

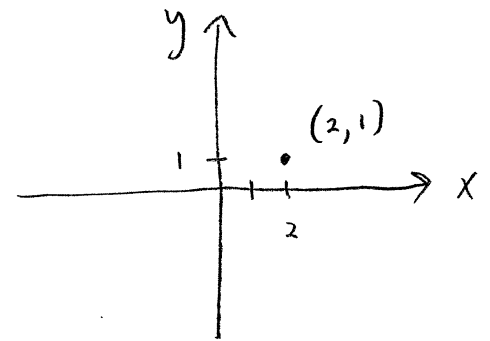
## 7.1 Functions

### Learning Objectives

- Know the difference between a relation and a function
  - Identify if a relation is a function
  - Understand the meaning and use of function notation.
  - Use a graph to identify functional values
  - Determine if a graph is a function
  - Find functional values given the function's definition in terms of the independent variable
- 

Defn A mathematical relation is any set of ordered pairs.

Recall that  $(2, 1)$  is an ordered pair located in the cartesian coordinate system.



Ex  $R$  is the relation with 3 ordered pairs

$$R = \{ (1, 2), (3, 4), (5, 6) \}$$

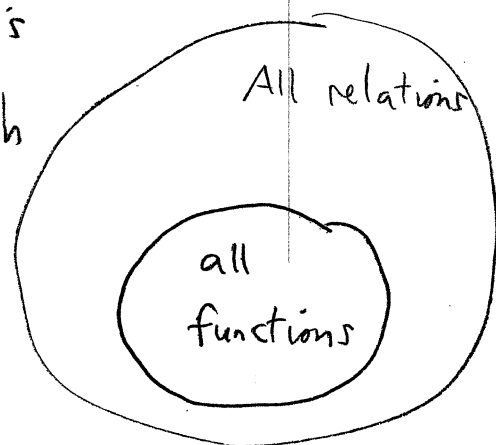
Defn A mathematical relation is correspondence of the numbers from a first set, called a domain, to the numbers of a second set, called the range.

Defn The domain of a relation is the set formed by all of the first coordinates (x), and the range of a relation is the set of all (y) second coordinates.

Ex Recall the relation  $R = \{(1,2), (3,4), (5,6)\}$  from before. What is the range and domain of  $R$ ?

answer:  $\text{dom}(R) = \{1, 3, 5\}$  and  $\text{rng}(R) = \{2, 4, 6\}$

Defn: (Function) A function is a special type of relation in which each number from the domain is paired with 1 and only 1 number from the range.



Ex Is  $R$  a function?

All functions are relations, but not all relations are functions.

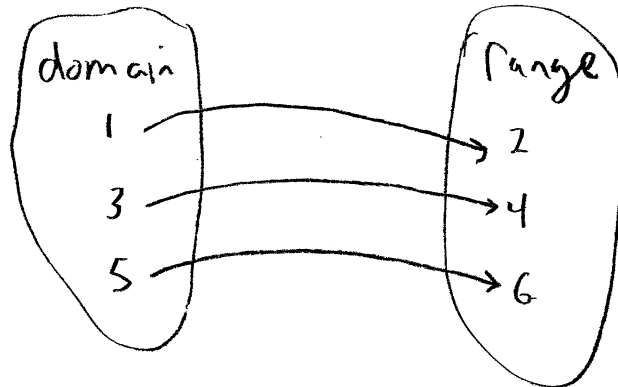
Answer: If we can find a value

from the domain which is paired with more than one number from the range, then we have a relation which is not a function.

Other wise, the relation is a function.

Ex Is  $R = \{ (1,2), (3,4), (5,6) \}$  a function?

One way to do an inspection is to make a relational diagram by taking the numbers from the domain and pointing them over to the range like so



Since we cannot find any  $x$  value that is paired with (or pointing to) more than one  $y$ -value,  $R$  is a function.

