Complex Numbers **Name:** ______________________

Definition 1. *A **complex number** is an expression of the form*

$$a + bi$$

where a and b are real numbers and $i^2 = -1$*. The **real part** of this complex number is a and the **imaginary part** is b. Two complex numbers are **equal** if and only if their real parts are equal and their imaginary parts are equal.*

Examples of complex numbers

$4 + 5i$	real part 4, imaginary part 4
$1 - i$	real part 1, imaginary part -1
$6i$	real part 0, imaginary part 6
-7	real part -7, imaginary part 0

Addition	**Description**
$(a + bi) + (c + di) = (a + c) + (b + d)i$	To add complex numbers, add the real parts and the imaginary parts.
Subtraction	
$(a + bi) - (c + di) = (a - c) + (b - d)i$	To subtract complex numbers, subtract the real parts and the imaginary parts.
Multiplication	
$(a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i$	Multiply complex numbers like binomials, using $i^2 = -1$.

Express the following in the form $a + bi$**.**

1. $(5 + 2i) + (3 + 8i)$ 1. __________

2. $(1 - 2i) - (5 - 3i)$ 2. __________

3. $(1 - 2i)(5 - 3i)$ 3. __________

4. $(7 + 3i)(4 + 12i)$ 4. __________

5. i^9 5. __________

6. i^{22} 6. __________

Definition 2. *For a complex number $z = a + bi$, we define its* ***complex conjugate*** *to be $\bar{z} = a - bi$. Note that*

$$z \cdot \bar{z} = (a + bi)(a - bi) = a^2 + b^2$$

For example, the conjugate of $7 + 5i$, written as $\overline{7 + 5i}$, equals $7 - 5i$.

7. $\overline{1 - 2i}$ 7. ____________

8. $\overline{2 + 3i}$ 8. ____________

9. $\overline{-i}$ 9. ____________

10. $\overline{-2}$ 10. ____________

Definition 3. *Dividing Complex Numbers*

To simplify the quotient $\frac{a + bi}{c + di}$, multiply by 1 in the form of the denominator's conjugate divided by itself.

$$\frac{a + bi}{c + di} = \frac{a + bi}{c + di} \cdot \left(\frac{c - di}{c - di}\right) = \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$$

Divide.

11. $\frac{8 - 3i}{4 + i}$ 11. ____________

12. $\frac{5 + 2i}{8i}$ 12. ____________

Definition 4. *Square Roots of Negative Numbers If $-r$ is negative, then the positive (principal) square root of $-r$ is*

$$\sqrt{-r} = i\sqrt{r}$$

The two square roots of $-r$ are $i\sqrt{r}$ and $-i\sqrt{r}$

We usually write $i\sqrt{b}$ instead of $\sqrt{b}i$ to avoid confusion with $\sqrt{bi}$.

Simplify.

13. $\sqrt{-25}$ — 13. ____________

14. $\sqrt{-5}$ — 14. ____________

15. $-\sqrt{-169}$ — 15. ____________

CAUTION

$\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$ when both a and b are positive, this is not true when both a and b are negative. For instance

$$\sqrt{-2}\cdot\sqrt{-3} = i\sqrt{2}\cdot i\sqrt{3} = i^2\sqrt{6} = -\sqrt{6}$$

but

$$\sqrt{(-2)\cdot(-3)} = \sqrt{6}$$

so

$$\sqrt{-2}\cdot\sqrt{-3} \neq \sqrt{(-2)\cdot(-3)}$$

Evaluate and express in the form $a + bi$.

16. $(\sqrt{12} - \sqrt{-3})(\sqrt{3} + \sqrt{-4})$ — 16. ____________

17. $(\sqrt{3} - \sqrt{-5})(1 + \sqrt{-1})$ — 17. ____________

We have already seen that if $a \neq 0$, the solutions of the quadratic equation $ax^2 + bx + c$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac < 0$, then the equation has no real solution. But in the complex number system, this equation will always have solutions, because negative numbers have square roots in this expanded setting.

Solve each equation.

18. $9x^2 + 4 = 0$

18. ____________

19. $x^2 + 2x + 2 = 0$

19. ____________

20. $x^2 + \frac{1}{2}x + 1 = 0$

20. ____________