Objectives

- Understand the use and meaning of the notation used to define arithmetic combinations of functions.
- Find the domain for an arithmetic combination of two (or more) functions.

Definition 1 (Function Notation). *$f(x)$ is read "f of x."*
$f(x)$ is notation for the y value corresponding to a particular x value.

Example $f(1)$ is notation for the y value of the function f that corresponds to $x = 1$.

We often define two different functions, f and g, and then use the arithmetic operators ($+, -, \times, \div$) to combine the two functions together to make another function. We call these new functions ***arithmetic combinations of functions***. We use the definition of function notation to define the notation of arithmetic combinations of functions, given next.

Definition 2. *If f and g are functions and x is in the domain of both functions, then:*

1. $(f + g)(x) = f(x) + g(x)$
2. $(f - g)(x) = f(x) - g(x)$
3. $(f \cdot g)(x) = f(x) \cdot g(x)$
4. $(f/g)(x) = f(x)/g(x)$

Exercises

1. Assume $f(x) = 2x - 1$ and $g(x) = x^2 + 1$. Find each of the following.

a) $(f + g)(3)$ b) $(f - g)(x)$ c) $(f - g)(-2)$

d) $(f/g)(x)$ e) $(f/g)(-3)$ f) $(f \cdot g)(-2)$

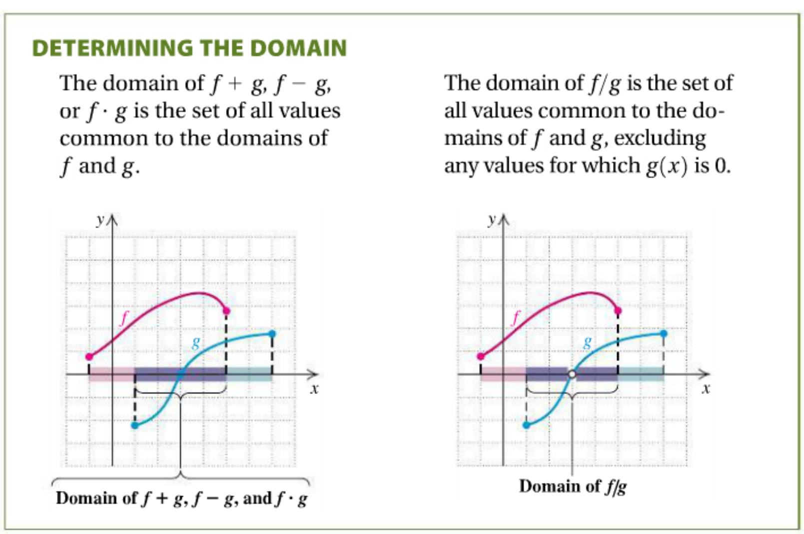

Figure 1: Elementary and Intermediate Algebra, 6E, Bittinger, Ellenbogen, and Johnson, page 474.

Use the functions F and G graphed below to answer the questions.

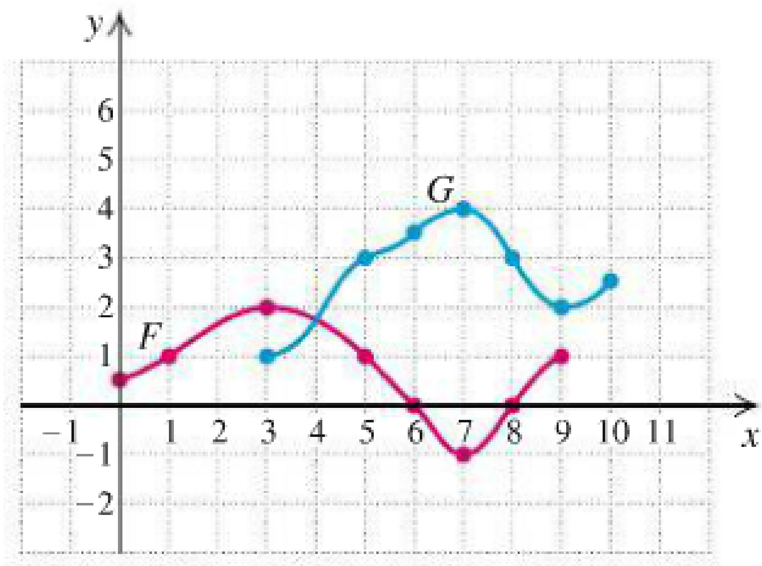

Figure 2: Elementary and Intermediate Algebra, 6E, Bittinger, Ellenbogen, and Johnson, page 477.

2. Find $(F + G)(5)$.
3. Find $(F \cdot G)(6)$.
4. Find $(G - F)(7)$.
5. Find the domains of $(F + G)(x)$ and $(G/F)(x)$.
6. For each pair of functions, find the domain of the sum, difference, product and quotient of the two functions.

   a) $f(x) = x + 4$

   $g(x) = x - 3$

   b) $f(x) = x^2 - 1$

   $g(x) = \dfrac{x}{x - 1}$

Answers: 1a) 15, b)$(f - g)(x) = {}^-x^2 + 2x - 2$, c) $^-10$, d) $(f/g)(x) = \dfrac{2x - 1}{x^2 + 1}$, e) $-\dfrac{7}{10}$, f) $^-25$, 2) 4, 3) 0, 4) 5, 5) $\text{dom}(F + G) = [3, 9]$, $\text{dom}(G/F) = [3, 6) \cup (6, 9]$ 6a) $\text{dom}(f + g) = (^-\infty, \infty)$, $\text{dom}(f - g) = (^-\infty, \infty)$, $\text{dom}(f \cdot g) = (^-\infty, \infty)$, $\text{dom}(f/g) = (^-\infty, 3) \cup (3\infty)$, 6b) $\text{dom}(f + g) = (^-\infty, 1) \cup (1, \infty)$, $\text{dom}(f - g) = (^-\infty, 1) \cup (1, \infty)$, $\text{dom}(f \cdot g) = (^-\infty, 1) \cup (1, \infty)$, $\text{dom}(f/g) = (^-\infty, 0) \cup (0, 1) \cup (1, \infty) = \{x| \quad x \neq 0, 1\}$,