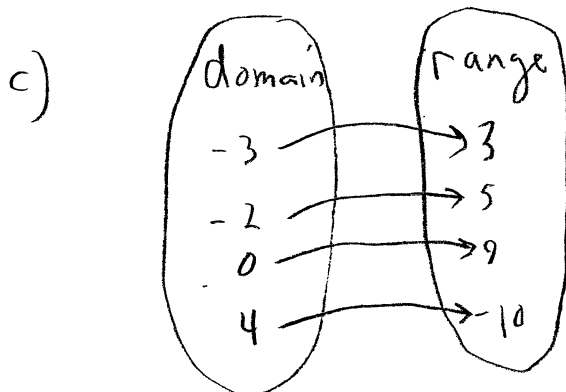
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Do Problems 9-14 all on page 448 (next page)

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#21 a)  $\text{dom} = \{ -3, -2, 0, 4 \}$

b)  $\text{rng} = \{ 3, 5, 9, -10 \}$



Since we cannot find an  $x$  value which points to more than 1  $y$ -value, it is a function.

**SOLUTION** We find  $c(8)$ :

7. The area of a square with side  $s$  is given by  $A(s) = s^2$ . What is the area of a square with sides of 9 m?

$$c(8) = \frac{24(8)}{8 + 12} = \frac{192}{20} = 9.6.$$

The dosage for a typical 8-year-old child is 9.6 mg.

YOUR TURN

## 7.1 EXERCISE SET

FOR EXTRA HELP

MyMathLab Math XL

PRACTICE



READ



### Vocabulary and Reading Check

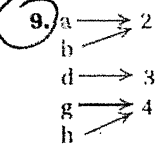
Choose the word from the following list that best completes each statement. Words may be used more than once.

- |                |            |
|----------------|------------|
| correspondence | horizontal |
| domain         | range      |
| exactly        | vertical   |
| " $f$ of 3"    |            |

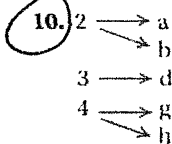
- A function is a special kind of \_\_\_\_\_ between two sets.
- In any function, each member of the domain is paired with \_\_\_\_\_ one member of the range.
- For any function, the set of all inputs, or first values, is called the \_\_\_\_\_.
- For any function, the set of all outputs, or second values, is called the \_\_\_\_\_.
- When a function is graphed, members of the domain are located on the \_\_\_\_\_ axis.
- When a function is graphed, members of the range are located on the \_\_\_\_\_ axis.
- The notation  $f(3)$  can be read \_\_\_\_\_.
- The \_\_\_\_\_-line test is used to determine whether or not a graph represents a function.

### Domain and Range

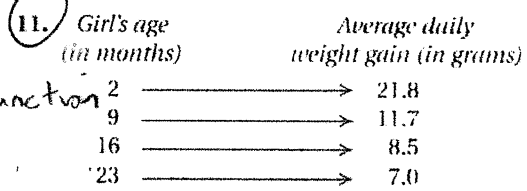
Determine whether each correspondence is a function.



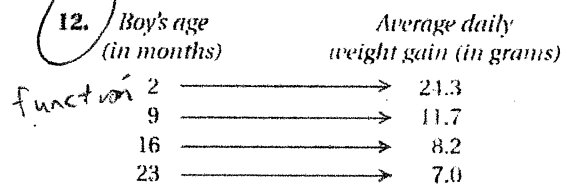
function



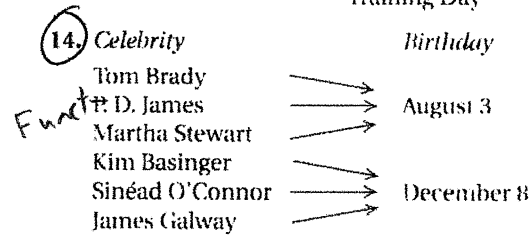
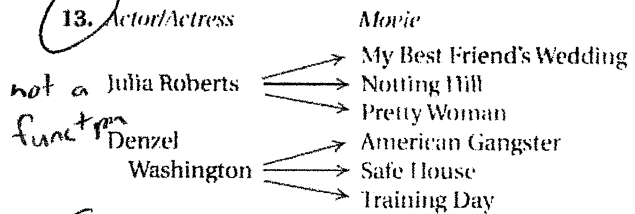
not a function



