Review Topics

Elementary Algebra Review Topics

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Mathematical Theorems

Definition

A theorem is a statement that has been proven on the basis of previously established statements.

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Basic Set Definitions

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Example 1

{ 1,2,3,4} denotes a set containing the four numbers 1, 2, 3 and 4.

Basic Set Definitions

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Then $A \cup B$ is equal to what set?

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$A \cap B = \{2,4\}$

Number Types

Definition

The natural numbers,

$$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$$

consists of the counting numbers, where the ellipsis $(\ldots)$ indicates that the set goes on to infinity, or that there is no upper bound (largest number) in the set.

Number Types

Definition

The set of whole numbers,

$$\mathbb{W} = \{0, 1, 2, 3, 4, \ldots\}$$

is the set of natural numbers unioned with zero, written $\mathbb{W} = \mathbb{N} \cup \{0\}$.

Number Types

Definition

The set of integers,

$$\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

is also known as all the positive and negative whole numbers.

Review Topics

Number Types

Definition

A rational number is any number that can be written as a fraction where both the numerator and denominator are integers (and the denominator is not zero). The set of rational numbers is written symbolically as

$$\mathbb{Q} = \left\{ \frac{a}{b} \,\middle|\, \text{a and b are any integers, and } b \neq 0 \right\}$$

Note that any integer “a” is a rational number since $a = \frac{a}{1}$.

Number Types

HUGE note: When a rational number (fraction) is represented as a decimal number, then

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1. it has a finite number of digits to the right of the decimal point; for example, $\frac{5}{4} = 1.25$, OR
2. it has an infinite number of digits to the right of the decimal point *AND* and those digits have a repeating pattern, for example $\frac{1}{3} = 0.\overline{3}$ and $\frac{2}{37} = 0.054054054\cdots = 0.\overline{054}$.

Number Types

Definition

Real numbers that are not rational, for example, $\sqrt{2}$, $\sqrt[3]{3}$, and π are called irrational numbers. The set of irrational numbers is denoted $\mathbb{I}$.

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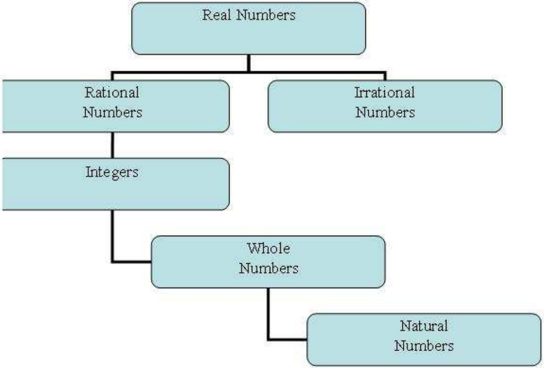
HUGE note: When an irrational number is represented as a decimal number, then

1. it has an infinite number of digits to the right of the decimal point,
2. and those digits do not have a repeating pattern.

Number Types

Definition

The set of real numbers, denoted $\mathbb{R}$, is the set $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$, that is the set of rationals unioned with the irrationals. Each real number can be uniquely represented as a decimal, and we associate each real number with a distinct point on a coordinate (number) line.


Real Numbers
Rational Numbers
Irrational Numbers
Integers
Whole Numbers
Natural Numbers

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Rational Expressions

Opposites

Theorem

Every real number has an opposite. The sum of a real number and its opposite is zero. Opposites are often called additive inverses.

Example The opposite of 5 is -5, and $5 + (-5) = 0$

Reciprocals

Theorem

Every non-zero real number has a reciprocal. The product of a real number and its reciprocal is one. Reciprocals are often called multiplicative inverses.

Example The multiplicative inverse of 5 is $\frac{1}{5}$, and $5 \cdot \frac{1}{5} = 1$

Absolute value

Definition (Absolute Value)

The absolute value of a real number is its distance from 0 on the number line. If $|x|$ represents the absolute value of x, then

$$| \, x \, | = \begin{cases} x, & \text{if } \; x \geq 0; \\ -x, & \text{if } \; x < 0. \end{cases}$$

The absolute value of a real number is never negative.

Example $\;| \, 5 \, | = 5$ and $| -5 \, | = 5$.

Theorem (Properties of Absolute value)

For any real numbers a and b:

1. $|a| \geq 0$
2. $|-a| = |a|$
3. $|a \cdot b| = |a| \cdot |b|$
4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ *provided* $b \neq 0$

Addition

To add two real numbers with *the same sign:*

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- Add the absolute values and use the common sign.

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Examples

$7 + 5 = 12$

$-7 + (-5) = -12$

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Examples

$7 + (-5) = 2$

$-7 + 5 = -2$

Subtraction

We use addition to define subtraction.

Definition

Suppose a and b represent any two real numbers. Then

$$a - b = a + (-b)$$

To subtract b, add the opposite of b.

Multiplication

To multiply two real numbers:

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Division

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Definition

Suppose a and b represent any two real numbers, and that $b \neq 0$. Then

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Suppose a and b represent any two real numbers, and that $b \neq 0$. Then

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Example

$$\frac{3}{4} = 3 \cdot \frac{1}{4}$$

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Properties of Real Numbers

Closure

Suppose a and b represent any real numbers, then $a + b$ and $a \cdot b$ are real numbers too.

	For Addition	For Multiplication
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$a + (b + c) = (a + b) + c$	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Identity	$0 + a = a$	$1 \cdot a = a$
Inverse	$a + (-a) = 0$	$a \cdot \left(\frac{1}{a}\right) = 1$
Mult. Prop of Zero	$0 \cdot a = 0$	

The Distributive Property

Theorem (Distributive Property of Multiplication)

Multiplication distributes over addition. For example,

$$\begin{tikzcd}[column sep=tiny] \color{red}{a} \arrow[rr, bend left, red] \arrow[rrrr, bend left, red] & \cdot (& b & + & c) = \color{red}{a} \cdot b + \color{red}{a} \cdot c \end{tikzcd}$$

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and

$$3 \cdot 2 + 3 \cdot 5 = 6 + 15 = 21$$

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and

$$3 \cdot 2 + 3 \cdot 5 = 6 + 15 = 21$$

Therefore,

$$3(2+5) = 3 \cdot 2 + 3 \cdot 5$$

Notice in the expression $3 \cdot (2+5)$ that each number inside the parenthesis is multiplied by 3.

The Distributive Property

Example Use the Distributive Property on the algebraic expression, $3 \cdot (x - 1)$. Assume x represents a real number.

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Example Use the Distributive Property on the algebraic expression, $3 \cdot (x - 1)$. Assume x represents a real number.

Solution:

$$3 \cdot (x - 1) = 3 \cdot \Big(x + (-1)\Big) \qquad \text{Definition of Subtraction}$$

$$= 3 \cdot x + 3 \cdot (-1) \qquad \text{Distributive Property}$$

$$= 3x + (-3)$$

$$= 3x - 3 \qquad \text{Definition of Subtraction}$$

Theorem

Suppose a and b are any real numbers. Then

$$-\frac{a}{b} = \frac{-a}{b} \text{ and } -\frac{a}{b} = \frac{a}{-b}$$

Exponential Notation

Recall the following terminology related to multiplication.

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Recall the following terminology related to multiplication.

$$\underset{\substack{\uparrow \\ \text{factor}}}{4} \times \underset{\substack{\uparrow \\ \text{factor}}}{6} = \underset{\substack{\uparrow \\ \text{product}}}{24}$$

The numbers we are multiplying are called **factors**.

Exponential Notation

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$$\underset{\substack{\uparrow \\ \text{factor}}}{4} \times \underset{\substack{\uparrow \\ \text{factor}}}{6} = \underset{\substack{\uparrow \\ \text{product}}}{24}$$

The result of multiplying the factors is called the **product**.

Exponential Notation

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(Read as "two to the fifth power.")

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base

This is called **exponential notation**. The **exponent**, 5, indicates how many times the **base**, 2, is a factor.

Exponential Expressions

Helpful Hint

An exponent applies only to its base. For example, $4 \cdot 2^3$ means $4 \cdot 2 \cdot 2 \cdot 2$

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Helpful Hint

Dont forget that 2^4, for example is not $2 \cdot 4$. The expression 2^4 means repeated multiplication of the same factor.

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16 \text{ whereas } 2 \cdot 4 = 8$$

The Order of Operations

Example: Simplify $6 + 2 \cdot 30$. Do you multiply or add first?

The Order of Operations

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3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

The Order of Operations

When evaluating or simplifying an algebraic or arithmetic expression, we always use the ORDER OF OPERATIONS. In other words we evaluate the expression in the following order:

$$\begin{aligned} &P-Parenthesis\\ &E-Exponents\\ &M-Multiplication\\ &D-Division\\ &A-Addition\\ &S-Subtraction \end{aligned}$$

The operations $+, -, \times, \div$ must be performed from left to right! The acronym

PEMDAS helps us to recall the ORDER OF OPERATIONS.

Example

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Solution:

$$5 - 10^2 = 5 + (-10^2) \qquad \text{Definition of Subtraction}$$

$$= 5 + \left((-1) \cdot 10^2\right) \qquad \text{Since } -a = (-1) \cdot a$$

$$= 5 + \left((-1) \cdot 10 \cdot 10\right) \qquad \text{Since } 10^2 = 10 \cdot 10$$

$$= 5 + (-100) = -95$$

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- Ex: How do you solve $2x - 4 = 0$ for x?

  use the addn./subt. and mult./div. props of equality—Section 1.1

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- A **CONTRADICTION** is a false equation, such as $4 = 9$.

Solving Linear Equations

Definition

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$$a \cdot x + b = c$$

where a, b and c are constants (numbers) and $a \neq 0$.

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For example, $2 \cdot x + 3 = -1$ is a linear equation in one variable.

$2x$, 3 and -1 are called the **terms** of the equation.

$2x$ is a **variable term** and 3 and -1 are **constant terms**.

Definition

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Example The solution set for $4x - 2 = 10$ is $\{3\}$ since replacing x with 3 makes the equation a true statement.

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Solution:

$$4x - 2 = 10$$

$$4 \cdot (x) - 2 = 10$$

Associative Property of Multiplication

$$4 \cdot (3) - 2 = 10$$

Replace x with 3

$$12 - 2 = 10$$

The Order of Operations says that we multiply before subtracting.

Definition

Two or more equations with the same solution set are called equivalent equations.

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Example $2x + 3 = -1$, $x = -2$, $2x = -4$, and $x + 2 = 0$ are all equivalent equations since the solution set for each is $\{-2\}$.

Properties of Equality

Addition Property of Equality

For any three algebraic expressions A, B and C,

$$\text{If} \quad A = B$$

$$\text{then} \quad A + C = B + C$$

In words: Adding the same quantity to both sides of an equation will not change the solution set.

Properties of Equality

Multiplication Property of Equality

For any three algebraic expressions A, B and C, where $C \neq 0$,

$$\text{If} \quad A = B,$$

$$\text{then} \quad A \cdot C = B \cdot C$$

In words: Multiplying both sides of an equation by the same non-zero quantity will not change the solution set.

Classroom Examples: Solve the following equations for x.

1. $\frac{2}{3}x + 4 = -8$
2. $3x - 3 = -5x + 9$
3. $-x = 1$
4. $\frac{3}{5}x + \frac{1}{3} = -\frac{5}{6}$
5. $0.08x + 0.10(8000 - x) = 680$

Solving Linear Equations in One Variable

Step 1: **a.** Use the Distributive Property to separate terms, if necessary.

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c. Combine similar terms on each side of an equation.

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b. If fractions are present, consider multiplying both sides by the LCD to eliminate the fractions. If decimals are present consider multiplying both sides by a power of 10 to clear the equation of decimals.

c. Combine similar terms on each side of an equation.

Step 2: Use the Addition Property of Equality to get all variable terms on one side of the equation and all constant terms on the other side.

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b. If fractions are present, consider multiplying both sides by the LCD to eliminate the fractions. If decimals are present consider multiplying both sides by a power of 10 to clear the equation of decimals.

c. Combine similar terms on each side of an equation.

Step 2: Use the Addition Property of Equality to get all variable terms on one side of the equation and all constant terms on the other side.

Step 3: Use the Multiplication Property of Equality to get the variable by itself on one side of the equation.

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Step 4: Check your solution in the original equation to be sure that you have not made a mistake in the solution process.

Classroom Examples: Solve the following equations for x.

1. $6 - 2(5x - 1) + 4x = 20$
2. $3(5x + 1) = 10 + 15x$ Special Case 1
3. $-4 + 8x = 2(4x - 2)$ Special Case 2

Linear Inequalities in One Variable

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    1. less than $<$,
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Linear Inequalities in One Variable

Definition

Replacing the equal sign in the general linear equation $a \cdot x + b = c$ by any of the symbols $<, \leq, >$ or $\geq$ gives a **linear inequality in one variable**.

For example, $2 \cdot x - 1 \leq 0$ and $3x + 5 > 8$ are two different linear inequalities in a single variable, x.

Solving Linear Inequalities

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The solution to any linear inequality is a <u>SET</u> of real numbers.

For example, $\{x \mid x < -2\}$ is shorthand notation for the set of real numbers less than -2.

$$\{x \quad \mid \quad x < -2\}$$

$$\swarrow \qquad \downarrow \qquad \searrow$$

$$x \;\; \overbrace{\text{such that}} \; \overbrace{x \text{ is any real number less than } -2}$$

Addition Property for Inequalities

For any three algebraic expressions A, B and C,

$$\text{If} \quad A < B$$

$$\text{then} \quad A + C < B + C$$

In words: Adding the same quantity to both sides of an inequality will not change the solution set.

We can use the Addn. Prop. to write ***equivalent inequalities***.

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

Tim Busken

Review Topics
- Set Definitions
- Number Types
- Opposites
- Absolute Value
- Operations
- Properties of Real Numbers
- Distributive Property
- Exponential Notation
- The Order of Operations
- Linear Equations
- Linear Equations
- Linear Inequalities
- Coordinate System
- Lines
- slope
- Perpendicular & Parallel Lines
- Algebraic Expressions
- Polynomials
- Poly Add/Subtract
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$$5x + 4 < 4x + 2$$

$$5x + 4 + (-4) < 4x + 2 + (-4)$$

Addition Prop. of Inequalities

$$5x + \left(4 + (-4)\right) < 4x + \left(2 + (-4)\right)$$

Associative Prop. of Addition

$$5x + 0 < 4x + (-2)$$

Additive Inverse & Closure Props.

$$5x < 4x - 2$$

Additive Identity & the Defn. of Subtraction

Example 1 Solve the inequality, $5x + 4 < 4x + 2$, then graph the solution.

Solution:

$5x < 4x - 2$	
$5x + (-4x) < 4x - 2 + (-4x)$	Addition Prop. of Inequalities
$5x + (-4x) < 4x + (-4x) - 2$	Commutative Prop. of Addn.
$\left(5x + (-4x)\right) < \left(4x + (-4x)\right) - 2$	Associative Prop. of Addn.
$(5 - 4) \cdot x < 0 - 2$	Distributive & Additive Inverse Props.
$1 \cdot x < -2$	Closure & Additive Identity Props.
$x < -2$	Multiplicative Identity Prop.

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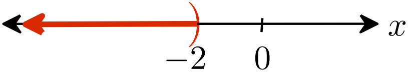

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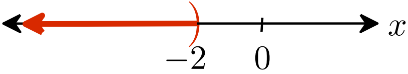

We use a left-opening parenthesis at −2 to indicate that −2 is not part of the solution set.

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An alternate and more compact way of writing the solution set is

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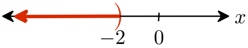

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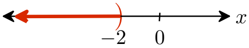
Set Notation	Line Graph	Interval Notation
$\{x \mid x < -2\}$		$(-\infty, -2)$

Properties of Inequalities

Multiplication Property of Inequalities

For any three algebraic expressions A, B and C, where $C \neq 0$,

$$\begin{array}{rll} \text{If} & A < B, & \\ \text{then} & A \cdot C < B \cdot C & \text{if } C \text{ is positive } (C > 0) \\ \text{or} & A \cdot C > B \cdot C & \text{if } C \text{ is negative } (C < 0) \end{array}$$

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In words: Multiplying both sides of an inequality by a positive quantity always produces an equivalent inequality. Multiplying both sides of an inequality by a negative number produces an equivalent inequality BUT it reverses the direction of the inequality symbol.

Example 2 Determine what set is the solution to

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Solution:

$$-2x - 3 \leq 3$$

$$-2x - 3 + 3 < 3 + 3$$ Addition Prop. of Inequalities

$$-2x \leq 6$$ Additive Inverse & Identity Props

$$\left(-\frac{1}{2}\right) \cdot (-2x) \geq \left(-\frac{1}{2}\right) \cdot 6$$ Multiplication Prop. of Inequalities

$$x \geq -3$$ Closure

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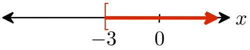

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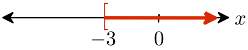
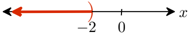
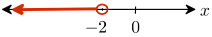


Interval Notation
$[-3, \infty)$

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
$x < -2$	$(-\infty, -2)$		

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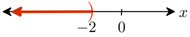
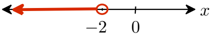
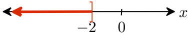
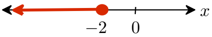
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		

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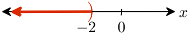
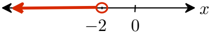
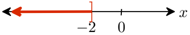
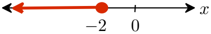
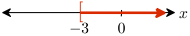
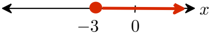
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		
$x \geq -3$	$[-3, \infty)$		

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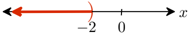
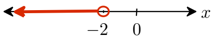
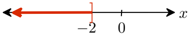
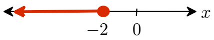
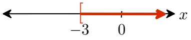
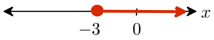
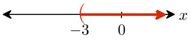
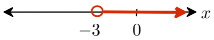
$x < -2$	$(-\infty, -2)$		
$x \leq -2$	$(-\infty, -2]$		
$x \geq -3$	$[-3, \infty)$		
$x > -3$	$(-3, \infty)$		

Linear Inequalities in One Variable

Classroom Example: Solve the following inequality.

- $3(2x + 5) \leq -3x$

Linear Inequalities in One Variable

Classroom Examples: Take the next five minutes to work these 6 problems. Graph the solution set to the given inequality, then write the solution set using interval notation.

- $x \leq -6$
- $x > 5$
- $x \geq -1$
- $x > 10$

Classroom Examples: Solve each inequality. Graph the solution set, then write the solution set using interval notation.

