4.6 Applications **Name:** ________________________

1. Environmental scientists measure the intensity of light at various depths in a lake to find the "transparency" of the water. Certain levels of transparency are required for the biodiversity of the submerged macrophyte population. In a certain lake the intensity of light at depth x is given by

$$I = 10e^{-0.008x}$$

where I is measured in lumens and x in feet.

(a) Find the intensity I at a depth of 30ft.

(b) At what depth has the light intensity dropped to $I = 5$?

Compound Interest

If a principal P is invested at an interest rate r for a period of t years, then the amount A of the investment is given by

$A = P(1 + r)$	Simple interest (for one year)
$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$	Interest compounded n times per year
$A(t) = Pe^{rt}$	Interest compounded continuously

2. A sum of \$1500 is invested at an interest rate of 8% per year. Find the time required for the money to double if the interest is compounded according to the following method.

(a) Semiannually

(b) Continuously

Exponential Growth (Doubling Time) If the initial size of a population is n_0 and the doubling time is a, then the size of the population at time t is

$$n(t) = n_0 2^{t/a}$$

where a and t are measured in the same time units (minutes, hours, days, years, and so on).

3. Under ideal conditions a certain bacteria population doubles every three hours. Initially there are 200 bacteria in a colony.

   (a) Find a model for the bacteria population after t hours.

   (b) How many bacteria are in the colony after 15 hours?

   (c) When will the bacteria count reach 100,000?

4. A certain breed of rabbit was introduced onto a small island 8 months ago. The current rabbit population on the island is estimated to be 4100 and doubling every 3 months.

   (a) What was the initial size of the rabbit population?

   (b) Estimate the population one year after the rabbits were introduced to the island.

   (c) Sketch a graph of the rabbit population.

Exponential Growth (Relative Growth Rate) A population that experiences exponential growth increases according to the model

$$n(t) = n_0 e^{rt}$$

where
$n(t)$ = population at time t
n_0 = initial size of the population
r = relative rate of growth (expressed as a proportion of the population)
t = time

5. The population of a certain species of fish has a relative growth rate of 1.2% per year. It is estimated that the population in 2000 was 12 million.

   (a) Find an exponential model $n(t) = n_0 e^{rt}$ for the population t years after 2000.

   (b) Estimate the fish population in the year 2005.

   (c) Sketch a graph of the fish population.

6. It is observed that a certain bacteria culture has a relative growth rate of 12% per hour, but in the presence of an antibiotic the relative growth rat is reduced to 5% per hour. The initial amount of bacteria in the culture is 22. Find the projected population after 24 hours for the following conditions.

   (a) No antibiotic is present, so the relative growth rate is 12%.

   (b) An antibiotic is present in the culture, so the relative growth rate is reduced to 5%.

7. The bat population in a certain Midwestern county was 350,000 in 2009, and the observed doubling time for the population is 25 years.

(a) Find an exponential model $n(t) = n_0 2^{t/a}$ for the population t years after 2009.

(b) Find an exponential model $n(t) = n_0 e^{rt}$ for the population t years after 2009.

(c) Estimate when the population will reach 2 million.

Radioactive Decay Model If m_0 is the initial mass of a radioactive substance with half-life h, then the mass remaining at time t is modeled by the function

$$m(t) = m_0 e^{-rt}$$

where $r = \frac{\ln 2}{h}$

8. The half-life of cesium-137 is 30 years. Suppose we have a 10 gram sample.

(a) Find a function $m(t) = m_0 2^{-t/h}$ that models the mass remaining after t years.

(b) Find a function $m(t) = m_0 e^{-rt}$ that models the mass remaining after t years.

(c) How much of the sample will remain after 80 years?

(d) After how long will only 2 grams of the sample remain?

Newton's Law of Cooling If D_0 is the initial temperature difference between an object and its surroundings, and if its surroundings have temperature T_s, then the temperature of the object at time t is modeled by the function

$$T(t) = T_s + D_0 e^{-kt}$$

where	$n(t)$	= population at time t
	n_0	= initial size of the population
	k	= relative rate of growth (expressed as a proportion of the population)
	t	= time

9. Newton's Law of Cooling is used in homicide investigations to determine the time of death. The normal body temperature is 98.6°F. Immediately following death, the body begins to cool. It has been determined experimentally that the constant in Newton's Law of Cooling is approximately $k = 0.1947$, assuming that time is measured in hours. Suppose that the temperature of the surroundings is 60°F.

   (a) Find a function $T(t)$ that models the temperature t hours after death.

   (b) If the temperature of the body is now 72°F, how long ago was the time of death.