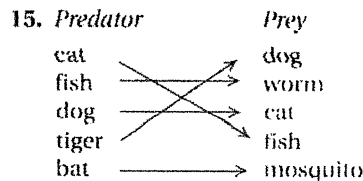
Source: American Family Physician, December 1993, p. 1435



Source: American Family Physician, December 1993, p. 1435



Source: www.leannesbirthdays.com



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16. State                      Neighboring state
- |          |   |            |
|----------|---|------------|
| Texas    | → | Oklahoma   |
|          | → | New Mexico |
| Colorado | → | Arkansas   |
|          | → | Louisiana  |

Determine whether each of the following is a function.

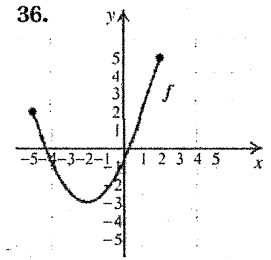
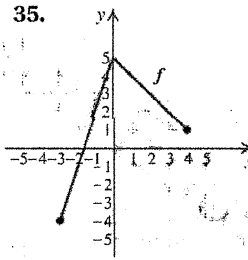
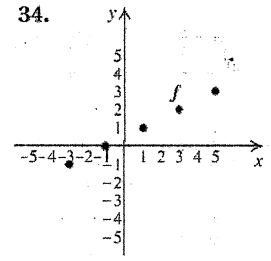
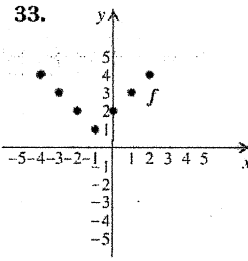
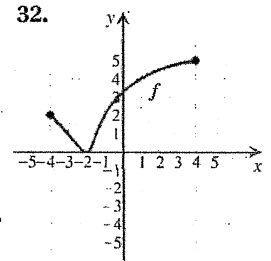
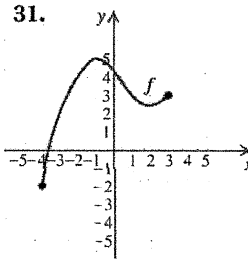
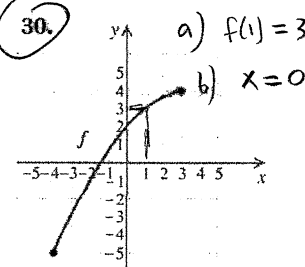
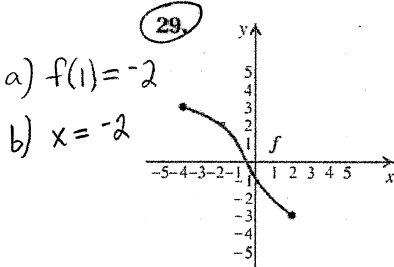
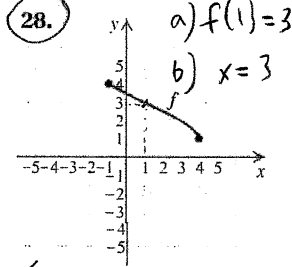
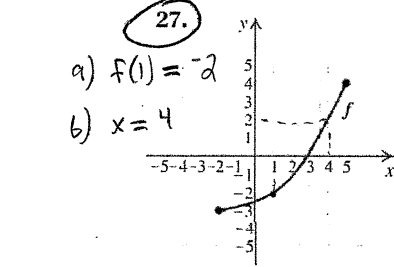
17. The correspondence that assigns to a USB flash drive its storage capacity
18. The correspondence that assigns to a member of a rock band the instrument the person can play
19. The correspondence that assigns to a player on a team that player's uniform number
20. The correspondence that assigns to a triangle its area

For each correspondence, (a) write the domain, (b) write the range, and (c) determine whether the correspondence is a function.