- $2x - 1 \leq -6$
- $-3x < 2x - 6$

Linear Inequalities in One Variable

Definition

A compound inequality is two or more simple inequalities {sets} joined by the terms 'and' or 'or' .

For Example, the set $\left\{ x \,\middle|\, 3x - 6 \leq -3 \quad \text{or} \quad 3x - 6 \geq 3 \right\}$ is a compound inequality.

The inequality statement $-7 < x < 7$ is to be read "x is in between -7 and 7."

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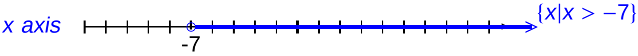

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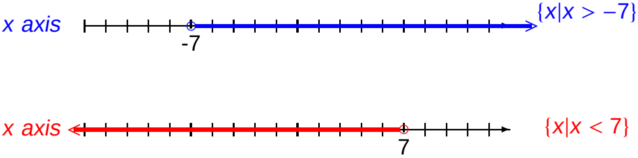

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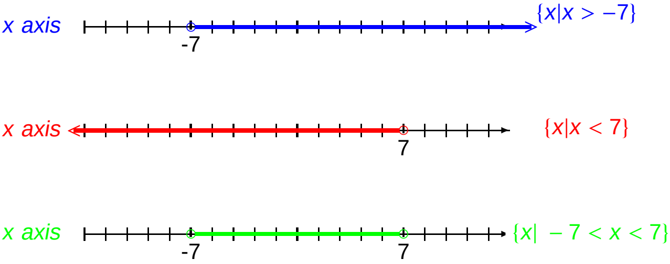

Linear Inequalities in One Variable

Classroom Examples: Solve the following compound inequalities. Graph the solution set on a number line, then write the solution set using interval notation.

- $-7 \leq 2x + 1 \leq 7$
- $3x - 6 \leq -3$ or $3x - 6 \geq 3$

Interval Notation and Graphing

Inequality Notation

$-4 < x < 3$

Interval Notation

$(-4, 3)$

Graph Using Parenthesis/Brackets

Graph using open and closed circles

Interval Notation and Graphing

Inequality Notation	Interval Notation	Graph Using Parenthesis/Brackets	Graph using open and closed circles
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Interval Notation and Graphing

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Review Topics

Interval Notation and Graphing

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Ordered Pairs

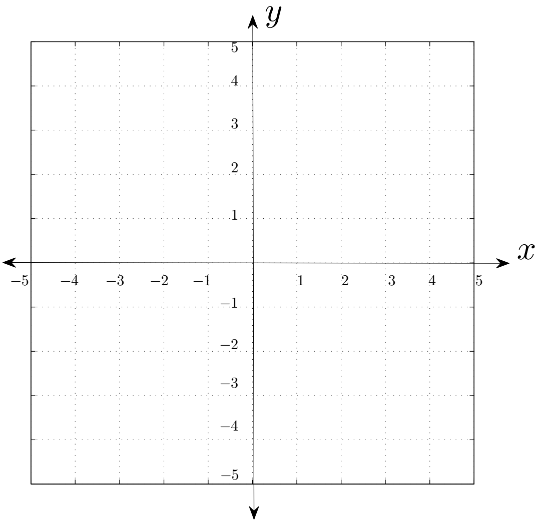
We now turn our attention to equations containing two variables, x and y. Paired data plays an important role in these type of equations.

Ordered Pairs

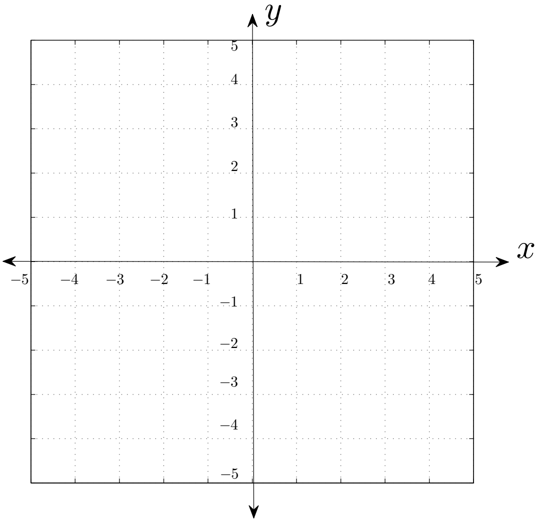
Definition

A pair of numbers enclosed in parenthesis and separated by a comma, such as $(-2, 1)$, is called an ordered pair of numbers. The first number in the pair is called the x-coordinate of the ordered pair; the second number is called the y-coordinate. For the ordered pair $(-2, 1)$, the x-coordinate is -2 and the y-coordinate is 1.

We use a rectangular coordinate system to visualize ordered pairs.



A rectangular coordinate system is made by drawing two real number lines at right angles to each other.



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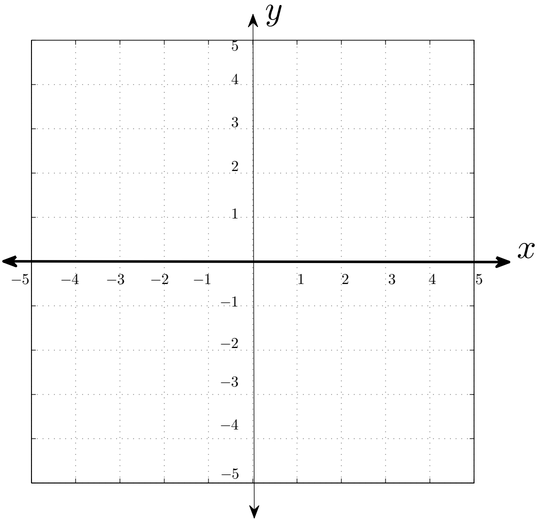

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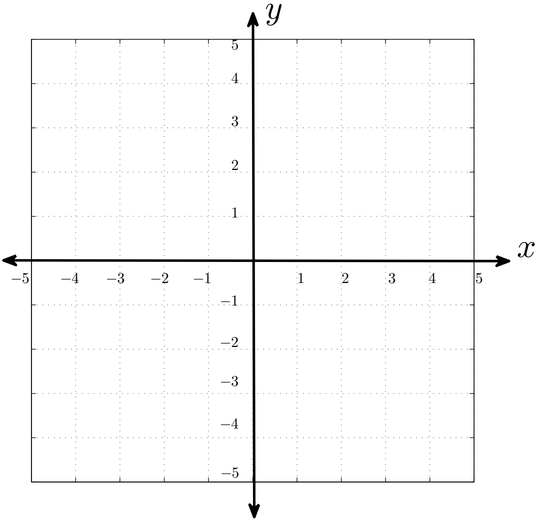
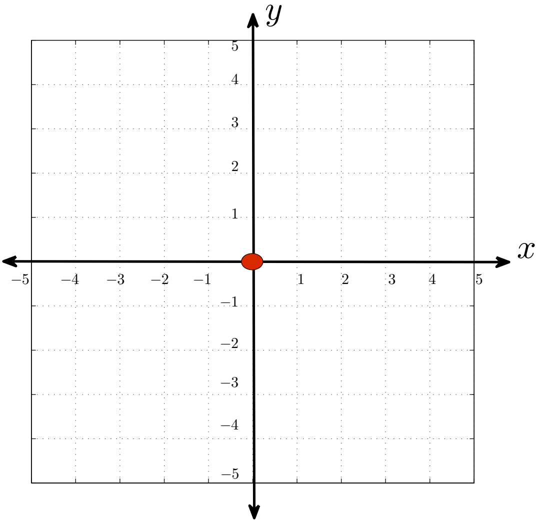
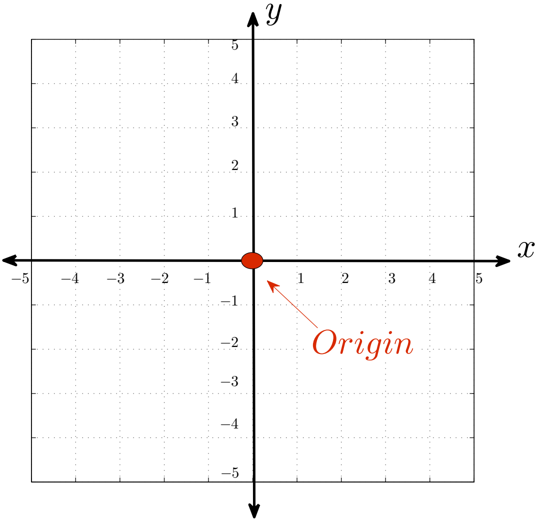


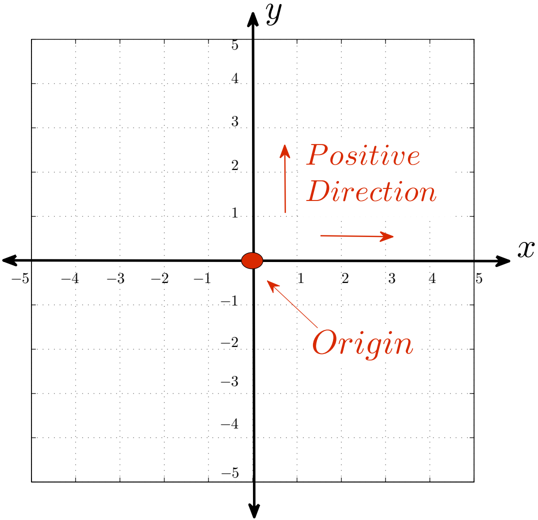
Two number lines, called axes, cross each other at zero.



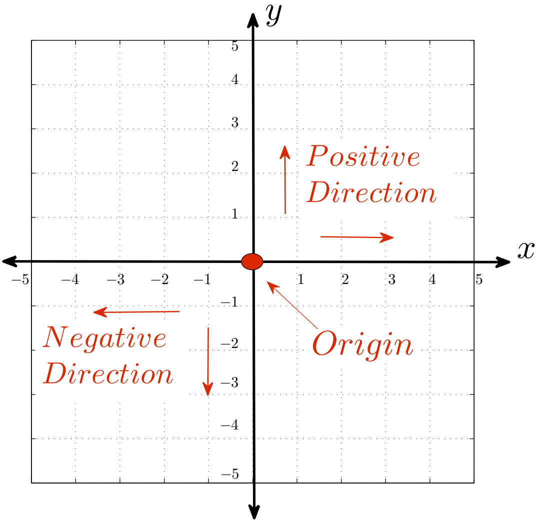
Two number lines, called axes, cross each other at zero. This point is called the *origin*.



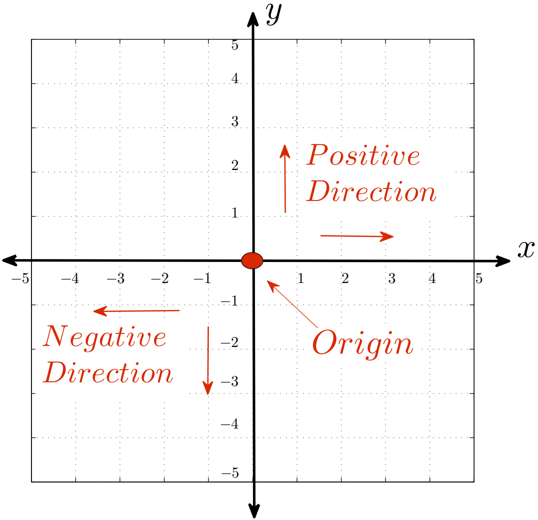
Relative to the origin, positive directions are to the right and up.



Negative directions are to the left and down.



The horizontal number line is called the x-axis



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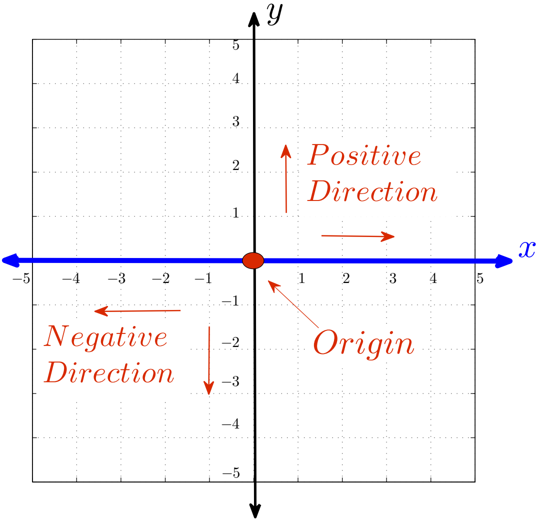
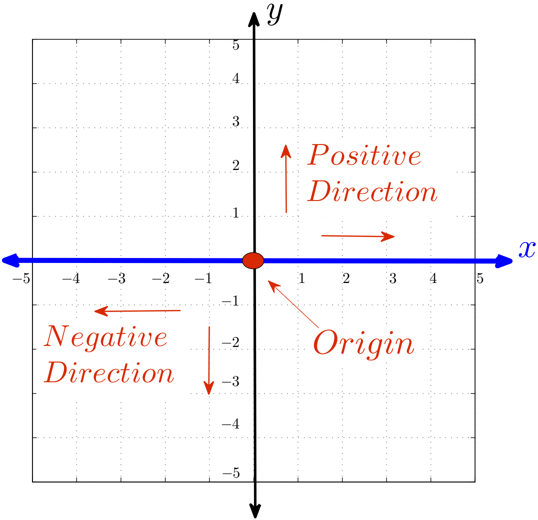


The horizontal number line is called the *x*-axis and the vertical number line is called the *y*-axis.



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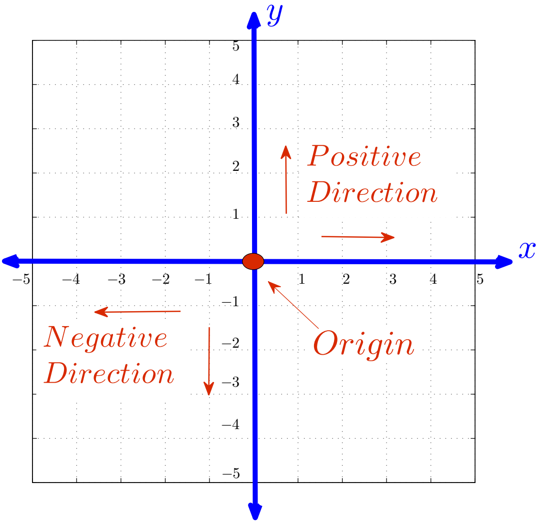
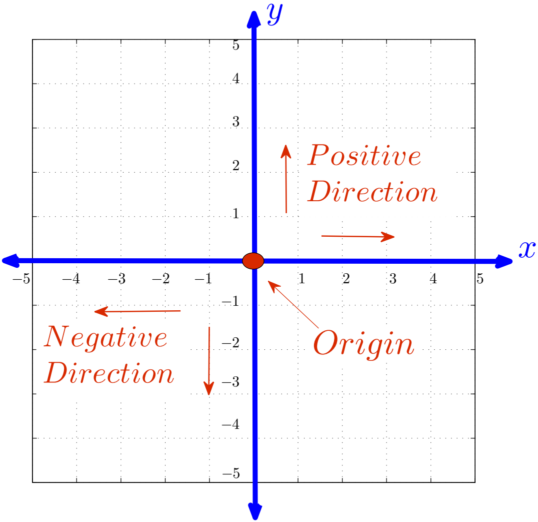


The two number lines divide the coordinate system into four quadrants, which we number I through IV in a counterclockwise direction.



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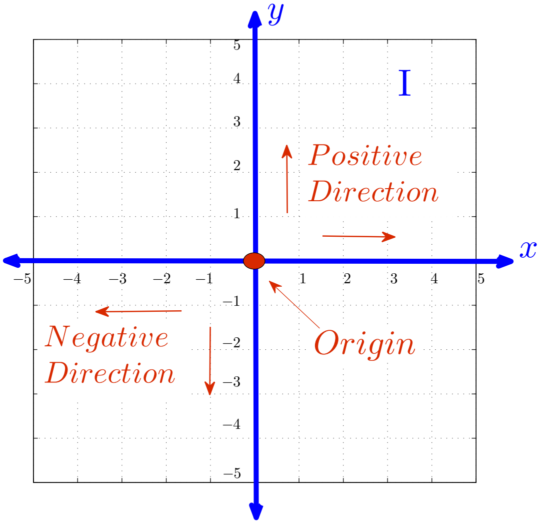

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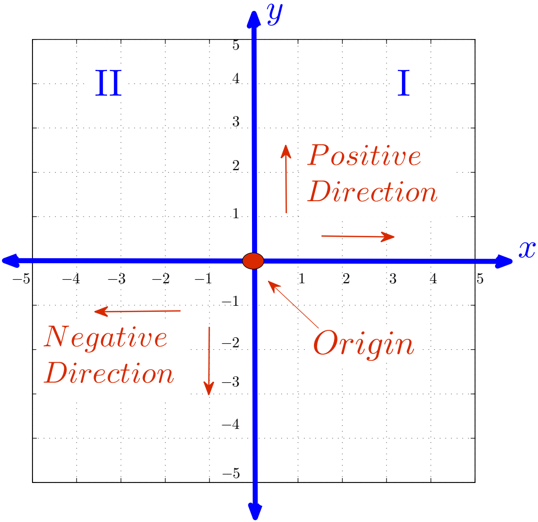

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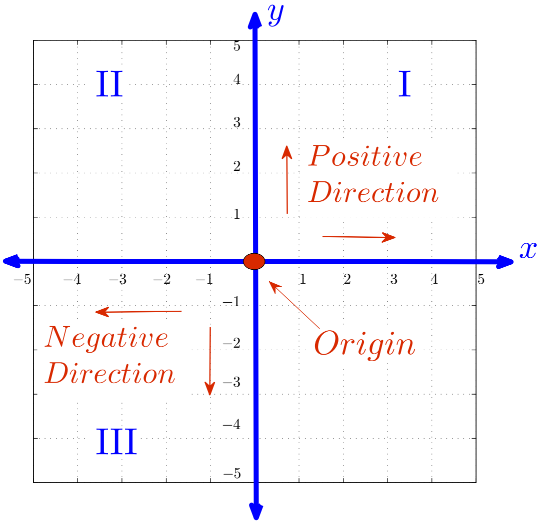

Review Topics

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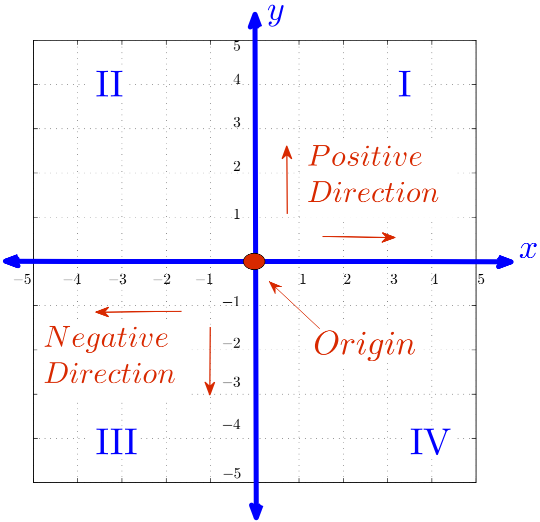

Graphing Ordered Pairs

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To graph the ordered pair (a, b) on the rectangular coordinate system, we:

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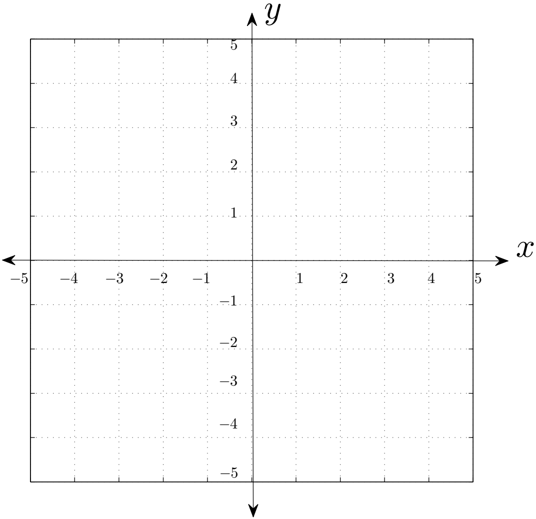
To graph the ordered pair (a, b) on the rectangular coordinate system, we:

1. begin at the origin and move along the x-axis a units right or a units left (right if a is positive and left if a is negative).
2. From that point we move b units up or down (up if b is positive and down if b is negative).
3. The point where we end up is the graph of the ordered pair.

