Math 51 Ch. 9 Test Name: Key

WRITE EACH SOLUTION SET USING INTERVAL NOTATION. Double check your answers! Use Algebraic Notation AND Show All of Your Work. You may not leave to use the restroom. You may use a calculator, but not any scratch paper. Students are not allowed to share calculators!

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1. (5 points) Solve $-2x + 7 \geq 9$

2. Solve $\frac{3}{4}(x - 7) \geq x + 2$

3. ***Multiple Choice*** Solve $-2(3x - 4) - 5 < 6x - 4(2 - x)$

   a) $\left(-\infty, -\frac{5}{4}\right)$ b) $\left(\frac{11}{16}, \infty\right)$ c) $\left(\frac{11}{4}, \infty\right)$ d) $\left(-\infty, \frac{11}{16}\right)$

4. Suppose $f(x) = -2x + 8$ and $g(x) = 3x + 5$. Find all x values for which $f(x) > g(x)$. Use interval notation in your answer.

5. ***Multiple Choice***

   Hans can rent a van for either \$75 per day with unlimited mileage or \$45 per day with 100 free miles and an extra charge of 15¢ for each mile over 100. For what numbers of miles traveled would the unlimited mileage plan save Hans money? 5. ________

   a) Less than 350 b) Less than 300
   c) More than 350 d) More than 300

6. Find the intersection: $\{3, 6, 8, 11, 14\} \cap \{25, 15, 8, 3\}$

7. Find the interval solution of $-1 \leq -3t + 2 \leq 7$

8. Solve $5 - x > 7$ *and* $2x + 3 \geq 13$

9. Find the solution set for $x + 9 < 0$ *or* $4x \geq -12$

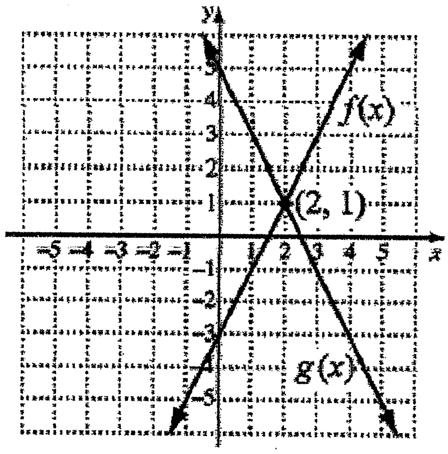
10. Solve $|5x - 7| + 8 = 1$

11. Solve $|2x-4| \leq 6$

12. Solve $|4-x| > 3$

13. Graph $\begin{cases} x+7 \geq -2 \\ x-y \geq 5 \end{cases}$

14. quad Using the graph, determine the solution of $f(x) \geq g(x)$



15. Solve $|2x+5| = 6$

16. Solve $3x < 20 + 2x < 2 + 3x$

① $-2x+7 \geq 9$

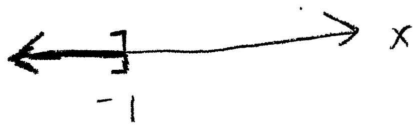
$-2x \geq 9-7$ subtract 7 from both sides

$-2x \geq 2$ Divide by -2. Change the direction of the inequality

$\frac{-2x}{-2} \leq \frac{2}{-2}$

$x \leq -1$

$\{x \mid x \leq -1\}$ set-builder notation



graph

$(-\infty, -1]$ interval notation

$\boxed{\text{Answer } (-\infty,-1]}$

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② $\frac{3}{4}(x-7) \geq x+2$ distribute

$\frac{3}{4}x - \frac{3}{4}\cdot\frac{7}{1} \geq x+2$

$\frac{3}{4}x - \frac{21}{4} \geq x+2$ Clear the inequality of fractions by multiplying both sides by 4

$4\left(\frac{3}{4}x - \frac{21}{4}\right) \geq 4\left(x+2\right)$ distribute

$\frac{4}{1}\cdot\frac{3}{4}x - \frac{4}{1}\cdot\frac{21}{4} \geq 4x+8$

$3x - 21 \geq 4x+8$

$$3x - 21 \geq 4x + 8$$ add 21 to both sides

(2)

(continued) $$3x - 21 + 21 \geq 4x + 8 + 21$$

$$3x \geq 4x + 29$$ subtract 4x

$$3x - 4x \geq 29$$ combine like terms

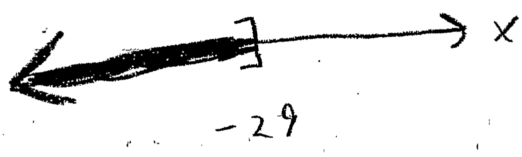
$$-1x \geq 29$$ divide by -1.

