8.3 Solving Applications of Systems **Math 51 Professor Busken**

Objectives

- Total-value and mixture problems
- Motion Problems

Exercises

1. Bob's copy center charges 25 cents for color copies and 10 cents for black and white copies. If Jilly's bill for 90 copies was \$14.25, how many copies of each kind were made?

2. Bob's Coffeehaus sells its original flavored coffee for \$10 per pound and its italian roast coffee for \$8 per pound. How much of each type should be used to make a 50-lb mixture that sells for \$8.80 per pound.

3. Bob's son's Boy Scout Troop had a pancake breakfast fundraiser to raise funds to send the troop to summer camp. The price of an adult breakfast ticket was \$7 and price of a kid's breakfast ticket \$5. If \$614 was collected from a total of 110 tickets, how many adult tickets were sold and how many kids tickets were sold?

4. Bob invests \$7000 in two accounts paying 6% and 8% annual interest. If the total interest earned for the year was \$520, how much was invested at each rate?

5. Bob's Gardening Company carries two brands of fertilizer containing nitrogen and water. "Gentle Grow" is 3% nitrogen and "Sun Saver" is 8% nitrogen. Bob needs to combine the two types of fertilizer into a 90-L mixture that is 6% nitrogen. How much of each brand should be used?

6. A car leaves Porterville and travels south at 56 mph. At the same time, a truck leaves Porterville and travels north at 70 mph. In how many hours will they be 441 miles apart?

7. Bob paddled for four hours with a 5 km/h current to reach a campsite. The return trip against the same current took 10 hr. Find the speed of Bob's boat in still water.

8. A train leaves Danville Union and travels north at 75 km/h. Two hours later, an express train leaves on a parallel track and travels north at 125 km/h. How far from the station will they meet?

Answers: 1) 35 color copies and 55 black and white copies 2) 20 lbs of original flavored and 30 lbs of italian roast 3) 78 kids tickets and 32 adult tickets 4) \$2000 at 6% and \$5000 at 8% 5) 36 L of Gentle Grow and 54 L of Sun Saver 6) 3.5 hours 7) $11.\overline{6}$ km/h 8) 375 km

(1) Givens: 25¢ for each color copy
10¢ for each black & white copy
90 total copies were made
$14.25 total bill

Unknown The number of color copies
The number of black & white copies

Let x represent the numb. of color copies and y represent the numb. of b & w copies. We know that

① total copies = 90
② total bill = $14.25

This translates to

① $x + y = 90$

② (amount spent on color copies) + (amount spent on black & white copies) = $14.25, or equivalently,

① $x + y = 90$
② $0.25x + 0.10y = 14.25$

Solve this system

① Solve eqn 1 for x
② clear eqn 2 of decimals by multiplying both sides by 100.

$$\begin{cases} x = 90 - y \\ 25x + 10y = 1425 \end{cases}$$

Now replace the x in eqn 2 with $(90 - y)$ and solve for y.

(1)

$$\left\{\begin{array}{l} x = 90 - y \\ 25x + 10y = 1425 \end{array}\right\} = \left\{\begin{array}{l} x = 90 - y \\ 25(90 - y) + 10y = 1425 \end{array}\right\}$$

$$= \left\{\begin{array}{l} x = 90 - y \\ 25(90) - 25y + 10y = 1425 \end{array}\right\} \text{ solve eqn 2 for } y$$

$$= \left\{\begin{array}{l} x = 90 - y \\ 2250 - 15y = 1425 \end{array}\right\} \text{ subtract 2250}$$

$$= \left\{\begin{array}{l} x = 90 - y \\ -15y = -825 \end{array}\right\} \text{ divide by } -15$$

$$= \left\{\begin{array}{l} x = 90 - y \\ y = 55 \end{array}\right\} \text{ Resubstitute } y = 55 \text{ into eqn 1}$$

$$= \left\{\begin{array}{l} x = 90 - 55 \\ y = 55 \end{array}\right\}$$

$$= \left\{\begin{array}{l} x = 35 \\ y = 55 \end{array}\right\}$$

Jilly had 35 color copies made and 55 b & w copies made.

(2) Given original flavored sells for $10 per lb.
italian roast sells for $8 per lb.

The mixture of the two blends totals to 50 lbs.
The mixture sells for $8.80 per pound.
The value of the mixture will be $(\$8.80)\cdot(50) = \$425$.

Unknown The number of pounds of original flavor used in the mixture, the number of pounds of italian roast

Let x represent the numb of pounds of original flavor and
let y represent " " " " italian roast.

We know that

① total wt of both coffees used = 50

② The value of the mixture is $(\$8.80)(50) = \$440$

This translates to

① $x + y = 50$, and

② (The value of the original flavored roast being used) + (The value of the italian roast being used in the mixture) = $\$440$

or equivalently,

$$\begin{cases} x+y=50 \\ \$10x + \$8y = \$440 \end{cases}$$

Solve eqn 1 for x. Then replace the x in eqn 2 with $(50-y)$ and solve eqn 2 for y.

