The Distance Formula & The Midpoint Formula

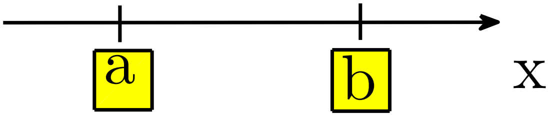
Professor Tim Busken

Mathematics Department

January 14, 2015

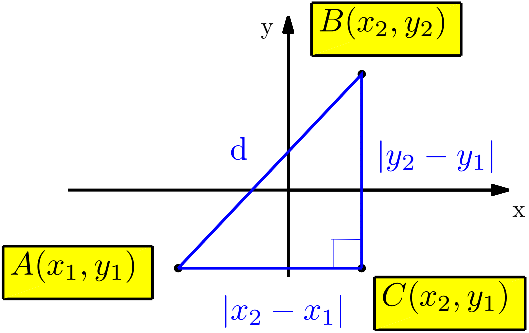
Theorem (Distance Formula: 1 dimension)

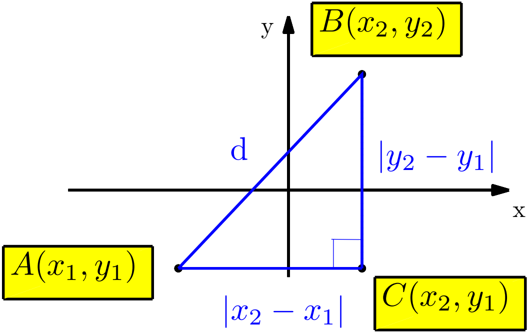
If a and b are real numbers, then the distance between them on a number line is $|a - b|$.



Distance Formula: 2 dimensions

Consider the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the figure below. Let d be the distance between points A and B (the HYPOTENUSE LENGTH of the right triangle). Since A and C lie on a horizontal line, the distance between them is $|x_2 - x_1|$. Likewise, $\overline{CB} = |y_2 - y_1|$.





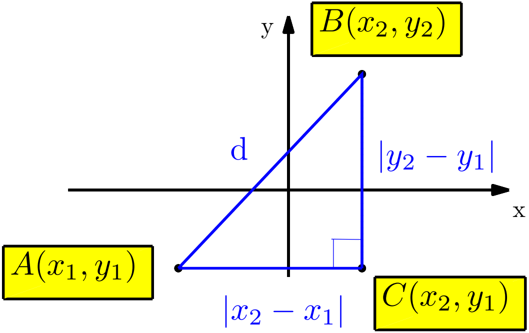
Since the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse (Pyth. thm), then from the diagram

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

Theorem (Distance Formula: 2 dimensions)

The distance d between the points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example: Find the exact distance between the points $(5, -3)$ and $(-1, -6)$

Solution

Let $(x_1, y_1) = (5, -$

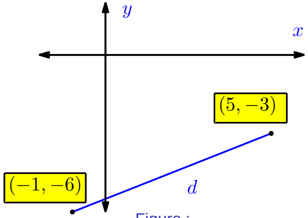


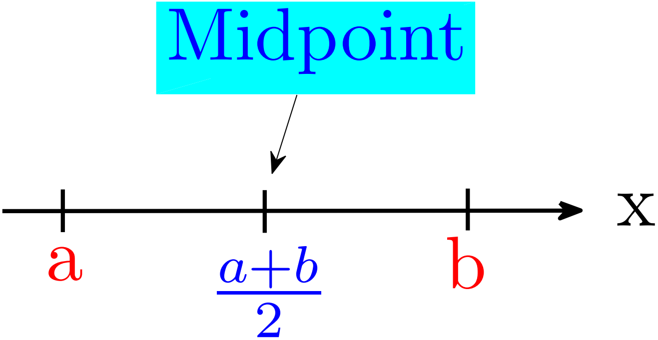
Figure :

Then

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1 - 5)^2 + (-6 - (-3))^2} \\ &= \sqrt{(-6)^2 + (-3)^2} \\ &= \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5} \end{aligned}$$

Theorem (Midpoint Formula: 1 dimension)

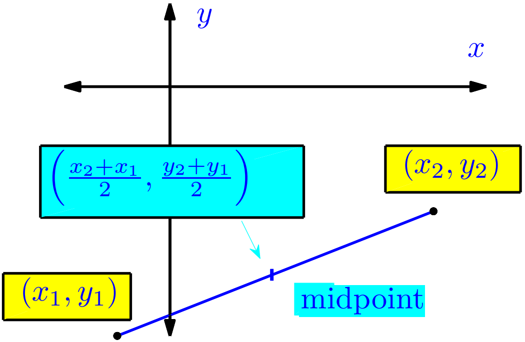
If a and b are real numbers, then the midpoint between them on a number line is $\frac{a+b}{2}$.