Change the direction of the inequality.

$$\frac{-1x}{-1} \leq \frac{29}{-1}$$

$$x \leq -29$$

$$\{x \mid x \leq -29\}$$ set-builder



graph

$$(-\infty, -29]$$ interval notation

$$\boxed{\text{Answer: } (-\infty, -29]}$$

(3) $-2(3x-4)-5 < 6x-4(2-x)$ (distribute)

$-6x+8-5 < 6x-8+4x$ (combine like terms)

$-6x+3 < 10x-8$ (subtract 3)

$-6x+3-3 < 10x-8-3$

$-6x < 10x-11$ (subtract 10x)

$-6x-10x < -11$

$-16x < -11$

(Divide by −16 now. Change the direction of the inequality.)

$$\frac{-16x}{-16} > \frac{-11}{-16}$$

$$x > \frac{11}{16}$$

$$\{x \mid x > \tfrac{11}{16}\}$$

Answer

$(11/16, \infty)$

(4) $f(x) > g(x)$ is equiv. to

$-2x + 8 > 3x + 5$ Subtract 8 now

$-2x > 3x + 5 - 8$

$-2x > 3x + (-3)$ Subtract $3x$

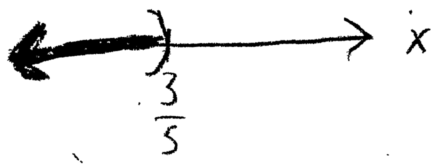
$-2x - 3x > -3$ combine like terms

$-5x > -3$ divide by -5 change the direction of the inequality symbol

$$\frac{-5x}{-5} < \frac{-3}{-5}$$

$$x < \frac{3}{5}$$

$$\{x \mid x < \tfrac{3}{5}\}$$



(5) Let x represent the number of miles traveled over (beyond) the 1st 100 miles. We are told that the

unlimited mileage plan is \$75 per day and the Per mile plan is \$45 + \$0.15x

We want the unlimited plan to cost less.

That is, we want

$$\begin{pmatrix}\text{Unlimited} \\ \text{mileage} \\ \text{plan}\end{pmatrix} < \begin{pmatrix}\text{Per} \\ \text{mile} \\ \text{plan}\end{pmatrix},$$

or equivalently,

$$\$75 < \$45 + \$0.15x$$

$$75 - 45 < 0.15x$$

$$30 < 0.15x$$

$$\frac{30}{0.15} < \frac{0.15x}{0.15}$$

$$200 < x$$

$$x > 200$$

$x > 200$. This says the number of miles traveled beyond the 1st 100 miles is greater than 200 miles. So the unlimited mileage plan is less than the per mile plan when miles traveled is greater than 300 miles.

(6) $\{3, 8\}$

(7) $-1 \le -3t+2 \le 7$ is equiv. to

$$-1 \le -3t+2 \quad \text{AND} \quad -3t+2 \le 7$$

$$-3t+2 \ge -1 \quad \text{AND} \quad -3t+2 \le 7$$ (subtract 2)

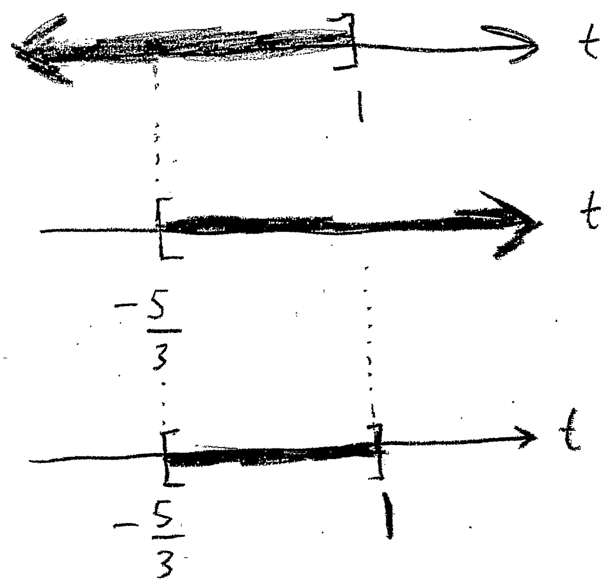
$$-3t \ge -1-2 \quad \text{AND} \quad -3t \le 7-2$$

$$-3t \ge -3 \quad \text{AND} \quad -3t \le 5$$

(Divide by -3. Change the direction of the inequality.)