Example 1: Plot (graph) the following ordered pairs:

$$(2,3),\ (-2,3),\ (-2,-3),\ (2,-3),$$



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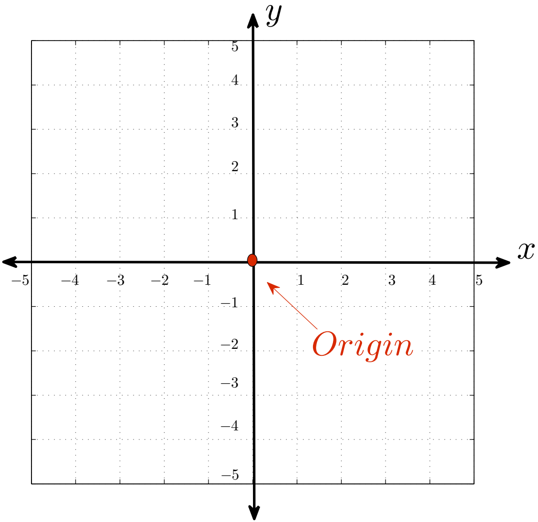

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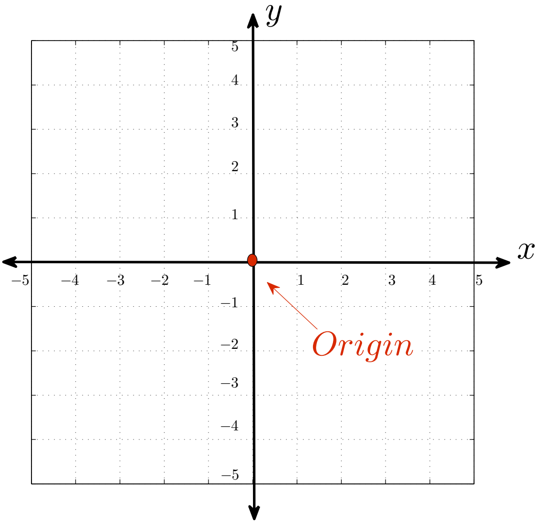

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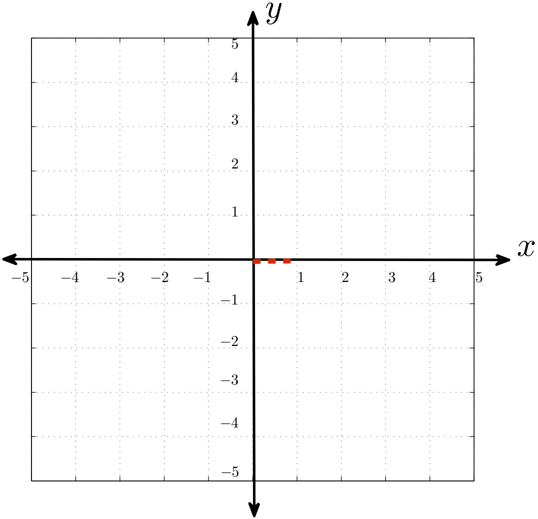

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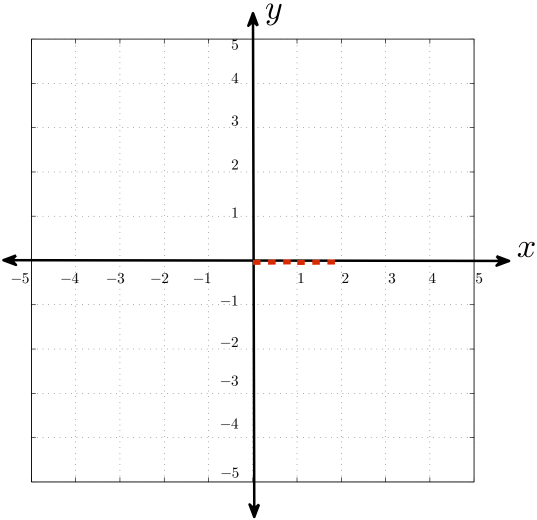

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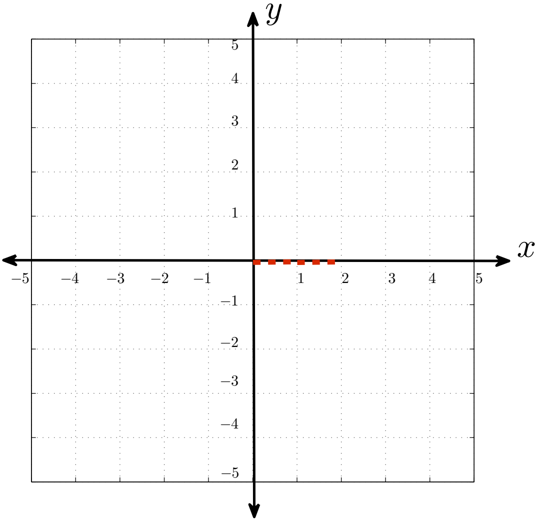

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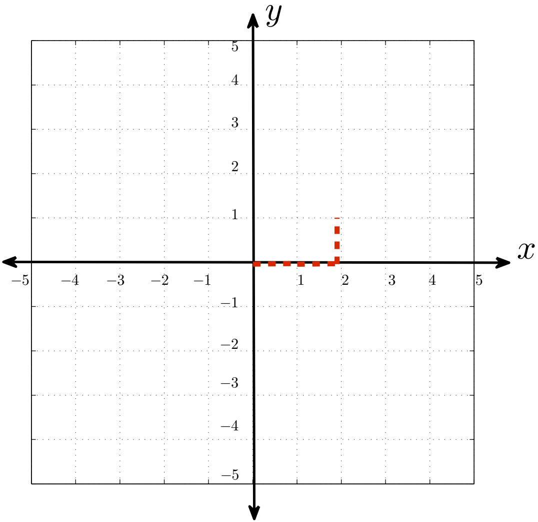

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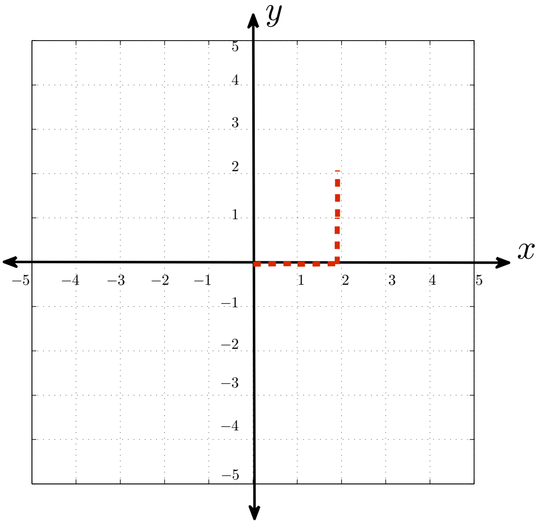

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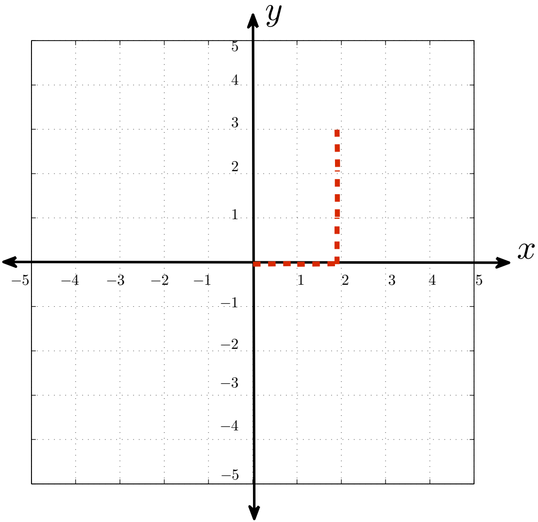

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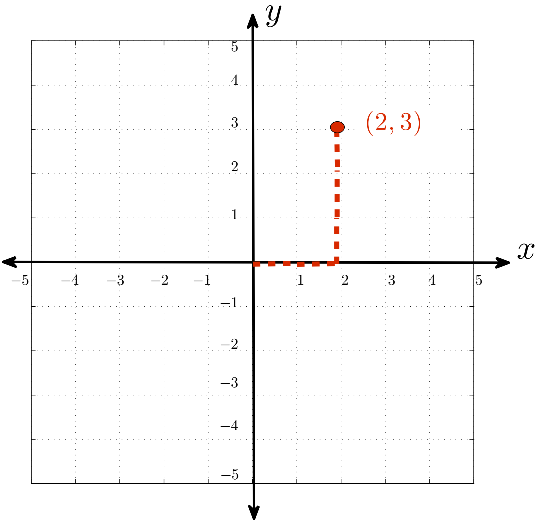

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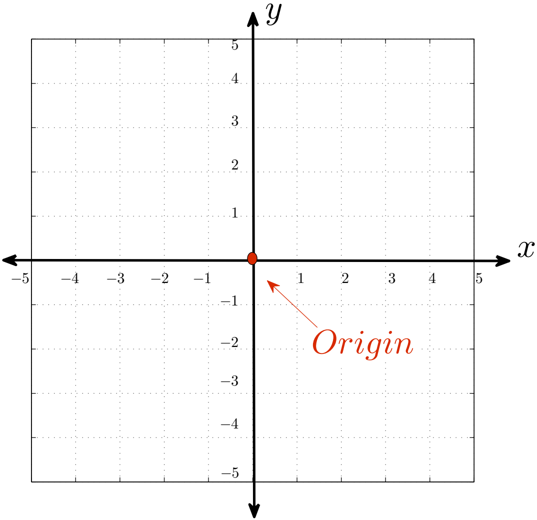

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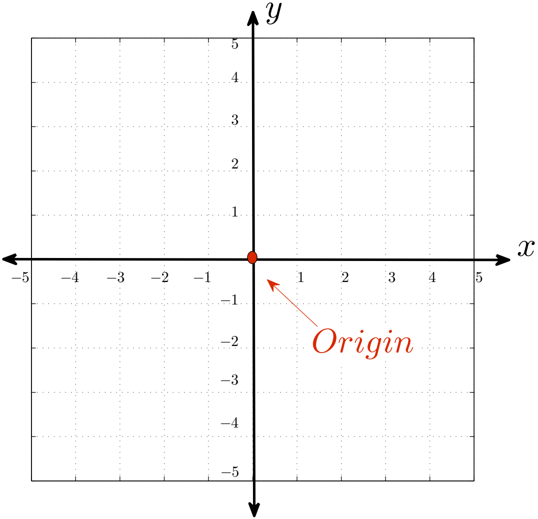

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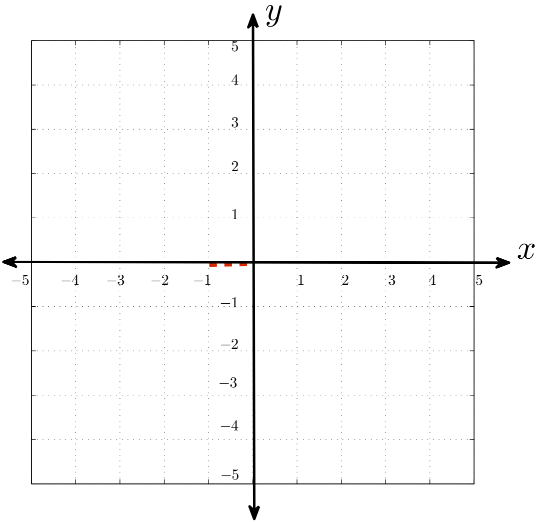

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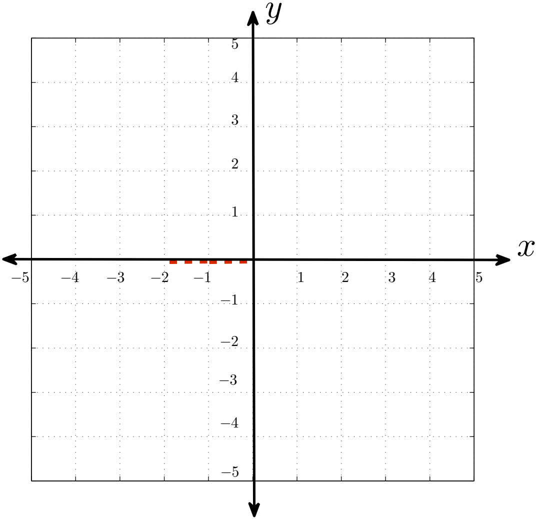

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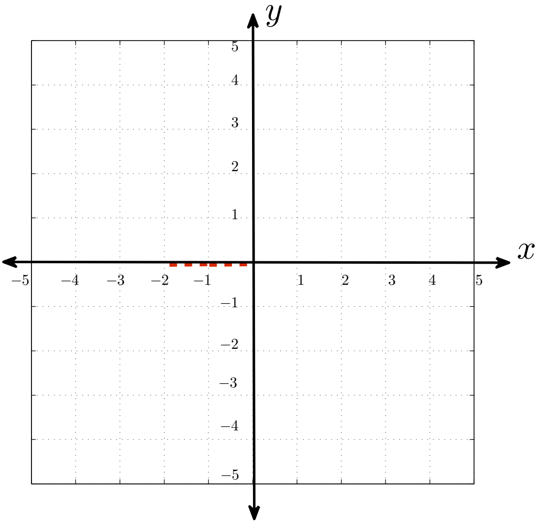

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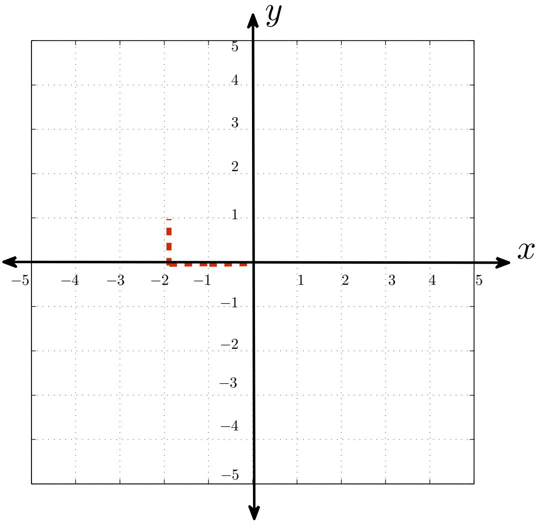

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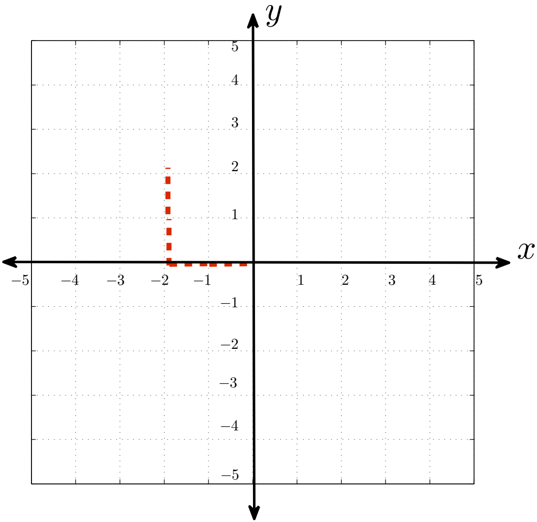

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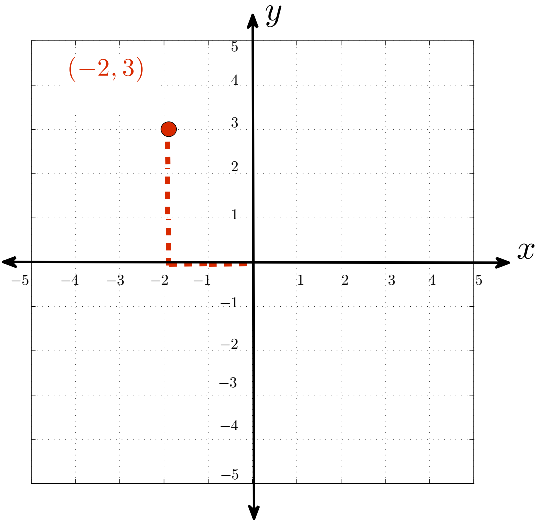

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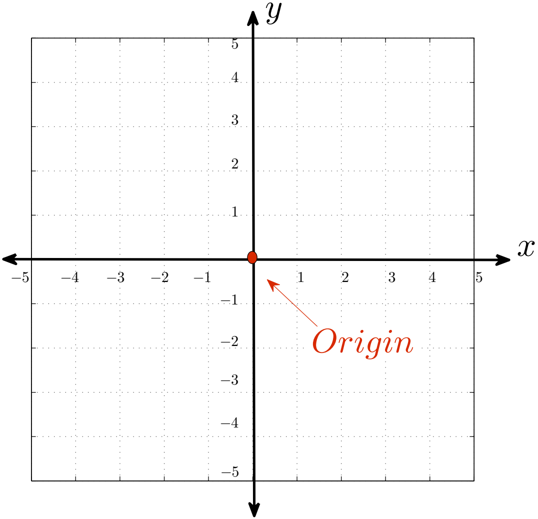