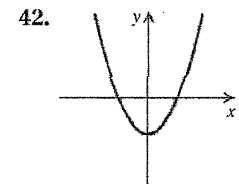
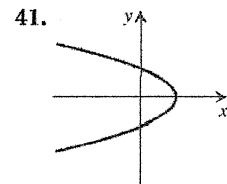
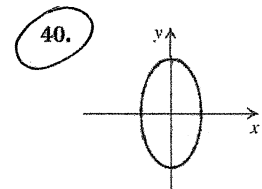
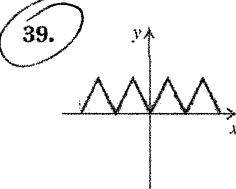
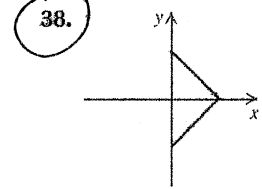
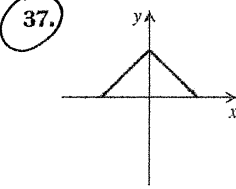
21.  $\{(-3, 3), (-2, 5), (0, 9), (4, -10)\}$
22.  $\{(0, -1), (1, 3), (2, -1), (5, 3)\}$
23.  $\{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1)\}$
24.  $\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5)\}$
25.  $\{(4, -2), (-2, 4), (3, -8), (4, 5)\}$
26.  $\{(0, 7), (4, 8), (7, 0), (8, 4)\}$

**Functions and Graphs**

For each graph of a function, determine (a)  $f(1)$  and (b) any  $x$ -values for which  $f(x) = 2$



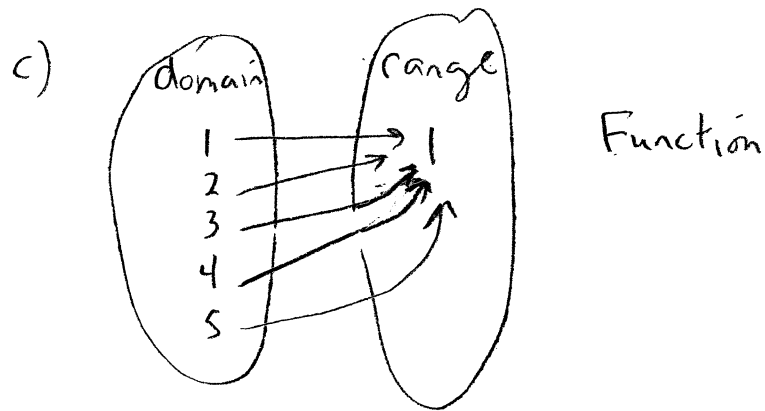
Determine whether each of the following is the graph of a function.