(2)

$$\left\{\begin{array}{l} x = 50 - y \\ 10(50-y) + 8y = 440 \end{array}\right\}$$ Solve eqn 2 for y now

$$= \left\{\begin{array}{l} x = 50 - y \\ 500 - 10y + 8y = 440 \end{array}\right\}$$ Distribute, then combine like terms.

$$= \left\{\begin{array}{l} x = 50 - y \\ 500 - 2y = 440 \end{array}\right\}$$ Subtract 500

$$= \left\{\begin{array}{l} x = 50 - y \\ -2y = -60 \end{array}\right\}$$ Divide by 2

$$= \left\{\begin{array}{l} x = 50 - y \\ y = 30 \end{array}\right\}$$ Now resubstitute $y=30$ into eqn 1.

$$= \left\{\begin{array}{l} x = 50 - 30 \\ y = 30 \end{array}\right\} = \left\{\begin{array}{l} x = 20 \\ y = 30 \end{array}\right\}$$

20 lbs of original roast was used and 30 lbs of italian roast was used.

(3) given Adult ticket price $7
Kids ticket price $5
total revenue = $614
total numb of tickets sold = 110

unknown numb. of adult tickets sold
numb. of kids tickets sold

This suggests we let x represent the numb. of adult tickets sold and let y represent the numb. of kids tickets.

We know ① total tickets sold = 110
② total revenue = $614. This translates to

① $x + y = 110$

② (dollar amount spent on adult tickets) + (dollar amount spent on kids tickets) = $614

or

$$\begin{cases} x+y = 110 \\ \$7x + \$8y = 614 \end{cases}$$

We can use substitution to solve this.

(4) given: total investment is $7000

total interest is $520

interest is accrued at a rate of 6% in one account and at 8% in the other

unknown: The amount invested in each account.

Let x = the amount, in dollars, invested in the 6% account and

Let y = " " " " " " " 8% "

Then, we know ① total investment = $7000

② total interest = $520

or ① $x + y = 7000$

② (interest earned from the first account) + (interest earned in the 2nd account) = $520

$$\left(\begin{array}{l}6\% \text{ of} \\ \text{the amount} \\ \text{invested} \\ \text{in the} \\ \text{first account}\end{array}\right) + \left(\begin{array}{l}8\% \text{ of the} \\ \text{amount} \\ \text{invest in the} \\ \text{2nd account}\end{array}\right) = \$520$$

or $$\left\{\begin{array}{l} x + y = 7000 \\ 0.06x + 0.08y = 520 \end{array}\right\}$$

(5) Known: Gentle grow is 3% nitrogen, and
Sun Saver is 8% nitrogen.
90 L is the total weight of the mixture
The total amount of nitrogen in the mixture is
6% of 90 L or 5.4 Liters.

Unknown The number of liters of "gentle grow"
and "Sun saver" being used in the mixture.

Let x represent the numb. of liters of gentle grow being used,
and let y represent the numb. of liters of sun saver being used.

We know

① total weight = 90 L
② total nitrogen = 5.4 L
} this translates to

① $x + y = 90$

② (amount of nitrogen in x liters of gentle grow) + (amount of nitrogen in y liters of Sun saver) $= 5.4$ L, or equivalently,

$$\begin{cases} x + y = 90 & ① \\ 0.03x + 0.08y = 5.4 & ② \end{cases}$$

Solve eqn 1 for x, then replace the x in eqn 2 and solve for y.

Clear eqn 2 of decimals by multiplying by 100.

$$= \begin{cases} x = 90 - y \\ 3x + 8x = 540 \end{cases}$$

(6)

	r	$\cdot$	t	$=$	d
car	56		t	$=$	$56t$
truck	70		t	$=$	$70t$

total distance = 441 miles

$$\begin{pmatrix}\text{distance traveled}\\ \text{by car}\end{pmatrix} + \begin{pmatrix}\text{distance}\\ \text{traveled by}\\ \text{truck}\end{pmatrix} = 441$$

$$56t + 70t = 441$$

$$126t = 441$$

$$\frac{126t}{126} = \frac{441}{126}$$

$$t = 3.5 \text{ hours}$$

(7) Let x = the speed of the boat in still water

	r	$\cdot$	t	$=$	d
upstream	$(x-5)$	$\cdot$	10	$=$	$10(x-5)$
downstream	$(x+5)$		4	$=$	$4(x+5)$

$$\begin{pmatrix}\text{distance traveled}\\ \text{downstream}\end{pmatrix} = \begin{pmatrix}\text{distance traveled}\\ \text{upstream}\end{pmatrix}$$

$$4(x+5) = 10(x-5)$$

$$4x + 20 = 10x - 50$$

$$-6x = -70$$

$$\frac{-6x}{-6} = \frac{-70}{-6}$$

$$x = 11.\overline{6} \text{ mph}$$

(8)

(8)

Problem 8

See page 524-525 in the etext. see Example 6