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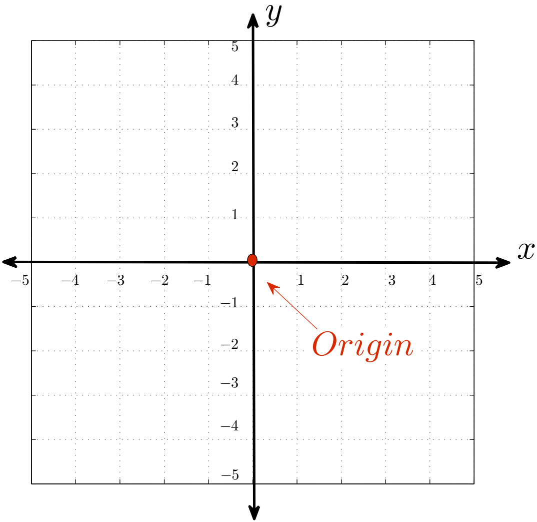

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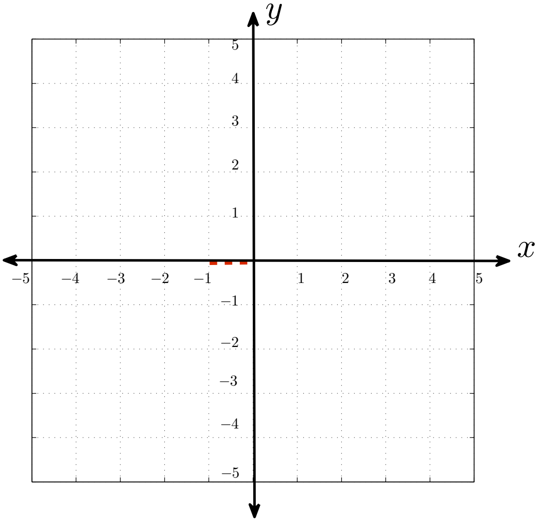

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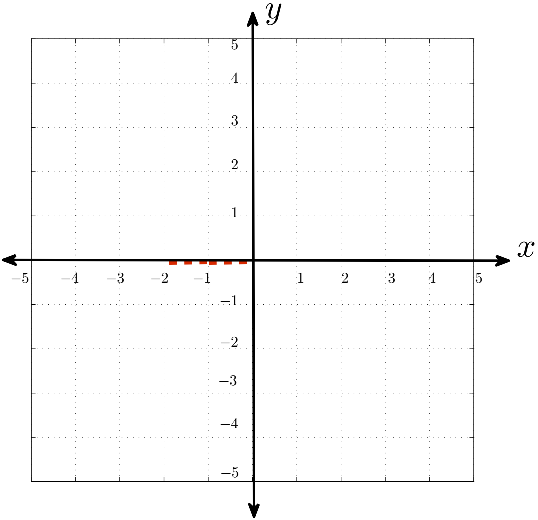

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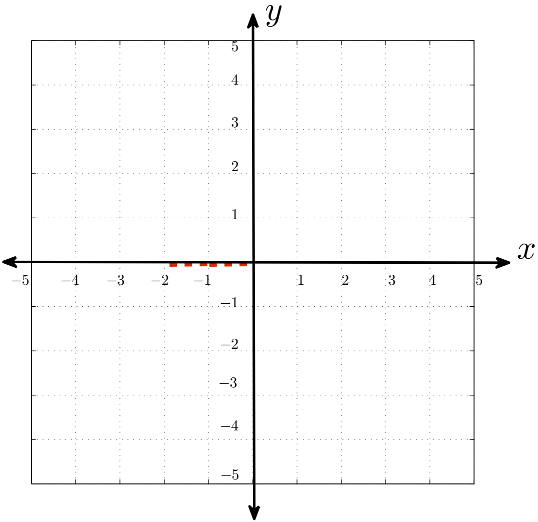

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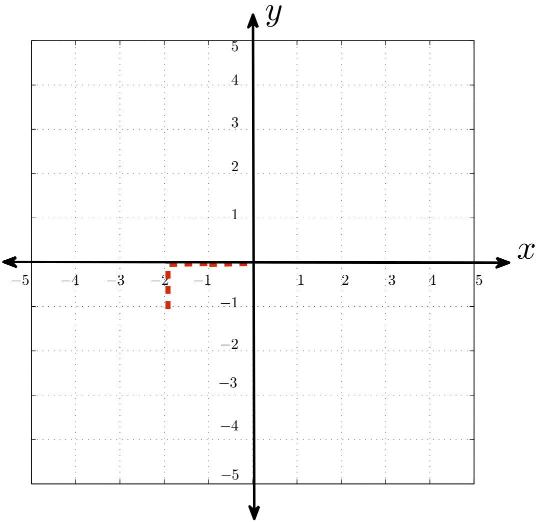

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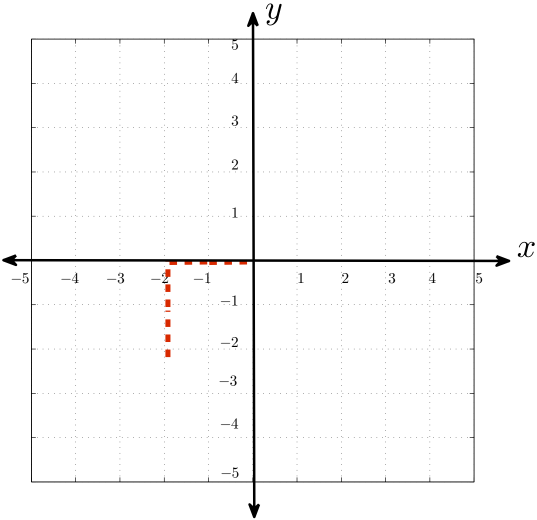

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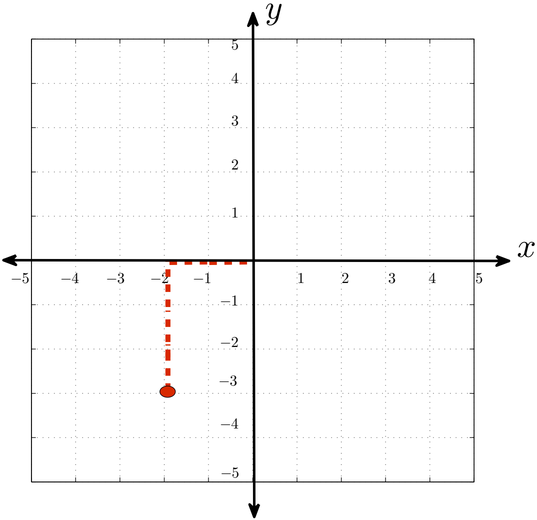

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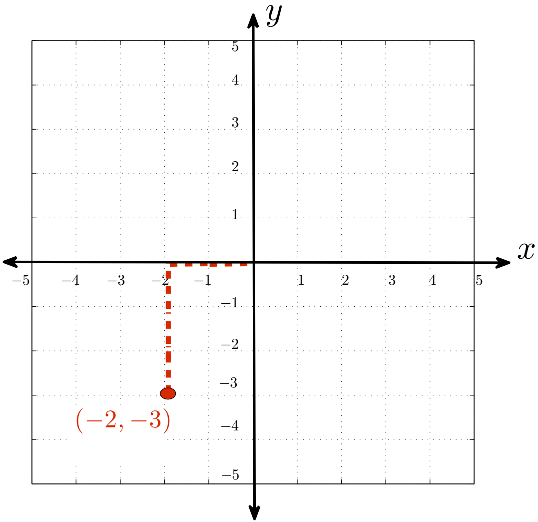

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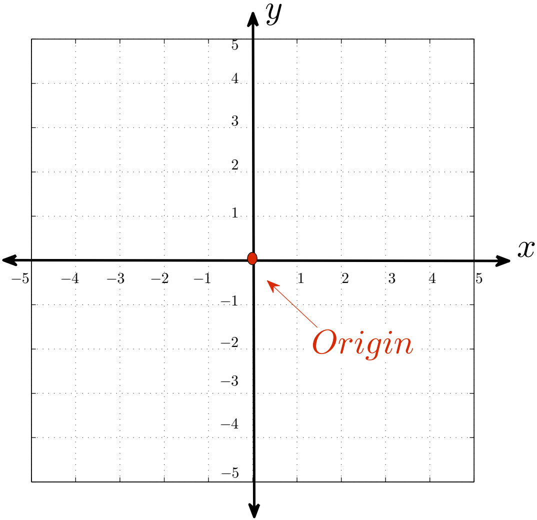

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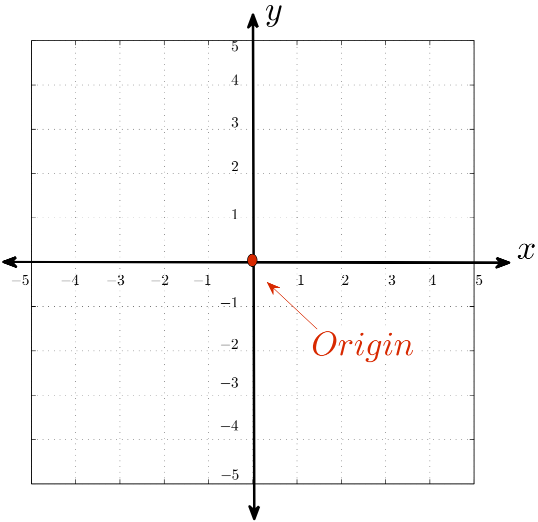

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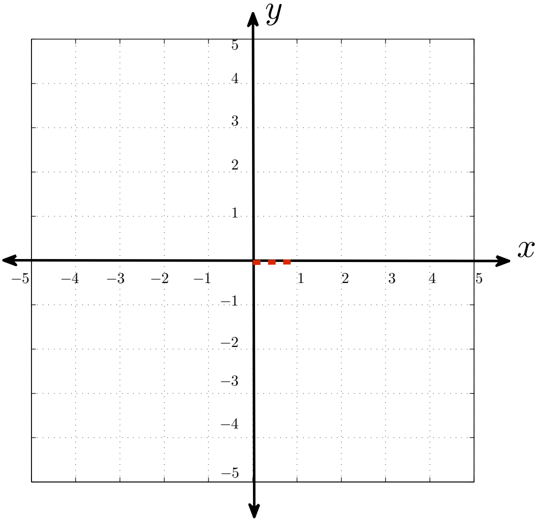

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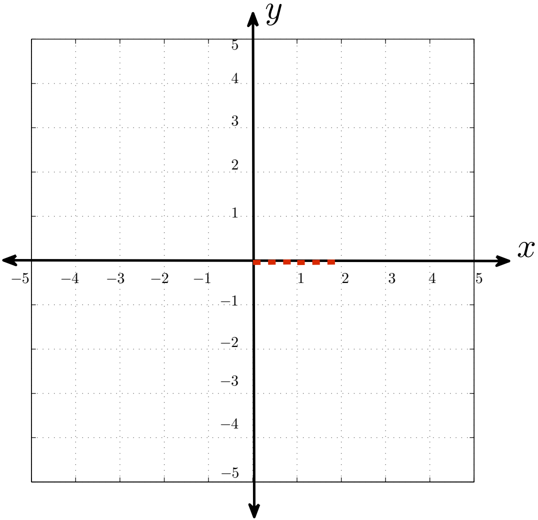

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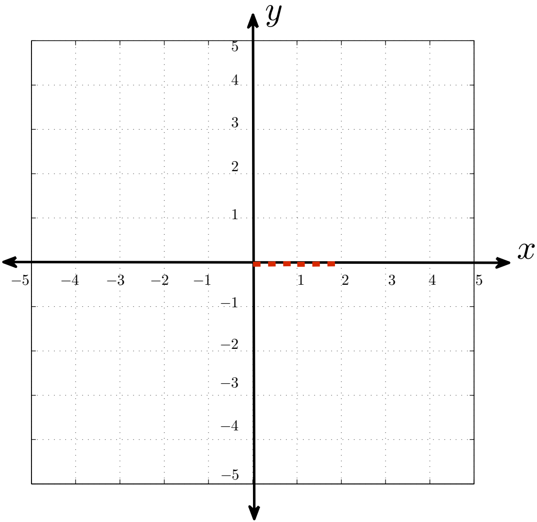

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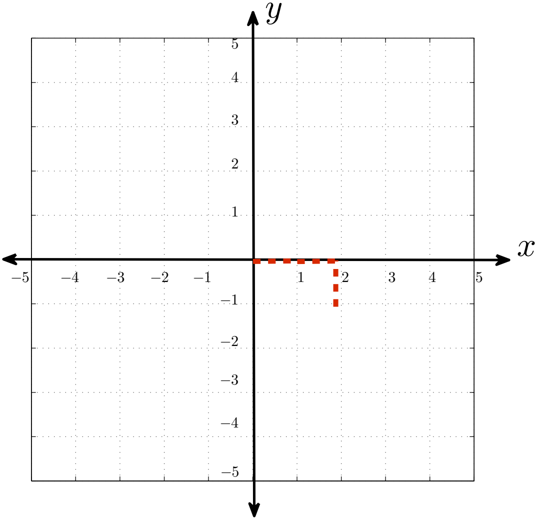

Review Topics

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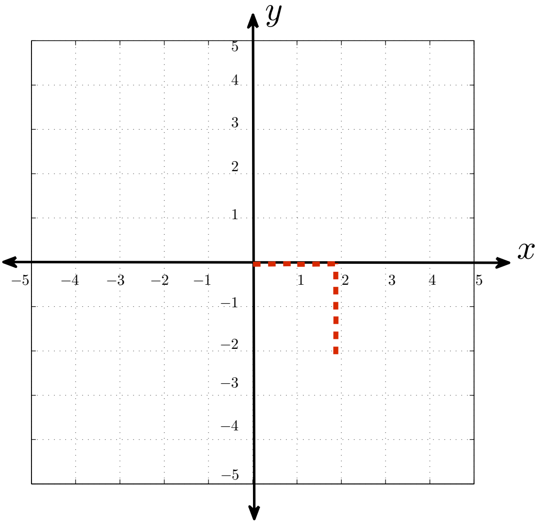

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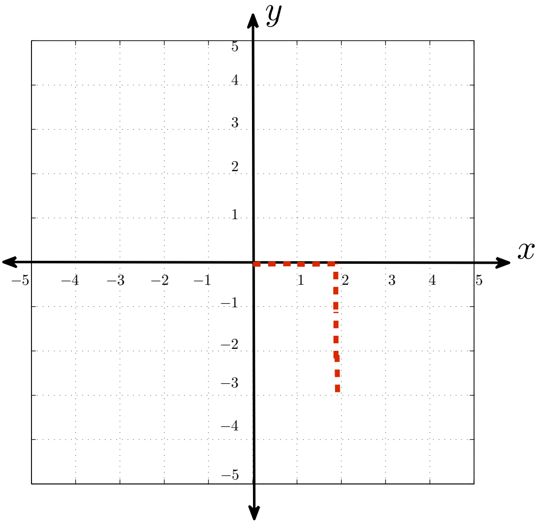

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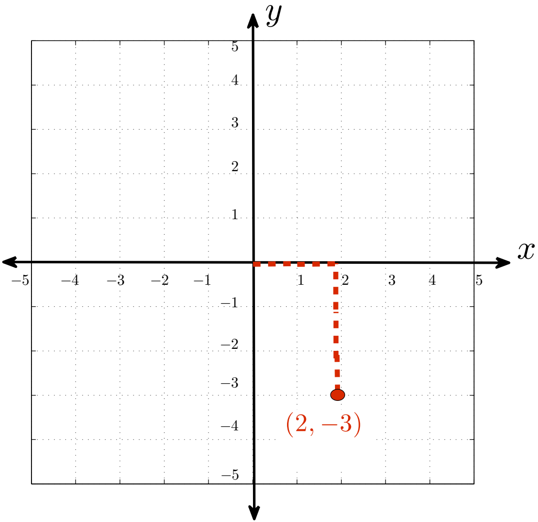
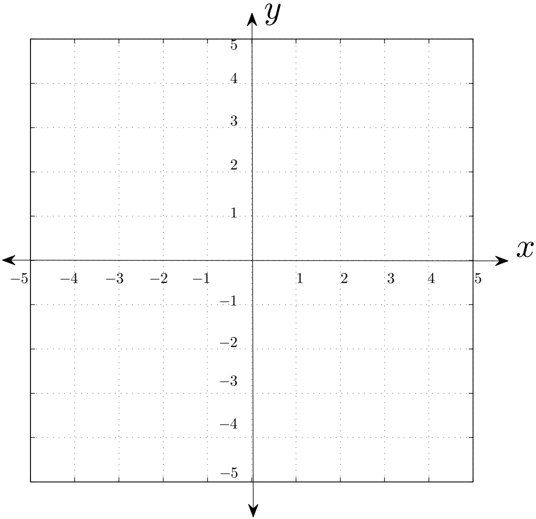


Example 2: Plot (graph) the following ordered pairs:

$$(3,0),\ (0,2),\ (-3,0),\ (0,-2),$$



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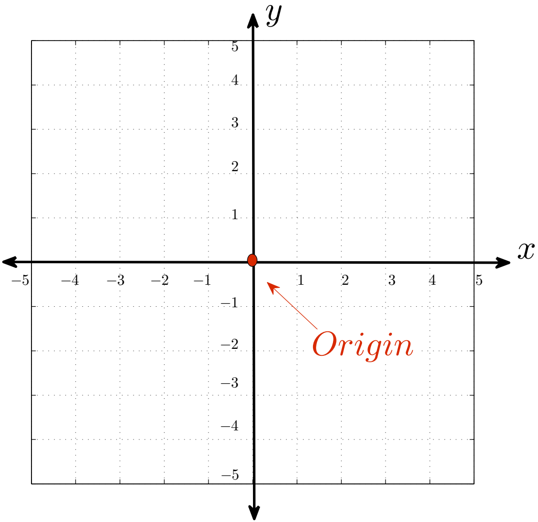


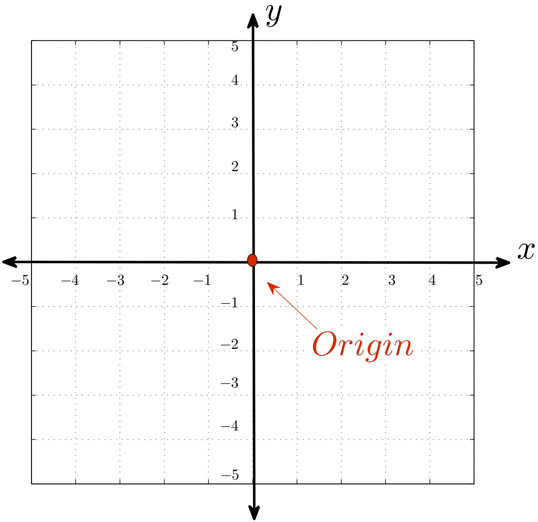
Tim Busken

Review Topics
Set Definitions
Number Types
Opposites
Absolute Value
Operations
Properties of Real Numbers
Distributive Property
Exponential Notation
The Order of Operations
Linear Equations
Linear Equations
Linear Inequalities
Coordinate System
Lines
slope
Perpendicular & Parallel Lines
Algebraic Expressions
Polynomials
Poly Add/Subtract
Combining Like Terms
Multiplying Poly's
Trinomial times Binomial
Factoring Polynomials
GCF
Factor By Grouping
ac-grouping method
Rational Expressions

Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

To plot $(3, 0)$, begin at the origin. Travel along the *x*-axis 3 units right, in the positive direction.