# 23

a)  $\text{dom} = \{1, 2, 3, 4, 5\}$

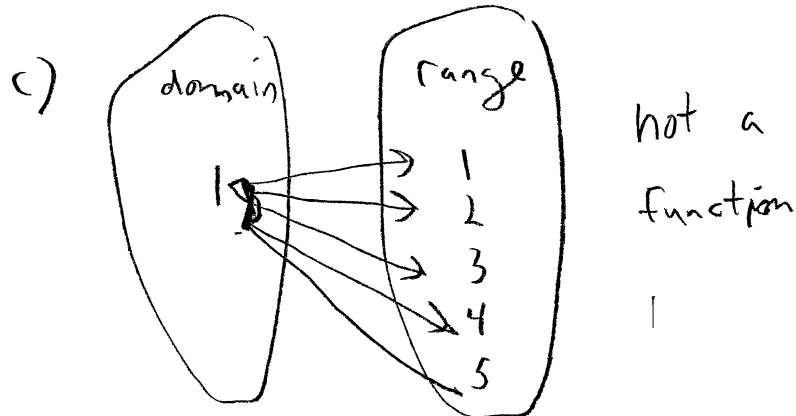
b)  $\text{rng} = \{1\}$



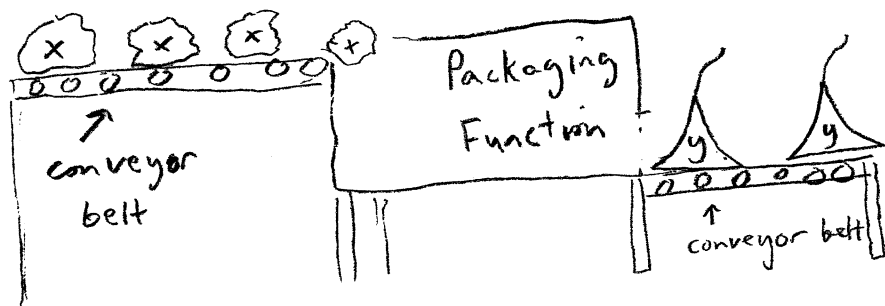
# 24

a)  $\text{dom} = \{1\}$

b)  $\text{rng} = \{1, 2, 3, 4, 5\}$



The defn of a function can be represented by a "function machine." In our machine diagram below, we make hershey's kisses. Chocolate chunks (x values) go into the packaging function and come out as packaged (and shaped) chocolate kisses (y-values). For each chocolate chunk going into the machine, we get 1 and only one packaged chocolate kiss coming out of the machine.



It is okay that distinctly different chocolate chunks (x-values) going into the machine are associated with chocolate kisses all having the same shape and packaging (the same y-value).<sup>that is,</sup> Having different x-values paired with the same y-value does not violate the definition of a function.

# Function Notation

" $f(x)$ " is called function notation.

" $f(x)$ " does not mean " $f$  times  $x$ "

" $f(x)$ " and  $y$  are synonymous, or having the same meaning

" $f(x)$ " is notation for the  $y$  value (or output value) of the function that is associated with a particular  $x$  value.

For example,

$f(1)$  is notation for the  $y$ -value associated with  $x=1$ .

$f(3)$  is notation for the  $y$ -value associated with  $x=3$ .

Example Consider the function  $f$  defined as

$$f = \{ (1, 2), (3, 4), (5, 6) \}$$

(a) What is  $f(1)$ ?

(b) What is  $f(3)$ ?

(c) What value of  $x$  is paired with  $f(x) = 6$ ?

Answers

$$f(1) = 2$$

$$f(3) = 4$$

$x = 5$  is paired with  $y = 6$ .

Now Do problems 27-30 all on page 449



The Vertical line test (VLT) is used to tell if a graph represents a function.

**The VLT** | If a vertical line intersects the graph of a relation in more than 1 point then the graph is not a function.

Use the VLT to do problems 37-40 on pg 449  
(page 5 of these notes)

# 43 Given  $g(x) = 2x + 5$

a)  $g(0) = 2 \cdot 0 + 5 = 0 + 5 = \boxed{5}$

b)  $g(-4) = 2 \cdot -4 + 5 = -8 + 5 = \boxed{-3}$

c)  $g(-7) = 2 \cdot -7 + 5 = -14 + 5 = \boxed{-9}$

d)  $g(8) = 2 \cdot 8 + 5 = 16 + 5 = \boxed{21}$

e)  $g(a+2) = 2 \cdot (a+2) + 5 = 2a + 2 \cdot 2 + 5$   
 $= 2a + 4 + 5 = \boxed{2a + 9}$

Now do problems 44, 47 and 61-64 on p 450 textbook  
(Next page)

(9)

**Function Notation and Equations**

Find the function values.