$$\frac{-3t}{-3} \le \frac{-3}{-3} \quad \text{AND} \quad \frac{-3t}{-3} \ge \frac{5}{-3}$$

$$t \le 1 \quad \text{AND} \quad t \ge -5/3$$



$t \le 1$

$t \ge -5/3$

The intersection set (the set of numbers common to both intervals) is the Answer

$[-5/3, 1]$

(8) Solve $5 - x > 7$ AND $2x + 3 \geq 13$

$$5 + (-x) > 7 \quad \text{AND} \quad 2x + 3 \geq 13$$

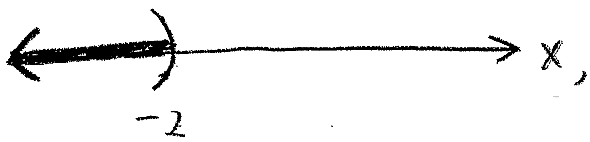
$$\begin{array}{rr} -5 & -5 \end{array} \qquad \begin{array}{rr} -3 & -3 \end{array}$$

$$-x > 2 \quad \text{AND} \quad 2x \geq 10$$

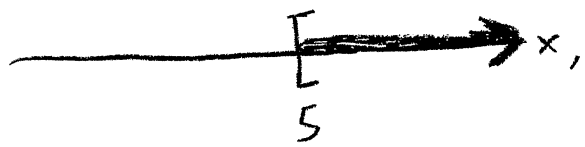
$$-1x > 2 \quad \text{AND} \quad 2x \geq 10$$

$$\frac{-1x}{-1} < \frac{2}{-1} \quad \text{AND} \quad \frac{2x}{2} \geq \frac{10}{2}$$

$$x < -2 \quad \text{AND} \quad x \geq 5$$



$x < -2$



$x \geq 5$

The intersection set is the set of numbers common to both intervals.

There are no numbers common to both intervals, so the answer is the empty set, $\{\}$. <u>Other acceptable solns:</u> $\emptyset$ or "no soln"

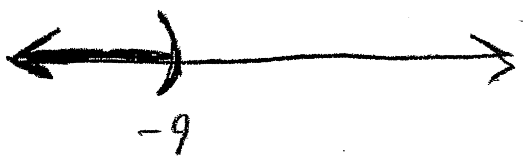
(9) $x+9<0$ OR $4x \geq -12$

$$\begin{aligned} x + 9 &< 0 \\ -9 \quad &-9 \end{aligned}$$

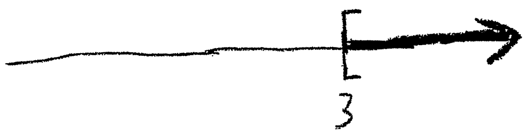
$$\frac{4x}{4} \geq \frac{-12}{4}$$

$x < -9$ OR $x \geq -3$

$$\boxed{x<-9 \quad \text{OR} \quad x \geq 3}$$



$x < 9$



$x \geq -3$

The Answer is the union set. Each number from both intervals is represented in the unioned set. Since both intervals are disjoint (not connected) we cannot write our answer as interval. We have to use the union symbol, $\cup$, as follows

$$\boxed{\underline{\text{Answer}}: (-\infty, -9) \cup [-3, \infty)}$$

(10) Solve $|5x-7| + 8 = 1$

Isolate the abs value expression by itself on one side of an equiv. eqn. In other words, subtract 8 from both sides. Then,

$$|5x-7| = 1-8 \quad , \quad \text{or}$$

$$|5x+7| = -7.$$

Then, the answer is no soln (or $\{\}$ or $\emptyset$).

The absolute value of a number ($5x-7$ is just a number) is that number's distance from zero on the number line. And distance is a non-negative quantity. The equation

$|5x+7| = -7$ means a number's distance from zero on the number line is equal to -7. This can't be since distance cannot be negative.