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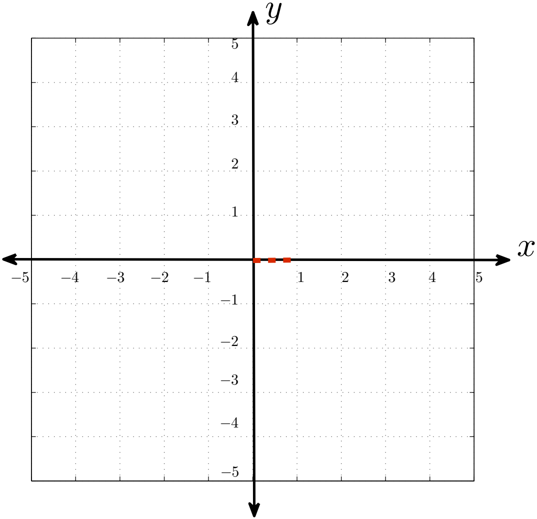

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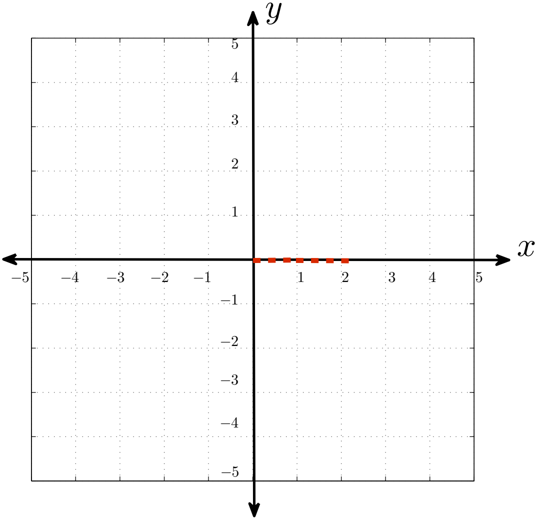

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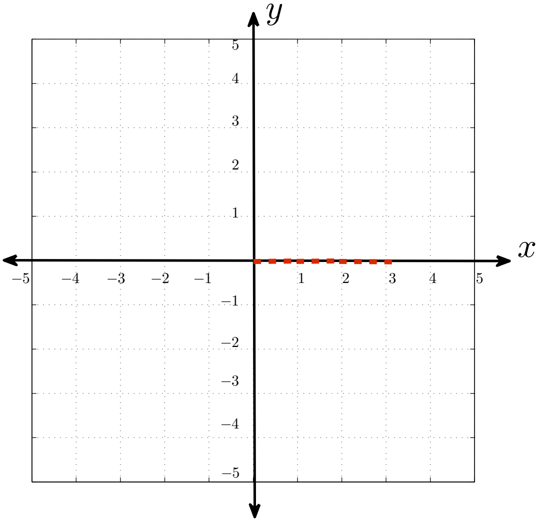

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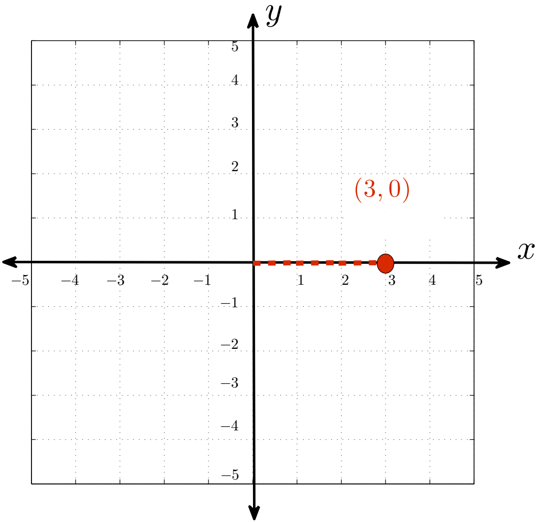

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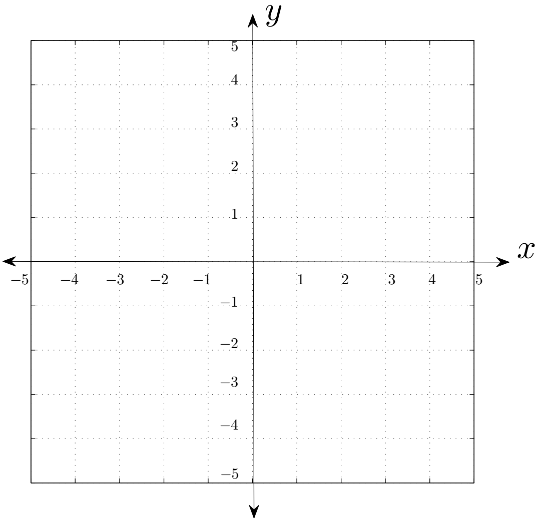

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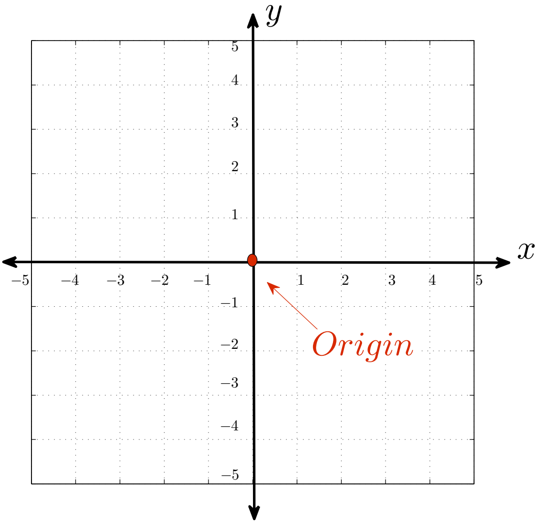

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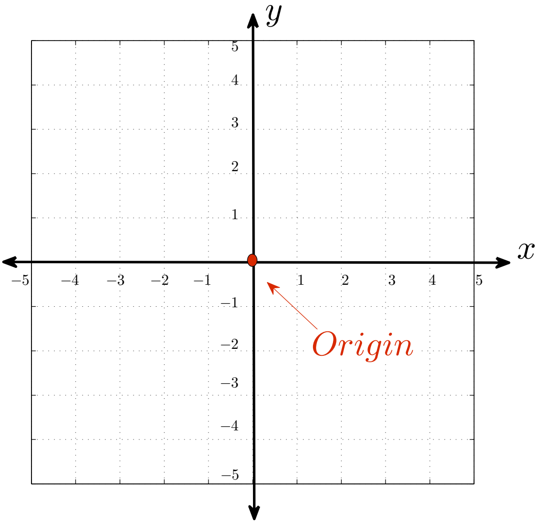
$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

From that point (the origin), move up 2 spaces in the positive *y* direction.



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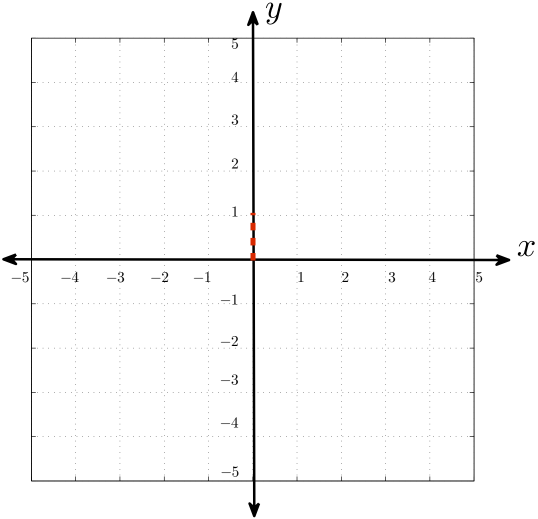

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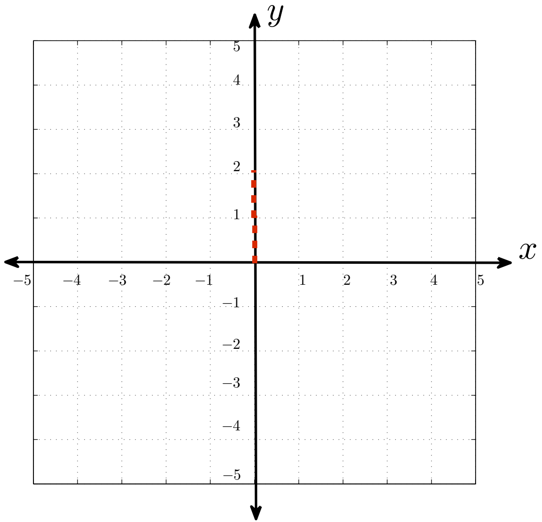

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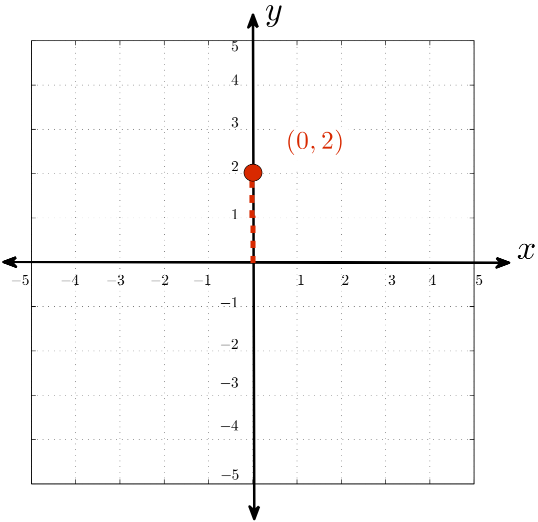

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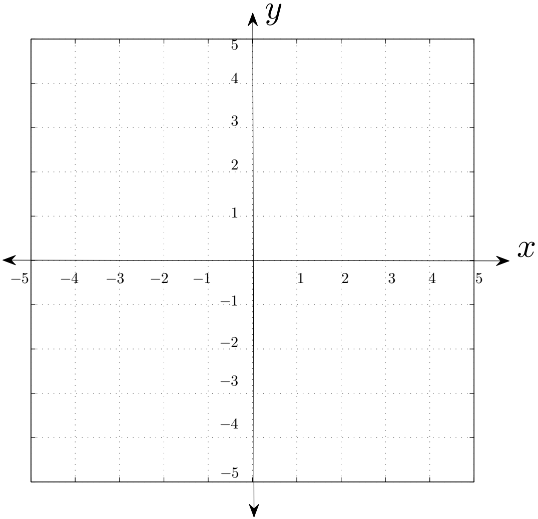

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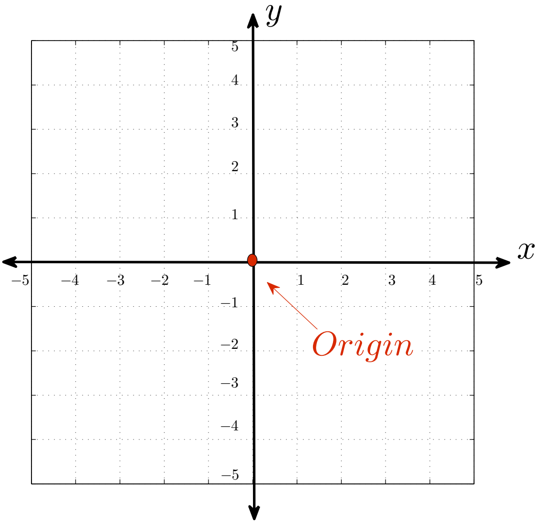

Example 2: Plot (graph) the following ordered pairs:

$(3, 0)$, $(0, 2)$, $(-3, 0)$, $(0, -2)$,

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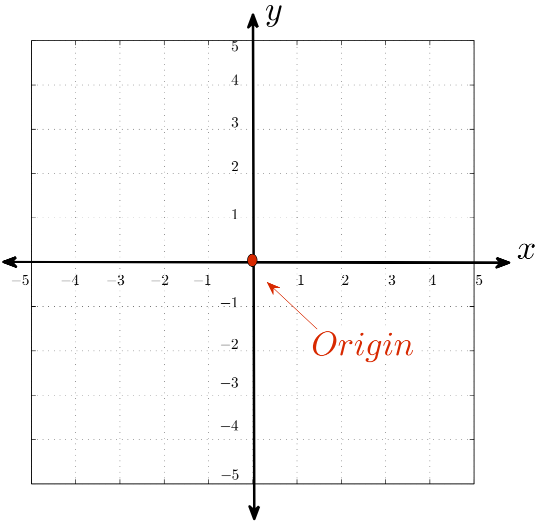

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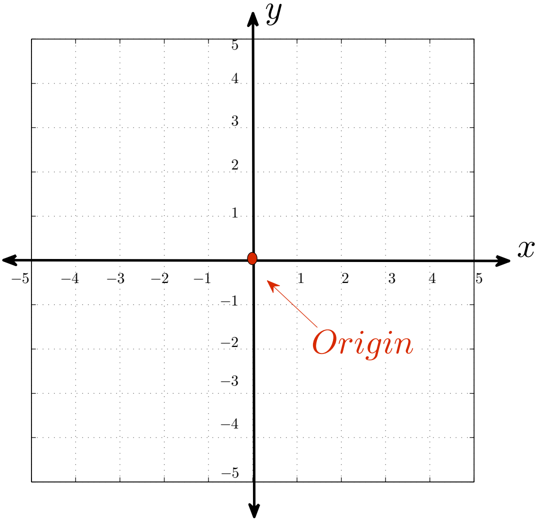

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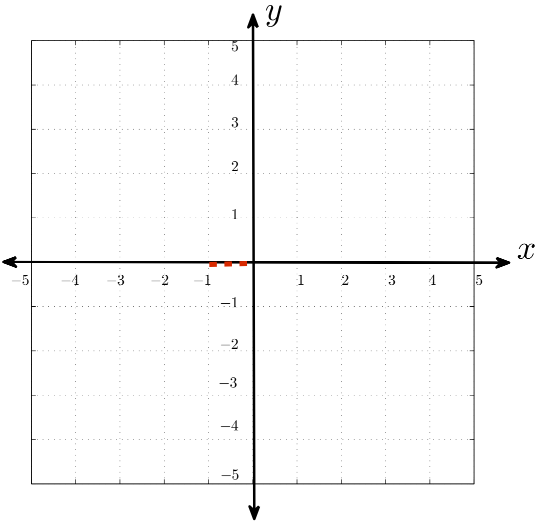

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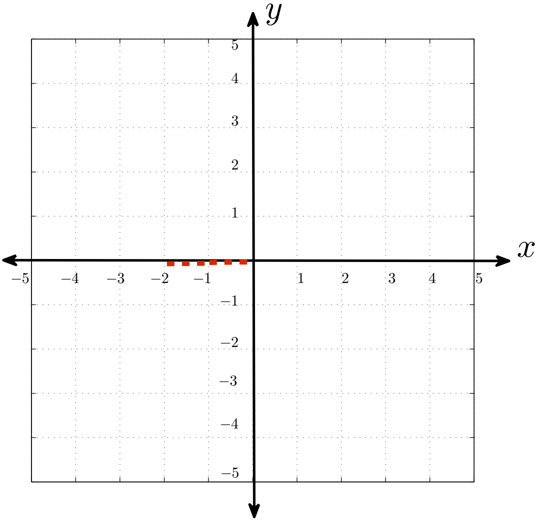

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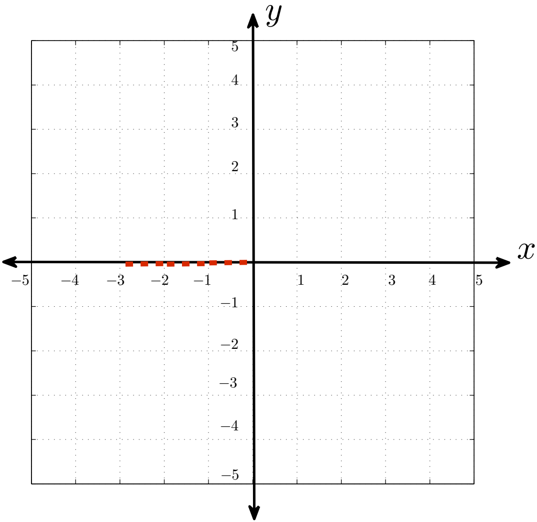

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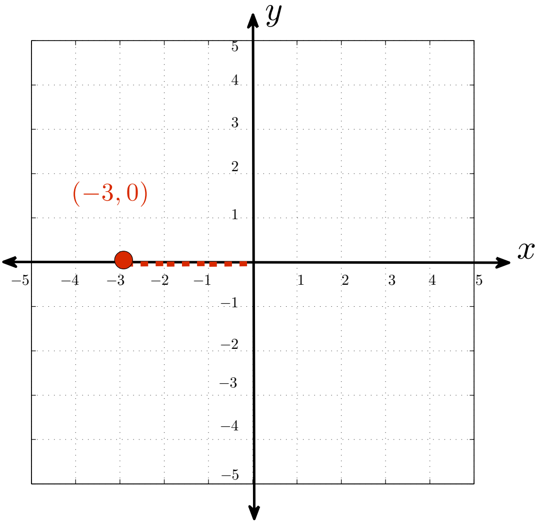
From that point (the origin), move up 0 spaces in the *y* direction.



