9.2 Intersections, Unions and Compound Inequalities **Math 55 Professor Busken**

Objectives

- Find solutions to compound inequalities and write those solutions using interval notation.

Definition 1. *The **intersection** of two sets is another set altogether, formed by the numbers common to both sets. We use the intersection symbol, $\cap$, to denote the intersection of two sets.*

Example: Suppose $A = \{10, 11, 12\}$ and $B = \{10, 20, 30\}$. Then, $A \cap B = \{10\}$.

Definition 2. *The **union** of two sets is another set altogether, formed by taking all the numbers from both sets and putting them in a larger set. That is, every number from both sets is represented in the unioned set. We use the union symbol, $\cup$, to denote the union of two sets.*

Example: Suppose $A = \{10, 11, 12\}$ and $B = \{10, 20, 30\}$. Then, $A \cup B = \{10, 11, 12, 20, 30\}$.

Definition 3. *A **compound inequality** is a statement that considers one or more inequalities simultaneously, separated by the word "and" or "or." The two types of compound inequalities we encounter here are **conjunctions** and **disjunctions**.*

Definition 4. *A **conjunction** is a statement that considers one or more inequalities simultaneously, separated by the word "and" or the intersection symbol, $\cap$.*

Definition 5. *A **disjunction** is a statement that considers one or more inequalities simultaneously, separated by the word "or" or the union symbol, $\cup$.*

Exercises

1. Find the indicated union or intersection set. Assume $A = \{1, 2, 3, 4\}$, $B = \{4, 9, 14\}$, $C = \{2, 4, 6, 8\}$, and $D = \{1, 3, 5, 7, 9\}$,

   a) $A \cap B$ b) $A \cup B$ c) $C \cup D$ d) $A \cap C$ e) $C \cap D$

2. Graph and write interval notation for each compound inequality.

   a) $x < -1$ *and* $x < 1$

   b) $-2x < -8$ *and* $x - 5 < 5$

   c) $2x + 4 \geq 0$ *and* $4x > 0$

   d) $\frac{5x}{3} - 5 \leq \frac{5}{3} \quad \cap \quad 3x - \frac{1}{2} \leq -\frac{7}{2}$

   e) $-\frac{1}{2} \leq \frac{4x - 1}{6} < \frac{5}{6}$

   f) $-\frac{1}{4} < \frac{6 - x}{12} < -\frac{1}{6}$

   g) $x \geq 1$ *or* $x \leq 0$

   h) $-5x \leq 10$ *or* $3x - 5 \geq 1$

   i) $x + 9 < 0$ *or* $4x > -12$

   j) $3(x - 1) - 5 < 7$ *or* $x + 7 > 10$

   k) $2x - 1 \geq 3$ *and* $-x > 2$

   l) $5(x - 1) + 1 \geq -4$ *or* $5 + x \leq 11$

Answers: 1a) $\{4\}$, b) $\{1, 2, 3, 4, 9, 14\}$, c) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, d) $\{2, 4\}$, e) $\emptyset$
2a) $(-\infty, -1)$ b) $(4, 10)$ c) $[0, \infty)$ d) $(-\infty, -1]$ e) $[-1/2, 3/2)$ f) $(8, 9)$
g) $(-\infty, 0) \cup [1, \infty)$ h) $[-2, \infty)$ i) $(-\infty, -9) \cup (-3, \infty)$ j) $(-\infty, \infty)$ k) $\emptyset$ l) $(-\infty, \infty)$