11. $|2x-4| \le 6$ is equiv to

$-6 \le 2x-4 \le 6$; which is shorthand for the conjunction

$-6 \le 2x-4$ AND $2x-4 \le 6$

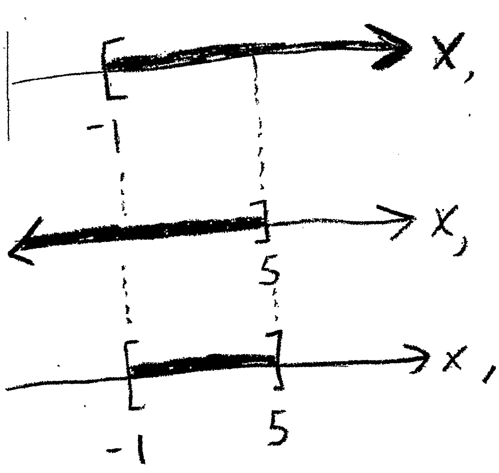
$2x-4 \ge -6$ AND $2x-4 \le 6$ (Add 4)

$$\begin{array}{rcl} 2x-4 & \ge & -6 \\ +4 & & +4 \\ \hline 2x & \ge & -2 \end{array} \qquad \begin{array}{rcl} 2x-4 & \le & 6 \\ +4 & & +4 \\ \hline 2x & \le & 10 \end{array}$$

$2x \ge -2$ AND $2x \le 10$ (Divide by 2 now)

$$\frac{2x}{2} \ge \frac{-2}{2} \qquad \frac{2x}{2} \le \frac{10}{2}$$

$$\boxed{x \ge -1 \quad \text{AND} \quad x \le 5}$$



$x \ge -1$

$x \le 5$

Soln set is the intersection set, the set of numbers found in both intervals.

$\boxed{\text{Answer } [-1, 5]}$

(12) Solve $|4-x| > 3$

$|4-x| > 3$ is equiv. to $4-x > 3$ OR $4-x < -3$

$4 - x > 3$ OR $4 - x < -3$

$4 + (-x) > 3$ OR $4 + (-x) < -3$

(Subtract 4)

$-x > 3-4$ OR $-x < -3-4$

$-x > -1$ OR $-x < -7$

$-1x > -1$ OR $-1x < -7$

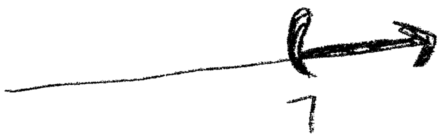
$\frac{-1x}{-1} > \frac{-1}{-1}$ OR $\frac{-1x}{-1} < \frac{-7}{-1}$

$x < 1$ or $x > 7$

Answer

$\boxed{(-\infty, 1) \cup (7, \infty)}$

$x < 1$



$x > 7$

(13) Graph $\begin{cases} x+7 \geq -2 & ① \\ x-y \geq 5 & ② \end{cases}$

① $x+7 \geq -2$

There is only one variable, and it's x. Solve for x. Replace the inequality symbol with an equals sign.

$$x+7=-2$$

Subtract 7

$$x=-2-7$$

$$x=-9$$

$$\boxed{x=-9}$$

② $x-y \geq 5$

Solve for y. Replace the inequality symbol with an equals sign.