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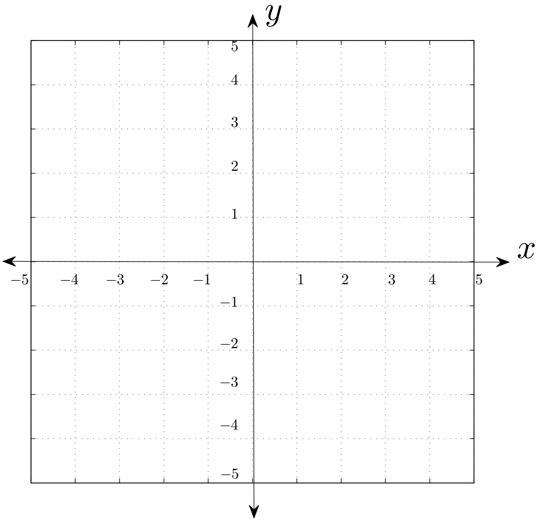

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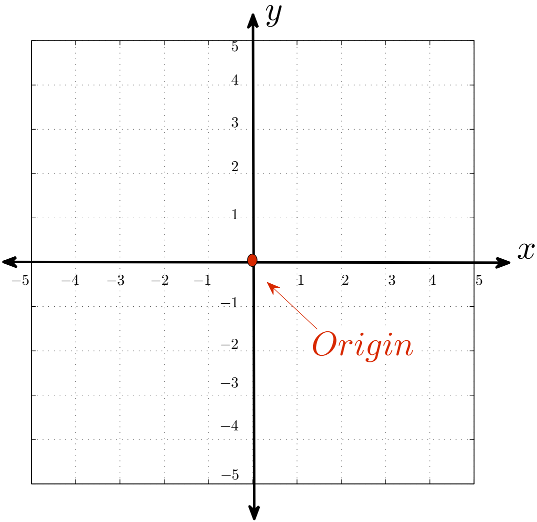

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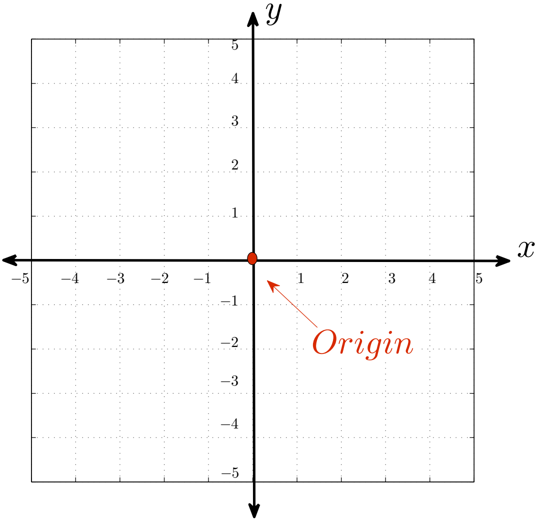
From that point (the origin), move up 2 spaces in the negative y direction (downwards).



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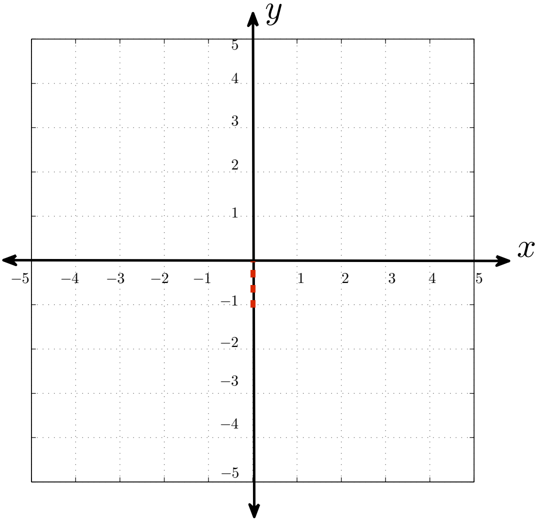

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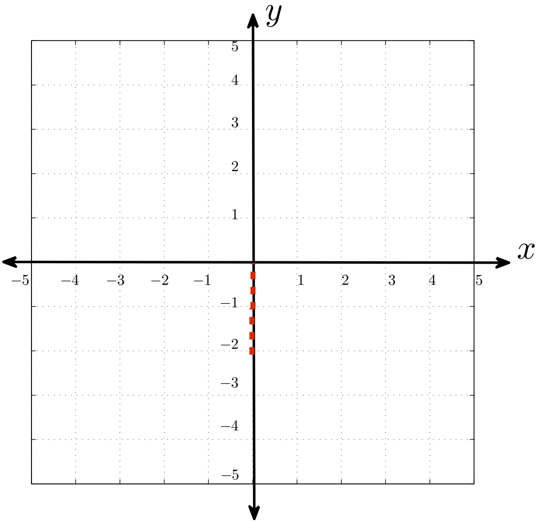

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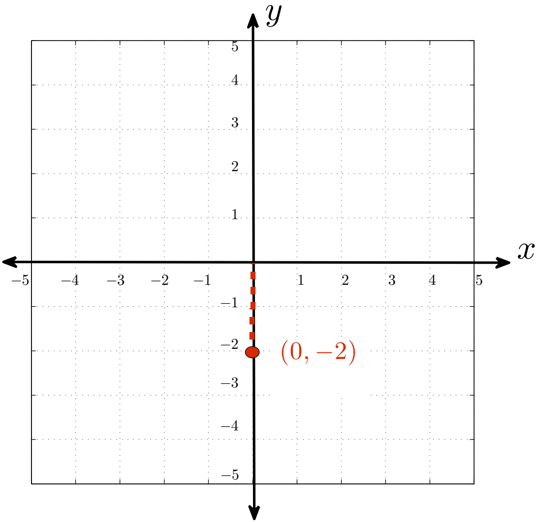
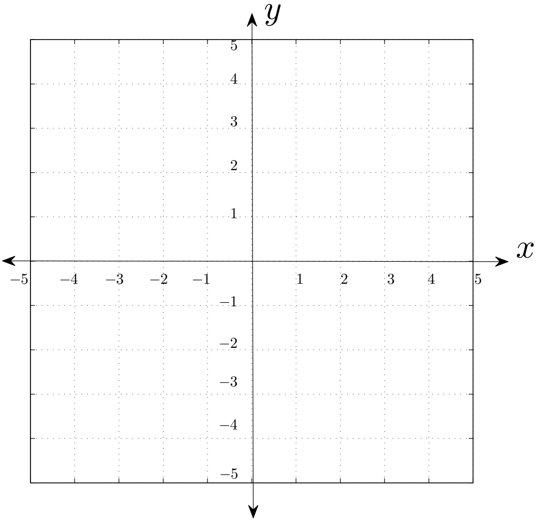


Concept Check

Plot (graph) $(3, 4)$, $(-4, 3)$, $(-1, -4)$ and $(5, -4)$



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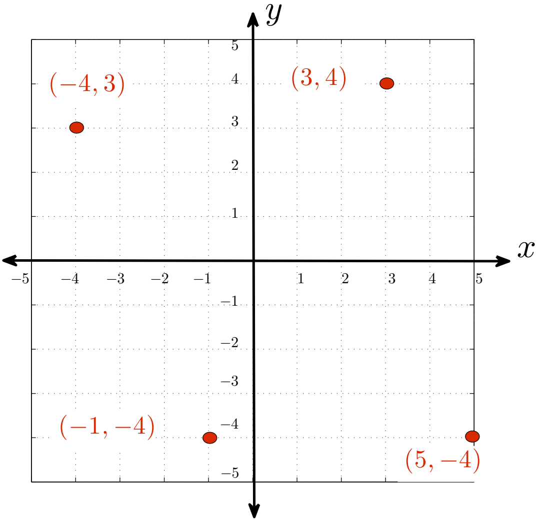
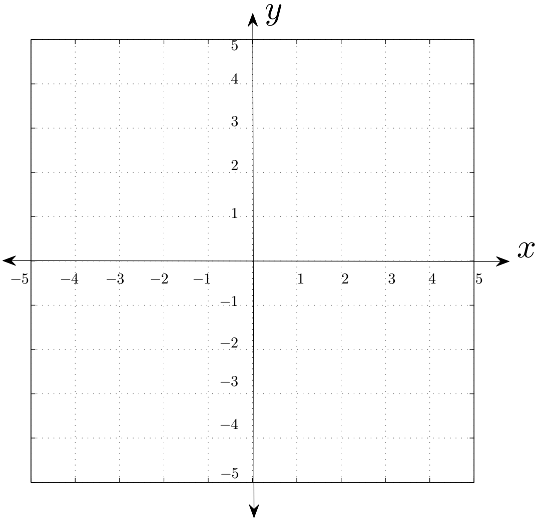


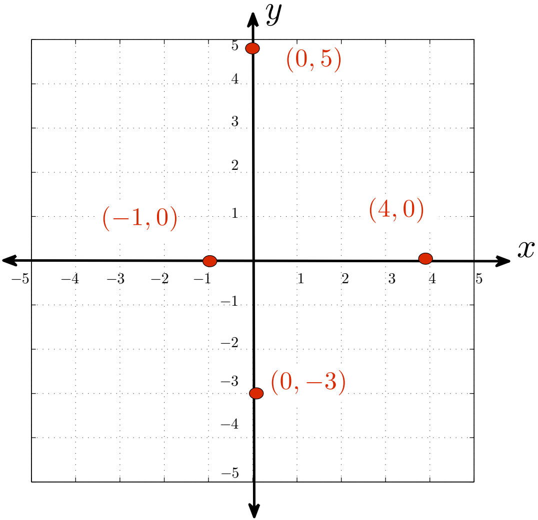
Concept Check

Plot (graph) $(4, 0)$, $(0, -3)$, $(-1, 0)$, and $(0, 5)$



Concept Check

Plot (graph) $(4, 0)$, $(0, -3)$, $(-1, 0)$, and $(0, 5)$



Definition

Suppose A, B and C represent any real numbers. A **linear equation in two variables** is an equation having the *form*

$$A\,x + B\,y = C,$$

For example, $2\,x + 3\,y = 1$ is a linear equation in the two variables x and y.

Solutions of a linear equation in two variables

Any linear equation in two variables always has in infinite number of solutions, and solutions come in the form of ordered pairs.

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Any linear equation in two variables always has in infinite number of solutions, and solutions come in the form of ordered pairs.

Terminology	Definition	Illustration
Solution of an equation in x and y	An ordered pair (a, b) that yields a true statement if $x = a$ and $y = b$	(1,4) is a solution of $y = 5x - 1$, since substituting x=1 and $y = 4$ renders the LHS = 4 and the RHS $= 5(1) - 1 = 4$

LHS is an abbreviation for "left-hand side" (of the equation)

RHS is an abbreviation for "right-hand side" (of the equation)

Equations and Graphs

Definition

For each ordered-pair solution, (a, b), of an equation in x and y there is a point (a, b) in a rectangular coordinate plane. The set of all such points is called a **graph of the equation**.

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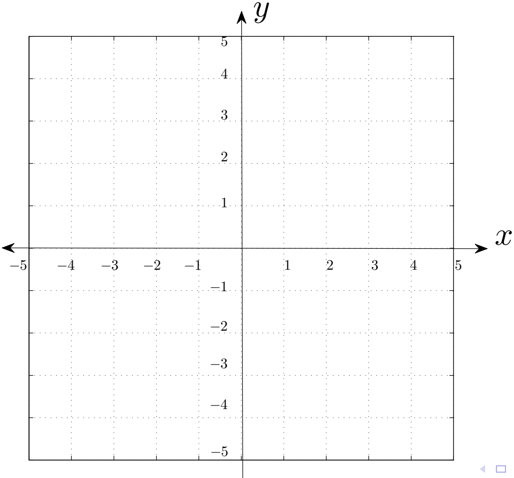
We can graph a linear equation by finding 3 ordered-pair solutions of the equation, plot the corresponding points on the rectangular grid, then draw a line between the three points.

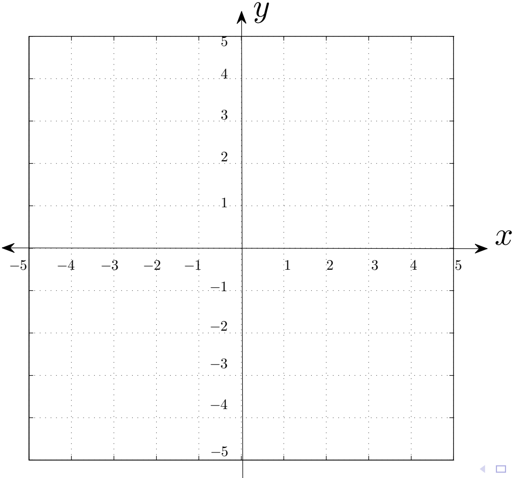
We use the third point for "insurance." If all three points line up in a straight we have not made a mistake!

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$



Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

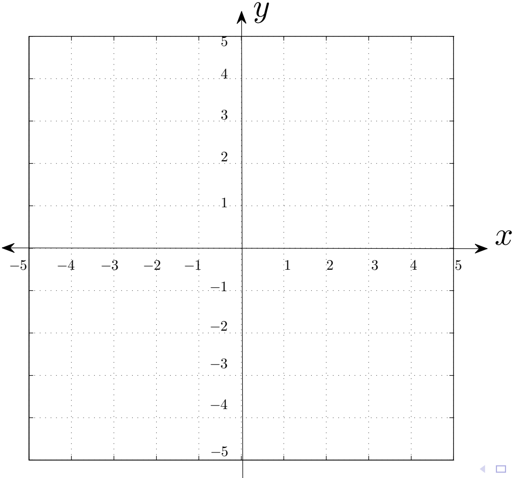
We begin by a making a table that summarizes x and y values. Since every value of x we substitute into the equation will be multiplied by $-\frac{1}{2}$, we use numbers for x that are divisible by 2.



x	y	$(x, y$
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

That way, when we multiply by $-\frac{1}{2}$, the result will be an integer.



x	y	$(x, y$
-2		
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Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We let $x = -2$ in the equation to find the y-value of the ordered pai
which is associated with x-coordinate -2.

x	y	$(x, y$
-2		
0		
2		

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$$\begin{aligned} y &= -\frac{1}{2} \cdot ({\color{red}x}) - 3 \\ &= -\frac{1}{2} \cdot ({\color{red}-2}) - 3 \\ &= 1 - 3 \\ &= -2 \end{aligned}$$

x	y	$(x, y$
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Upon simplification, we get the ordered pair solution (-2,-2)

$$\begin{aligned} y &= -\frac{1}{2} \cdot (x) - 3 \\ &= -\frac{1}{2} \cdot (-2) - 3 \\ &= 1 - 3 \\ &= -2 \end{aligned}$$

x	y	$(x, y$
-2	-2	(-2,-
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Next, we let $x = 0$ in the equation to find the y-value of the ordered pair which is associated with x-coordinate 0.

$$
\begin{aligned}
y &= -\frac{1}{2} \cdot (x) - 3 \\
&= -\frac{1}{2} \cdot (0) - 3 \\
&= 0 - 3 \\
&= -3
\end{aligned}
$$

x	y	$(x, y$
-2	-2	(-2,-
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

This gives us the ordered pair solution (0,-3)

$$\begin{aligned} y &= -\frac{1}{2}\cdot(x) - 3 \\ &= -\frac{1}{2}\cdot(0) - 3 \\ &= 0 - 3 \\ &= -3 \end{aligned}$$

x	y	$(x, y$
-2	-2	(-2,-
0	-3	(0,-3
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Afterwards, we let $x = 2$ in the equation.

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (2) - 3$$

$$= -1 - 3$$

$$= -4$$

x	y	$(x, y$
-2	-2	(-2,-
0	-3	(0,-3
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Upon simplification, we get the ordered pair solution (2,-4)

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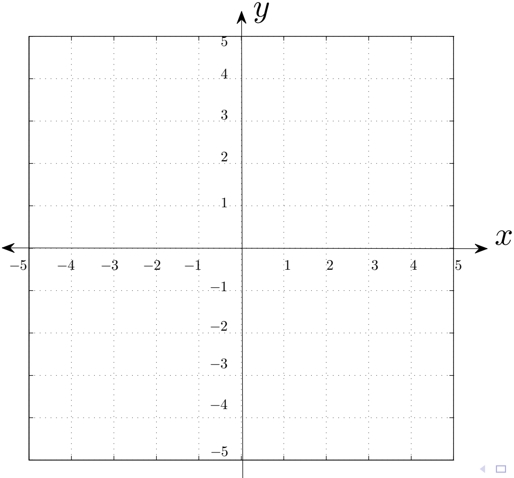
\begin{aligned}
y &= -\frac{1}{2} \cdot (x) - 3 \\
&= -\frac{1}{2} \cdot (2) - 3 \\
&= -1 - 3 \\
&= -4
\end{aligned}
$$

x	y	$(x, y$
-2	-2	(-2,-
0	-3	(0,-3
2	-4	(2,-4

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We now locate the three ordered pair solutions (points) on the rectangular coordinate grid, then draw a line through the solutions.



x	y	$(x, y$
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0	-3	(0,-3
2	-5	(2, -

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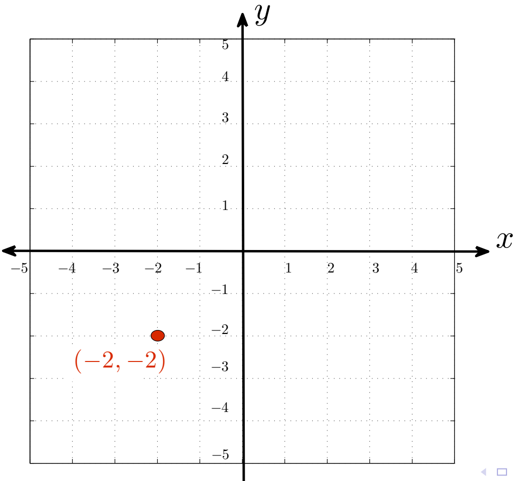

x	y	$(x, y$
-2	-2	(-2, -
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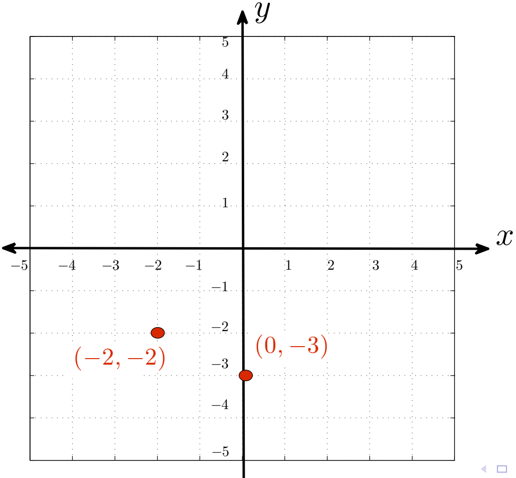

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-2	-2	(-2, -
0	-3	(0,-3
2	-5	(2, -

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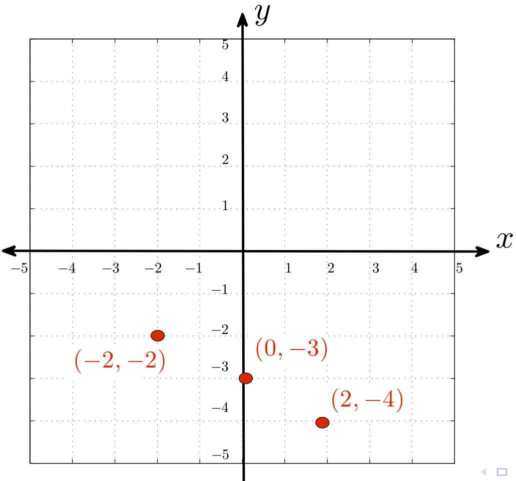

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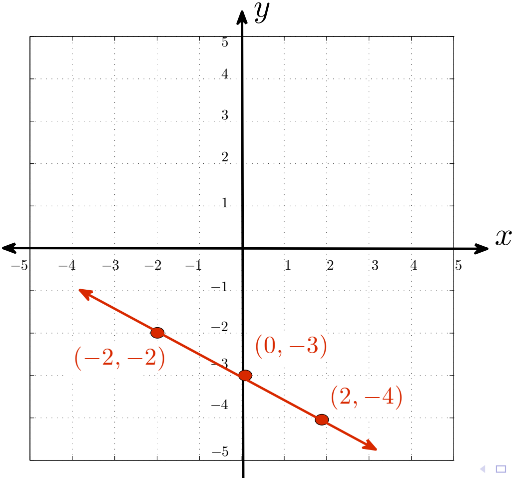
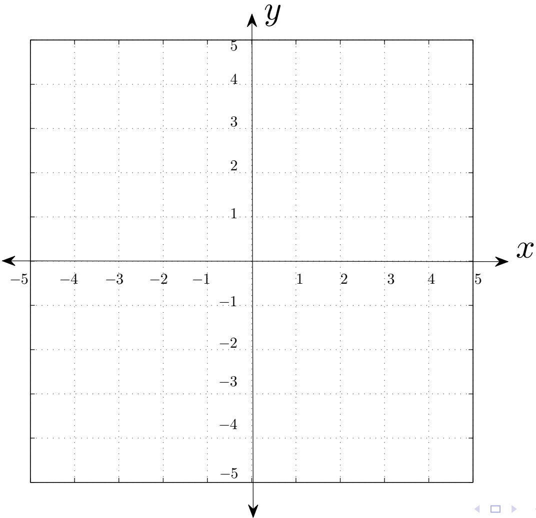
Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We now locate the three ordered pair solutions (points) on the rectangular coordinate grid, then draw a line through the solutions.