Theorem (Midpoint Formula: 2 dimensions)

Suppose (x_1, y_1) *and* (x_2, y_2) *are any two points in two-dimensional space. Then the midpoint of the line segment that joins them is:*

$$m = \left(\frac{(x_2 + x_1)}{2}, \frac{(y_2 + y_1)}{2} \right).$$



Example: Find the midpoint between the points (5, –3) and (–1, –6)

Solution

Let $(x_1, y_1) = (5, -$

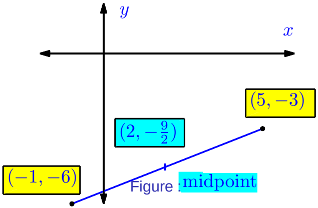


Figure : midpoint

Then

$$\begin{aligned} m &= \left(\frac{(x_2 + x_1)}{2}, \frac{(y_2 + y_1)}{2}\right) \\ &= \left(\frac{5 + (-1)}{2}, \frac{-3 + (-6)}{2}\right) \\ &= \left(\frac{4}{2}, \frac{-9}{2}\right) \\ &= \left(2, -\frac{9}{2}\right) \end{aligned}$$

The Circle

- An **ordered pair** is a solution to an equation in two variables if the equation is correct when the variables are replaced by the coordinates of the ordered pair.

The Circle

- An **ordered pair** is a solution to an equation in two variables if the equation is correct when the variables are replaced by the coordinates of the ordered pair.
- The **solution set** to an equation in two variables is the set of all ordered pairs that satisfy the equation.

The Circle

- An **ordered pair** is a solution to an equation in two variables if the equation is correct when the variables are replaced by the coordinates of the ordered pair.
- The **solution set** to an equation in two variables is the set of all ordered pairs that satisfy the equation.
- The **graph of (the solution set to) an equation** in two variables is a two-dimensional geometric object that gives us a visual image of an algebraic object.

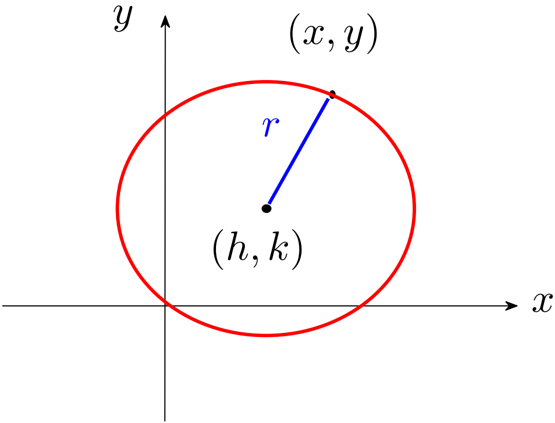
The Circle

- An **ordered pair** is a solution to an equation in two variables if the equation is correct when the variables are replaced by the coordinates of the ordered pair.
- The **solution set** to an equation in two variables is the set of all ordered pairs that satisfy the equation.
- The **graph of (the solution set to) an equation** in two variables is a two-dimensional geometric object that gives us a visual image of an algebraic object.

Definition (Circle)

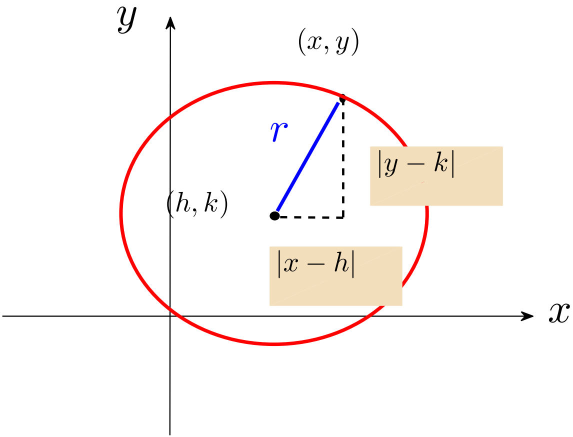
A *circle* is defined by the set of all points in the xy plane that lie a fixed distance from a given point (the center). The fixed distance is called the *radius*, and the given point is the center.

The distance formula can be used to write an equation for a circle with center (h, k) and radius r for $r > 0$.

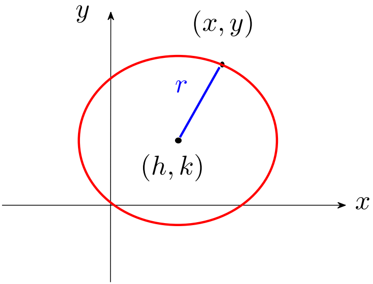


A point (x, y) is on the circle if and only if it satisfies the equation

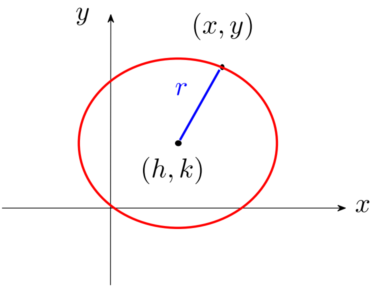
$$\sqrt{(x-h)^2 + (y-k)^2} = r.$$



Since both sides of the equation (previous slide) are positive, we can square each side to get the standard form for the equation of a circle.



Since both sides of the equation (previous slide) are positive, we can square each side to get the standard form for the equation of a circle.



Theorem (Equation for a Circle in Standard Form)

The equation for a circle with center (h, k) *and radius* r *(where* $r > 0$*) is*

$$(x - h)^2 + (y - k)^2 = r^2$$

A circle centered at the origin has equation $x^2 + y^2 = r^2$.

Example: Sketch the graph of the equation

$$(x-1)^2 + (y+2)^2 = 3$$

.