43.  $g(x) = 2x + 5$

- a)  $g(0)$       b)  $g(-4)$       c)  $g(-7)$   
 d)  $g(8)$       e)  $g(a + 2)$       f)  $g(a) + 2$

44.  $h(x) = 5x - 1$

- a)  $h(4)$       b)  $h(8)$       c)  $h(-3)$   
 d)  $h(-4)$       e)  $h(a - 1)$       f)  $h(a) + 3$

45.  $f(n) = 5n^2 + 4n$

- a)  $f(0)$       b)  $f(-1)$       c)  $f(3)$   
 d)  $f(t)$       e)  $f(2a)$       f)  $f(3) - 9$

46.  $g(n) = 3n^2 - 2n$

- a)  $g(0)$       b)  $g(-1)$       c)  $g(3)$   
 d)  $g(t)$       e)  $g(2a)$       f)  $g(3) - 4$

47.  $f(x) = \frac{x - 3}{2x - 5}$

- a)  $f(0)$       b)  $f(4)$       c)  $f(-1)$   
 d)  $f(3)$       e)  $f(x + 2)$       f)  $f(a + h)$

48.  $r(x) = \frac{3x - 4}{2x + 5}$

- a)  $r(0)$       b)  $r(2)$       c)  $r(\frac{4}{3})$   
 d)  $r(-1)$       e)  $r(x + 3)$       f)  $r(a + h)$

Fill in the missing values in each table.

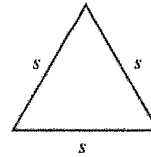
$f(x) = 2x - 5$	
$x$	$f(x)$
49. 8	
50.	13
51.	-5
52. -4	

$f(x) = \frac{1}{3}x + 4$	
$x$	$f(x)$
53.	$\frac{1}{2}$
54.	$-\frac{1}{3}$
55. $\frac{1}{2}$	
56. $-\frac{1}{3}$	

57. If  $f(x) = 4 - x$ , for what input is the output 7?  
 58. If  $f(x) = 5x + 1$ , for what input is the output  $\frac{1}{2}$ ?  
 59. If  $f(x) = 0.1x - 0.5$ , for what input is the output  $-3$ ?  
 60. If  $f(x) = 2.3 - 1.5x$ , for what input is the output 10?

**Applications**

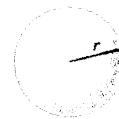
The function  $A$  described by  $A(s) = \frac{\sqrt{3}}{4}s^2$  gives the area of an equilateral triangle with side  $s$ .



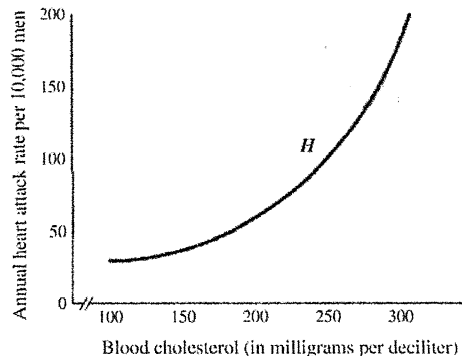
61. Find the area when a side measures 4 cm.  
 62. Find the area when a side measures 6 in.

The function  $V$  described by  $V(r) = 4\pi r^2$  gives the surface area of a sphere with radius  $r$ .

63. Find the surface area when the radius is 3 in.  
 64. Find the surface area when the radius is 5 cm.



**Heart Attacks and Cholesterol.** For Exercises 65–68, use the following graph, which shows the annual heart attack rate per 10,000 men as a function of blood cholesterol level.\*



65. Approximate the annual heart attack rate for those men whose blood cholesterol level is 225 mg/dl. That is, find  $H(225)$ .  
 66. Approximate the annual heart attack rate for those men whose blood cholesterol level is 275 mg/dl. That is, find  $H(275)$ .  
 67. Approximate the blood cholesterol level for an annual heart attack rate of 100 attacks per 10,000 men. That is, find  $x$  for which  $H(x) = 100$ .  
 68. Approximate the blood cholesterol level for an annual heart attack rate of 50 attacks per 10,000 men. That is, find  $x$  for which  $H(x) = 50$ .

\*Copyright 1989, CSPI. Adapted from *Nutrition Action Health-letter* (1875 Connecticut Avenue, N.W., Suite 300, Washington, DC 20009-5728, \$20 for 10 issues).

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