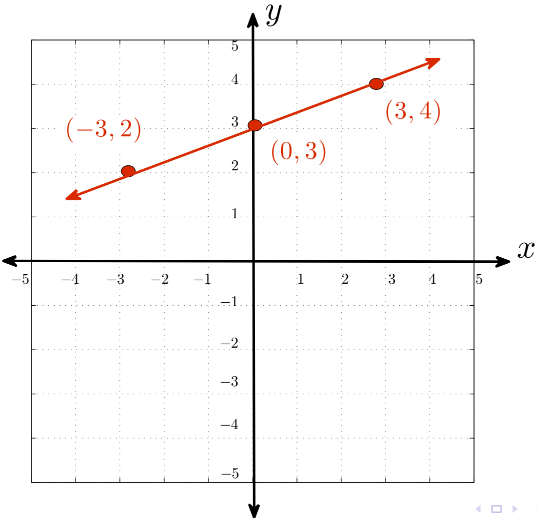
x	y	(x, y
-2	-2	(-2, -
0	-3	(0,-3
2	-5	(2, -

Concept Check: Graph $y = \frac{1}{3}x + 3$



x	y	$(x, y$

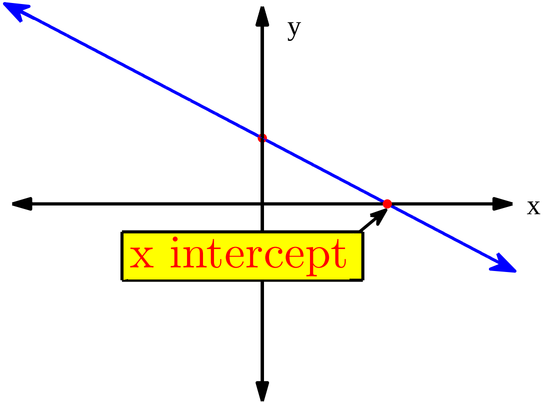
Concept Check: Graph $y = \frac{1}{3}x + 3$



x	y	$(x, y$
-3	2	(-3,2
0	3	(0,3)
3	4	(3,4)

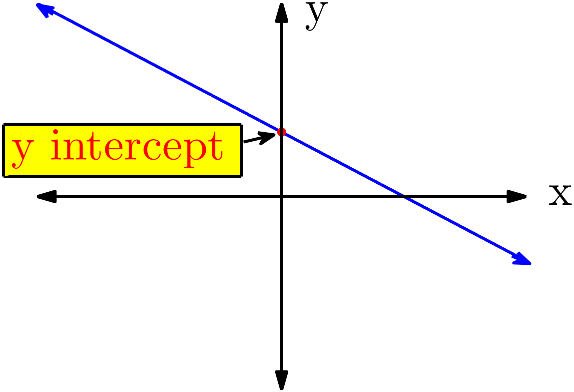
Definition

The graph of an equation has an **x-intercept** whenever the graph of the equation crosses the x axis. The x intercept always occurs when the value of y is equal to zero.



Definition

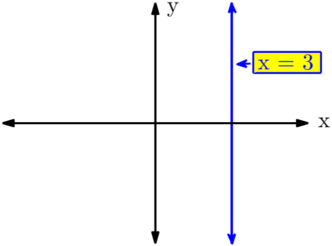
The graph of an equation has an **y-intercept** whenever the graph of the equation crosses the y axis. The y intercept always occurs when the value of x is equal to zero.



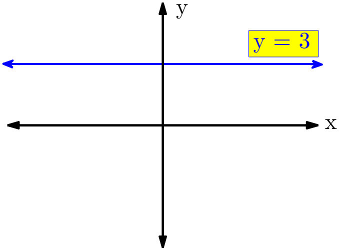
Example 4 Find the x- and y-intercepts for $5x - 7y = -35$, then graph the solution set.

Theorem

Suppose a and b are real numbers. Graphs of linear equations of the form $x = a$ are vertical lines and graphs of linear equations of the form $y = b$ are horizontal lines.



graph of $x = 3$



graph of $y = 3$

More Classroom Examples

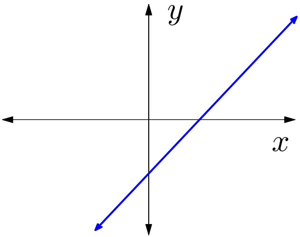
Graph each of the following lines:

- $y = \frac{1}{2}x$
- $x = -2$
- $y = -4$

Definition

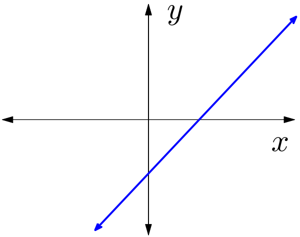
The slope of a line is a measure of the steepness of the line.

A line that rises from left to right has positive slope.

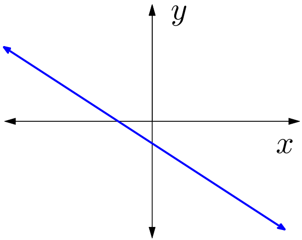


Positive Slope

A line that falls from left to right has negative slope.

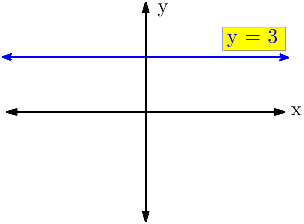


Positive Slope



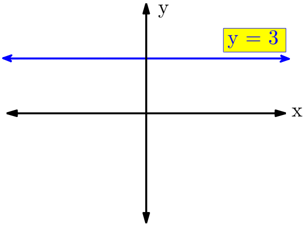
Negative Slope

Horizontal lines have zero slope

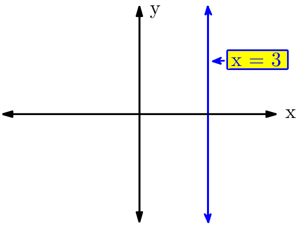


graph of $x = 3$

Horizontal lines have zero slope, and vertical lines have no slope.

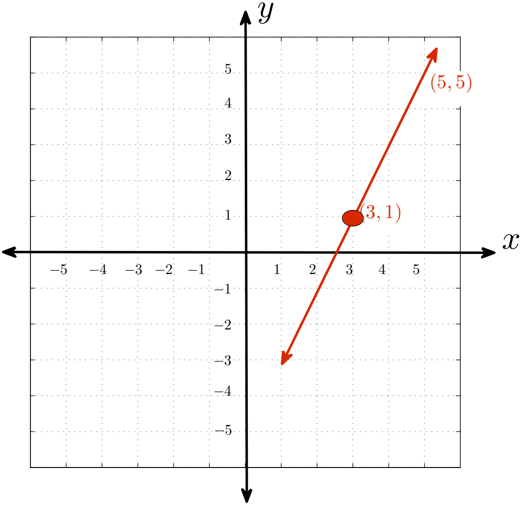


graph of $x = 3$



graph of $y = 3$

We can define the slope of a line as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



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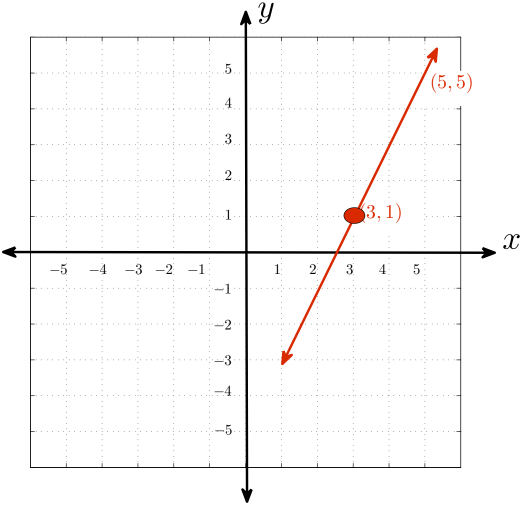

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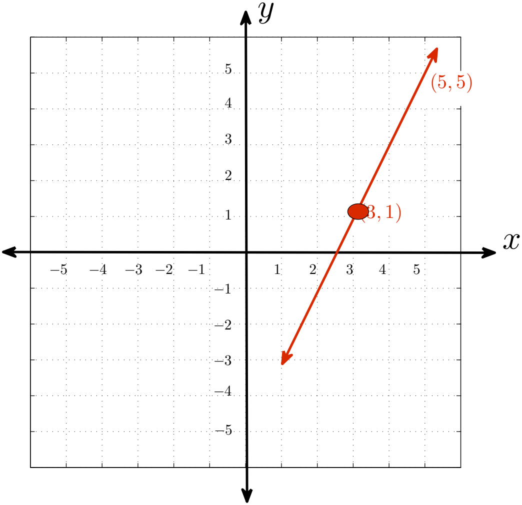

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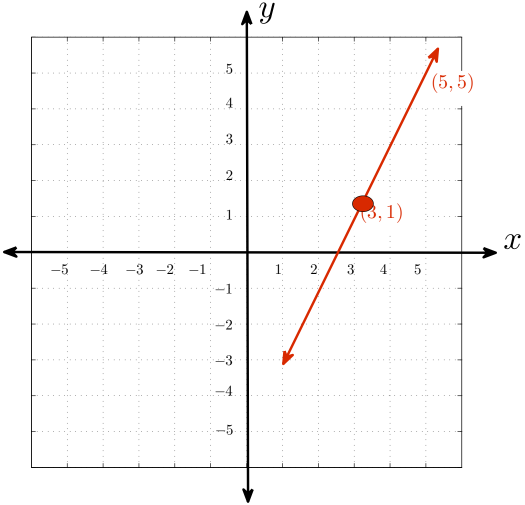

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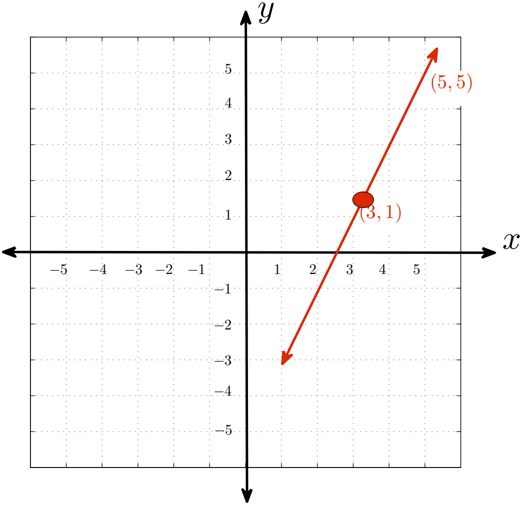

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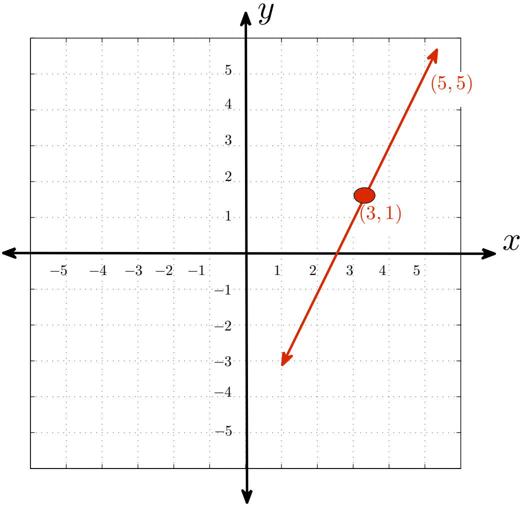

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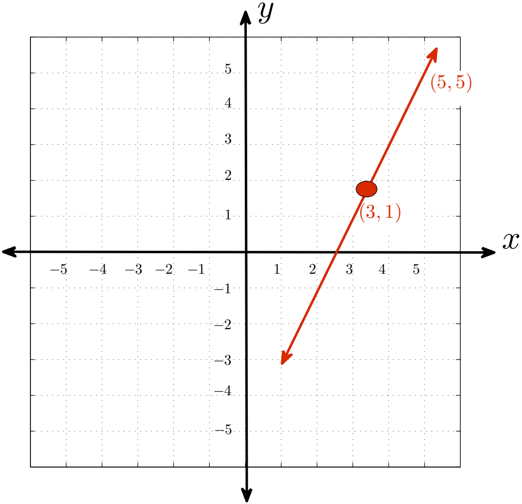

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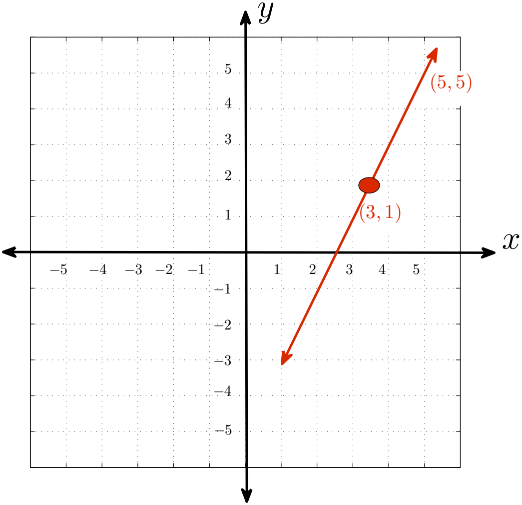

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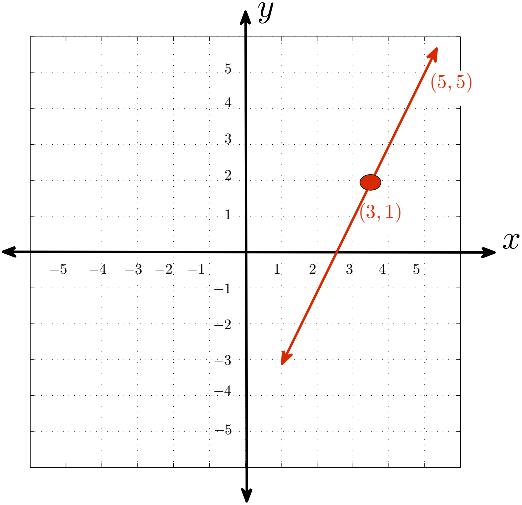

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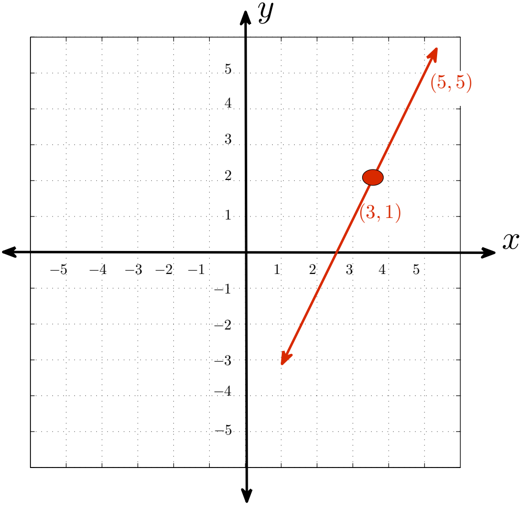

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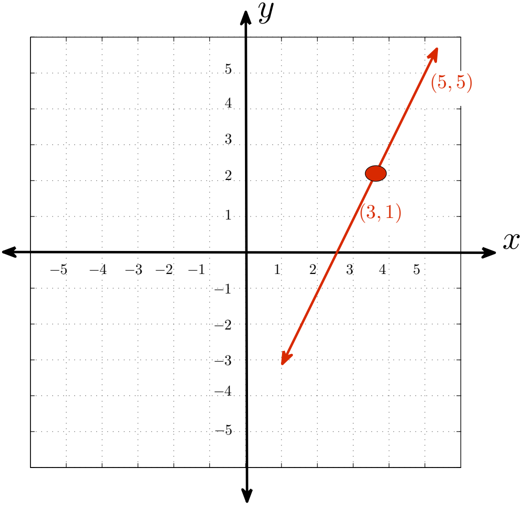

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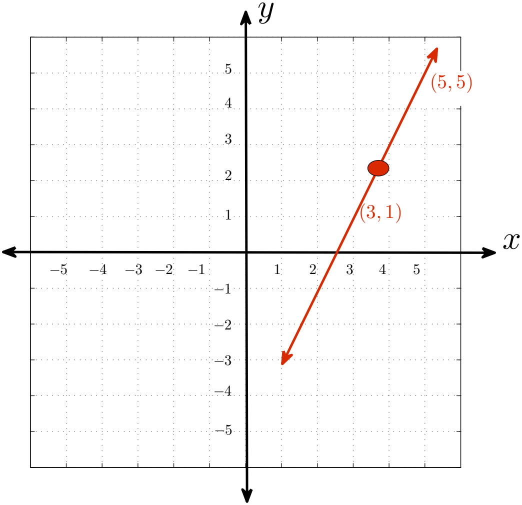

