11.1 Quadratic Equations and Functions **Math 51 Professor Busken**

Objectives

- Know when and how to apply the Square Root Property to solve a quadratic equation
- Solve quadratic equations by "Completing the Square" (CTS) and then using the Square Root Property
- Find x-intercepts of quadratic functions

> **The Square Root Property**
>
> Suppose x is any algebraic expression and k is a real number.
>
> $$\text{If } x^2 = k \quad \text{then} \quad x = \sqrt{k} \quad \text{or} \quad x = -\sqrt{k}$$

Exercises

1. **Solve. Find all complex number solutions.**

a) $x^2 = 144$ b) $x^2 = -144$ c) $2x^2 = -8$ d) $16x^2 + 26 = 1$

e) $(x+1)^2 = -4$ f) $5(x-7)^2 = -125$ g) $3(x+3)^2 = 54$ h) $(x-23)^2 = 53$

Answers: 1a) 12 or -12 1b) $12i$ or $-12i$ 1c) $2i$ or $-2i$ 1d) $\frac{5}{4}i$ or $-\frac{5}{4}i$
1e) $-1 \mp 2i$ 1f) $7 \mp 5i$ 1g) $-3 \mp 3\sqrt{2}$ 1h) $23 \mp \sqrt{53}$

Definition 1. *A* ***monomial*** *is any expression of the form $a \cdot x^n$, where n is a whole number and a is a real number. A monomial is a real number times a variable raised to a whole number power.*

For example, $2x$, $5x^3$, 8, and $\frac{3}{5}x^2$ are monomials, but $2x^{1/2}$, $5x^{-1}$, and $\frac{3}{5x}$ are not monomials.

Definition 2. *Expressions that are sums and/or differences of monomials are called* ***polynomials****. Any monomial is also a polynomial.*

For example, $2x$, $5x^3 - 2$, $x^2 + 3x - 1$ and $\frac{3}{5}x^3 + 3x^2 - x - 1$ are polynomials.

Definition 3. *A polynomial that is a sum or difference of two monomials is called a* ***binomial****. A polynomial that is a sum or difference of three monomials is called a* ***trinomial****.*

For example, $5x^3 - 2$ is a binomial and $x^2 + 3x - 1$ is a trinomial.

Definition 4. *A number is called a* ***perfect-square number*** *when it can be written as a rational number squared (raised to the second power).*

For example, 25 is a perfect-square number since it can be written as 5^2.

Definition 5. *An algebraic expression is called a* ***perfect-square expression*** *when it can be written as another expression squared (raised to the second power).*

For example, $25x^2$ is a perfect-square expression since it can be written as $(5x)^2$. $(x+1)^2$ is another example of a perfect-square expression.

Definition 6. *A trinomial is called a* ***perfect-square trinomial*** *when it factors as a binomial squared.*

For example, x^2+2x+1 is a perfect-square trinomial since it factors as $(x+1)(x+1)$ or $(x+1)^2$.

We can solve quadratic equations by "completing the square," then use the square root property.

Completing the Square (CTS) Formula

Suppose b is a real number. We add $\left(\frac{b}{2}\right)^2$ to binomials of the form $x^2+b\cdot x$ in order to **complete the square**. Once $\left(\frac{b}{2}\right)^2$ is added to $x^2+b\cdot x$, the resulting trinomial is a perfect-square trinomial, $x^2+bx+(b/2)^2$, that has factorization

$$x^2+bx+\left(\frac{b}{2}\right)^2=\left(x+\frac{b}{2}\right)^2$$

Example Find all complex number solutions of $x^2+12x+1=0$. Solve by completing the square (CTS), then using the square root property.

Solution Steps

1. Isolate x^2+bx on one side of an equivalent equation.
2. Identify the values of b, $b/2$ and $(b/2)^2$.
3. Add $(b/2)^2$ to both sides of the equation.
4. Write an equivalent equation, but rewrite the left hand side as $\left(x+\frac{b}{2}\right)^2$.
5. Apply the Square Root Property.
6. Factor all radicals and write complex solutions in the form $a+bi$.

$x^2+12x=-1$	1)	Subtract 1 from both sides.
	2)	$b=12$, $b/2=12/2=6$ and $(b/2)^2=6^2=36$.
$x^2+12x+36=-1+36$	3)	Add $(b/2)^2$ to both sides
$(x+6)^2=35$	4)	Write an equivalent equation, but rewrite the left hand side as $\left(x+\frac{b}{2}\right)^2$
$x+6=\sqrt{35}$ or $x+6=\sqrt{35}$	5)	Apply the Square Root Property.
$x=-6+\sqrt{35}$ or $x=-6-\sqrt{35}$	6)	Add -6.
$\boxed{x=-6\pm\sqrt{35}}$		two solutions

Exercises

2. **Find all complex number solutions. Solve by completing the square (CTS), then using the square root property.**

   a) $x^2+6x=7$ b) $x^2-4x+1=0$ c) $x^2+3x-2=0$

   d) $2x^2-x+6=0$ e) $4x^2+8x+3=0$

Answers: 2a) $-7,\ 1$ 2b) $2\pm\sqrt{3}$ 2c) $\frac{-3-\sqrt{17}}{2}$ *or* $\frac{-3+\sqrt{17}}{2}$ 2d) $\frac{1-i\sqrt{47}}{4}$ *or* $\frac{1+i\sqrt{47}}{4}$
2e) $-3/2,\ -1/2$