}{5-2x}$ 4c) $\frac{35\sqrt{6}+99}{43}$

5a) $\frac{-7}{12+6\sqrt{11}}$ 5b) $\frac{1}{3+2\sqrt{2}}$ 5c) $\frac{5}{7+3\sqrt{6}}$

6a) $\sqrt{x}$ 6b) $xy \sqrt[6]{xy^5}$ 6c) $\sqrt[12]{x^7y^{10}} - y \sqrt[20]{x^{13}y^2}$