$$x-y=5$$

$$x+(-y)=5 \quad (\text{subtract } x)$$

$$-y=5-x \quad (\text{multiply by } -1)$$

$$(-1)(-y)=(-1)(5-x)$$

$$y=-5+x$$

$$y=x-5$$

$$\boxed{y=\frac{1}{1}x-5}$$

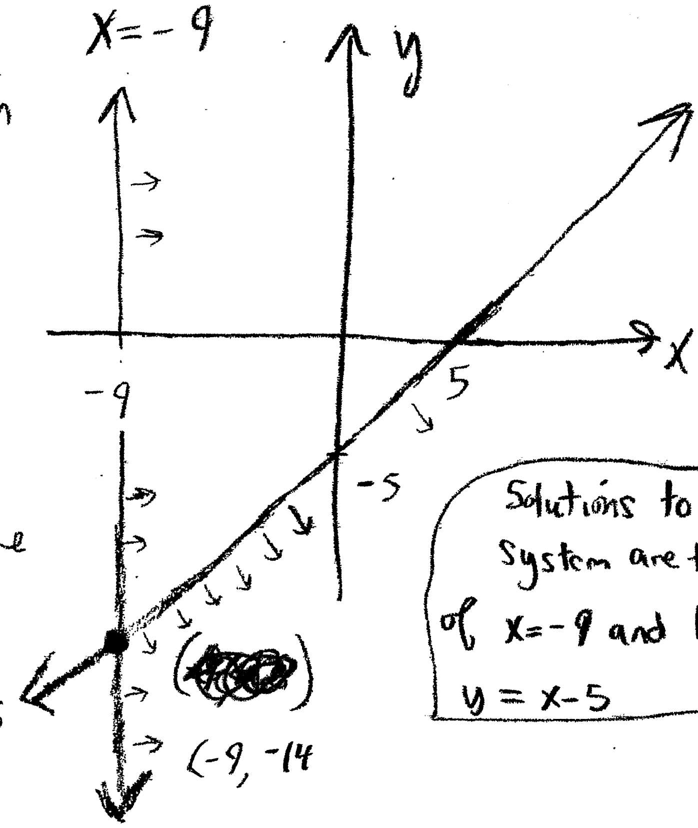
Now graph each boundaries

(13) Find the intersection point of the 2 boundaries.

Solve

$$\begin{cases} x = -9 & (1) \\ y = x - 5 & (2) \end{cases}$$

Use substitution. Replace the x in (2) with -9, then solve for y.



Solutions to the system are right of $x = -9$ and below $y = x - 5$

(2) $y = -9 - 5$

$y = -14$

So, the point of intersection is at $(x, y) = (-9, -14)$.

Now we use a test point not on either boundary. We can use the origin, $(0,0)$ as our test point. Substitute this value into

(1) $x + 7 \geq -2$

$0 + 7 \geq -2$

$7 \geq -2$ true, so the solns are right of the boundary at $x = -9$. Substitute $(0,0)$ into (2) $x - y \geq 5$

$0 - 0 \geq 5$

$0 \geq 5$ false, so solns are below $y = x - 5$

(14) From the graph we can see that $f(x) \geq g(x)$ (the f curve is above the g curve) for $x \geq 2$.

So, the solution set is the set $\{x \mid x \geq 2\}$ or the interval $[2, \infty)$.

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(15) Solve $|2x+5| = 6$

$|2x+5| = 6$ is equiv. to $2x+5 = 6$ or $2x+5 = -6$

Now solve

$$\begin{array}{lcll} 2x+5 = 6 & \text{or} & 2x+5 = -6 & \text{(subtract 5)} \\ \quad -5 \quad -5 & & \quad -5 \quad -5 & \\ \hline 2x = 1 & \text{or} & 2x = -11 & \text{(divide by 2)} \\ x = \frac{1}{2} & \text{or} & x = -\frac{11}{2} & \end{array}$$

$$\boxed{\text{Soln set } \{-11/2,\ 1/2\}}$$

(16) $3x < 20 + 2x < 2 + 3x$

$3x < 20 + 2x$ AND $20 + 2x < 2 + 3x$

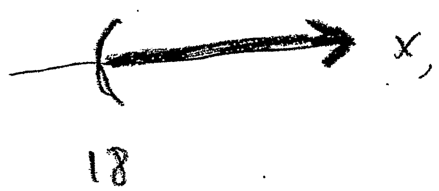
$-2x \qquad -2x$ $\qquad\qquad$ $-20 \quad -3x \qquad -20 \quad -3x$

$1x < 20$ AND $-1x < -18$

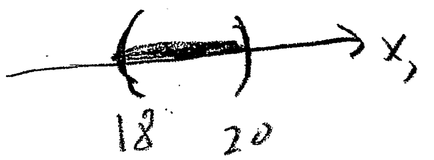
$x < 20$ AND $\dfrac{-1x}{-1} > \dfrac{-18}{-1}$

$\boxed{x < 20 \quad \text{AND} \quad x > 18}$

$x < 20$



$x > 18$



Soln is the intersection set, the interval $(18, 20)$

$\boxed{\text{Answer } (18, 20)}$