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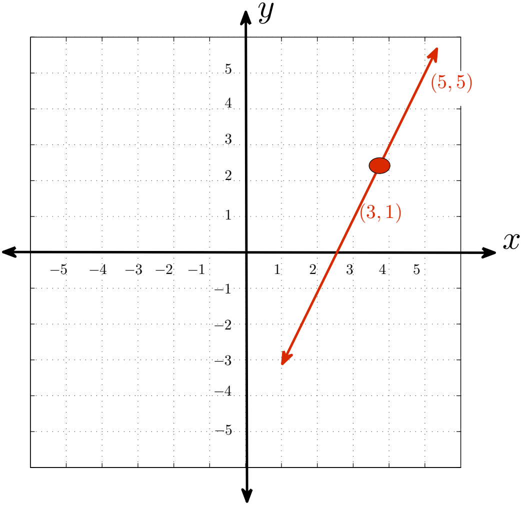

Review Topics

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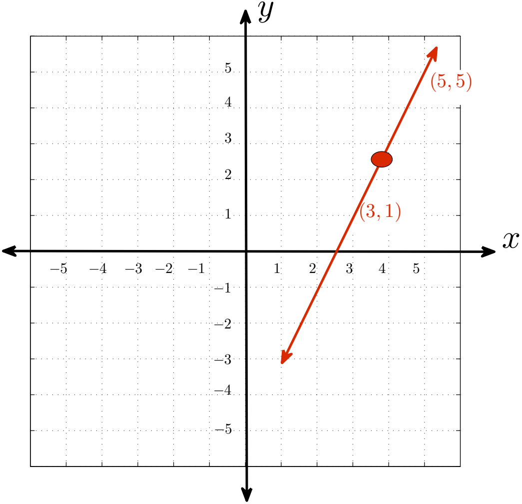

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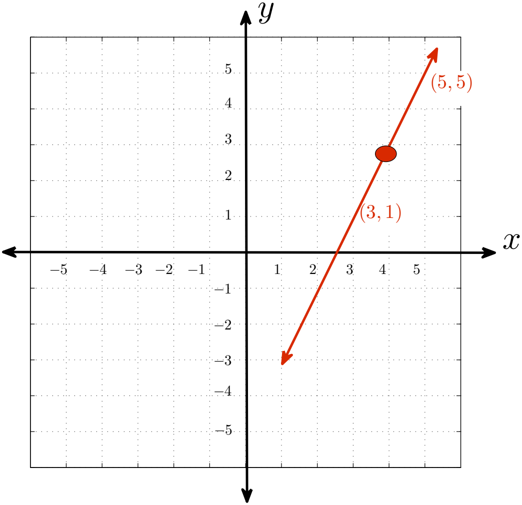

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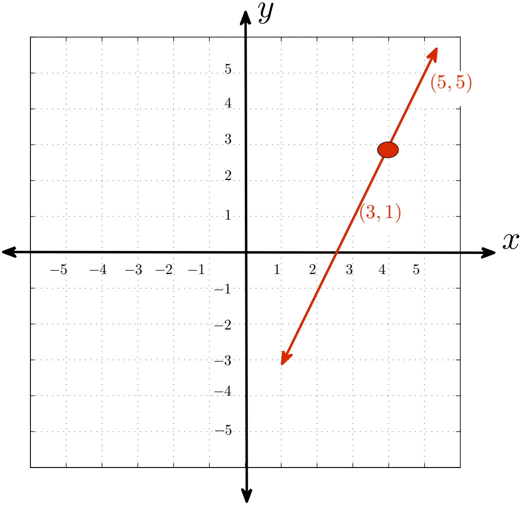

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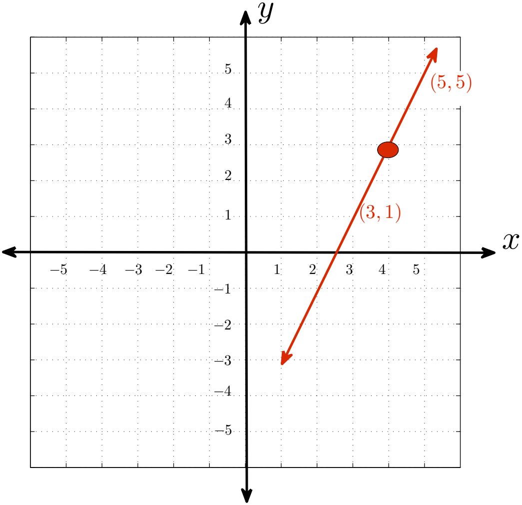

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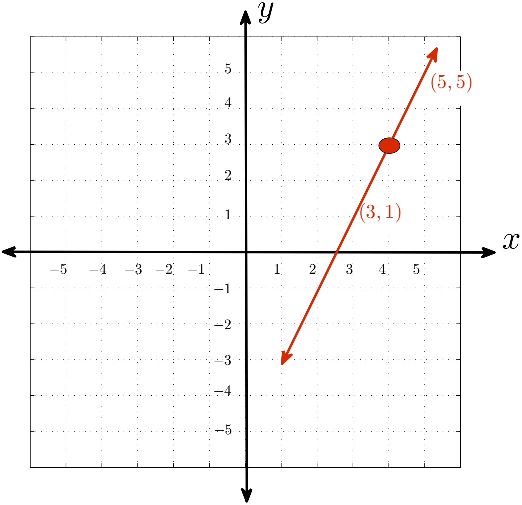

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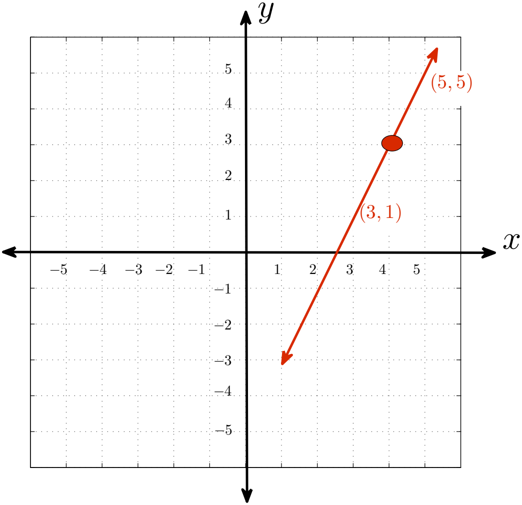

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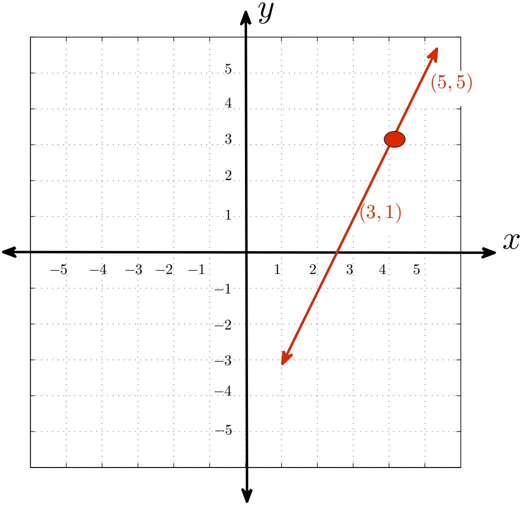

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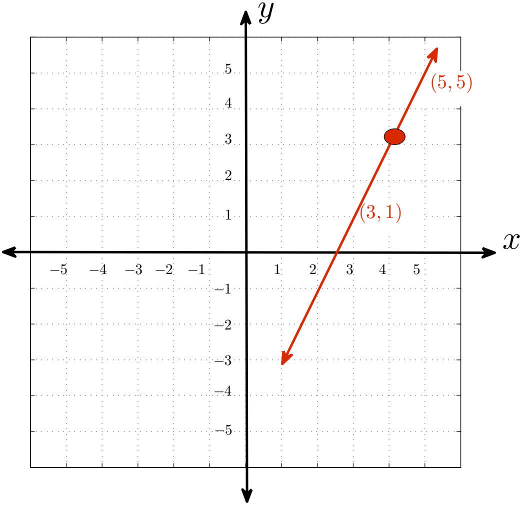

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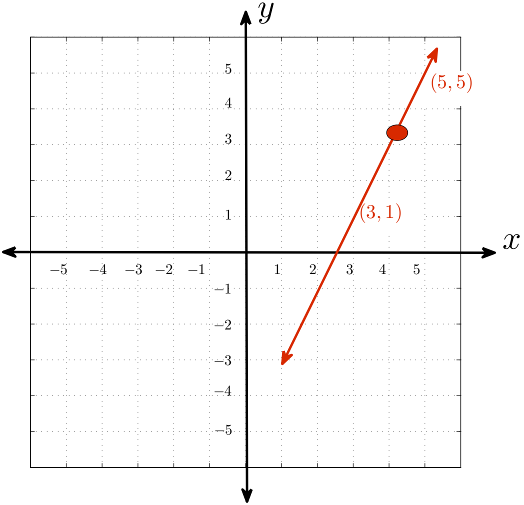

Review Topics
Set Definitions
Number Types
Opposites
Absolute Value
Operations
Properties of Real Numbers
Distributive Property
Exponential Notation
The Order of Operations
Linear Equations
Linear Equations
Linear Inequalities
Coordinate System
Lines
slope
Perpendicular & Parallel Lines
Algebraic Expressions
Polynomials
Poly Add/Subtract
Combining Like Terms
Multiplying Poly's
Trinomial times Binomial
Factoring Polynomials
GCF
Factor By Grouping
ac-grouping method
Rational Expressions

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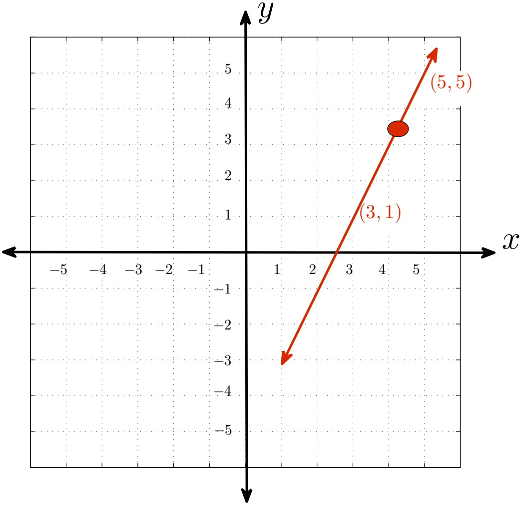

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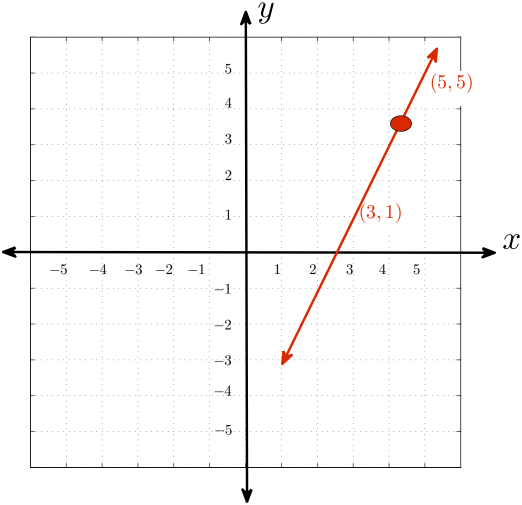

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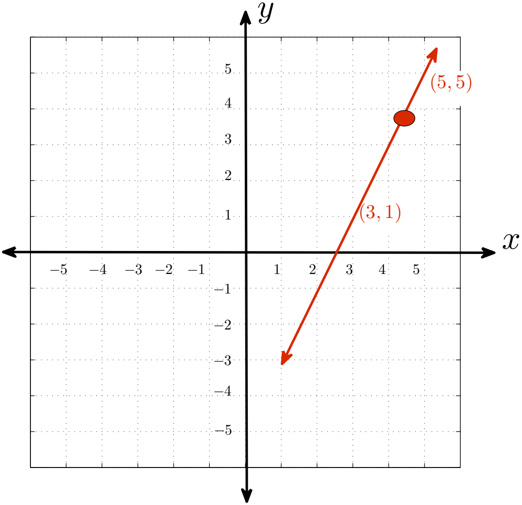

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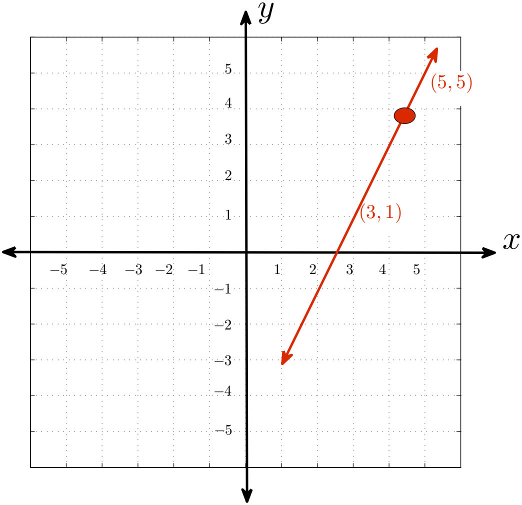

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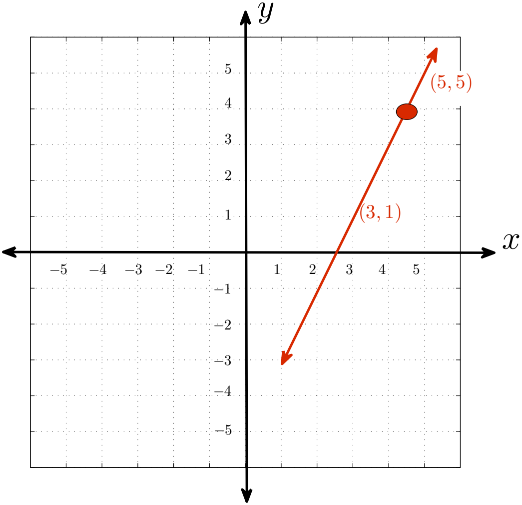

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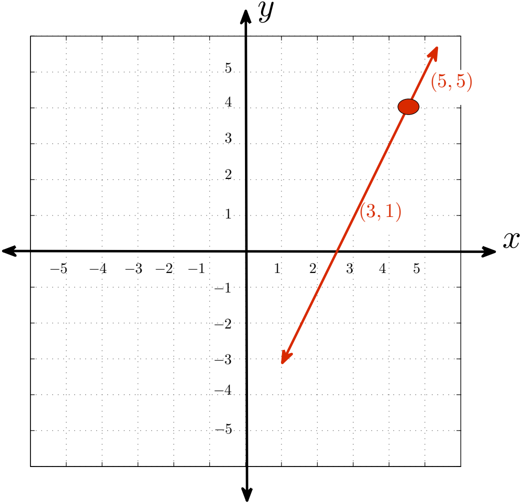

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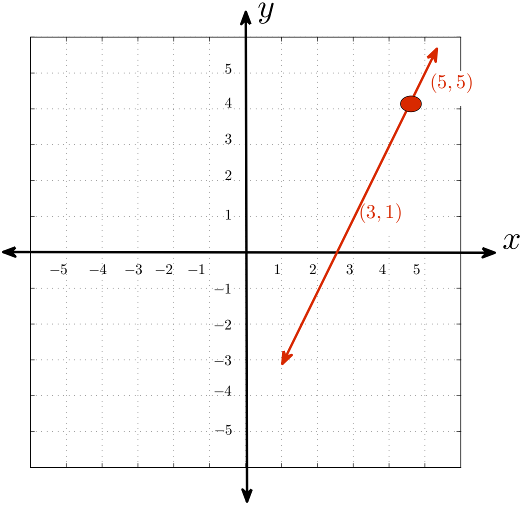

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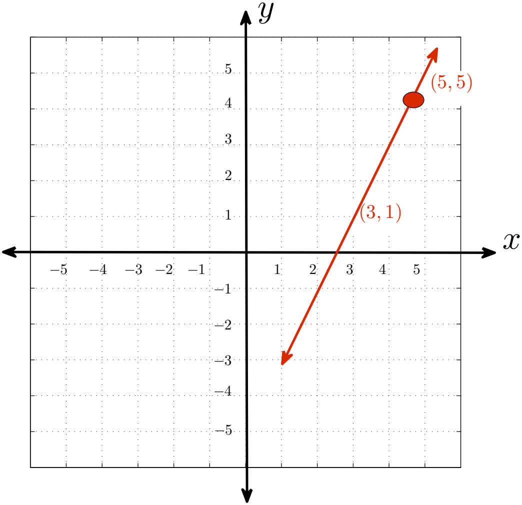

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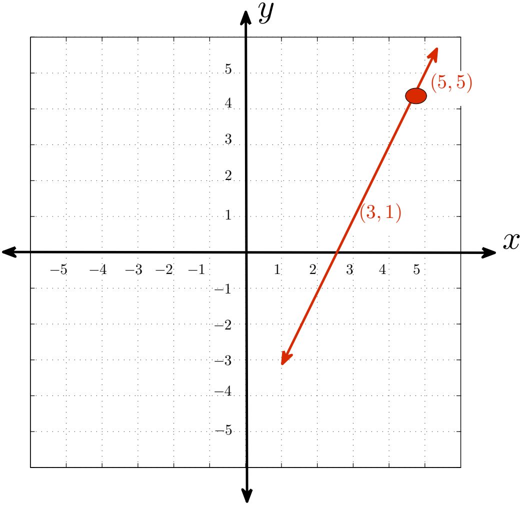

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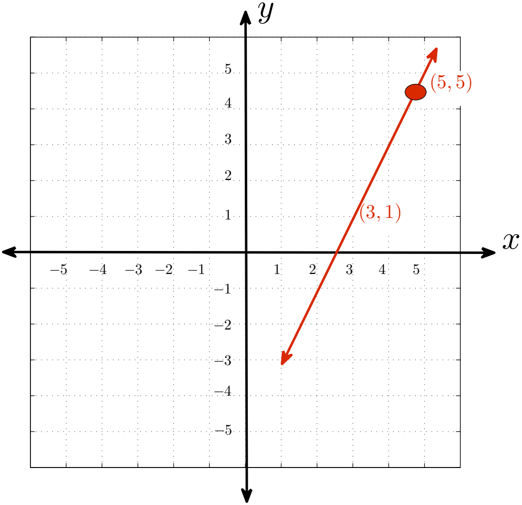

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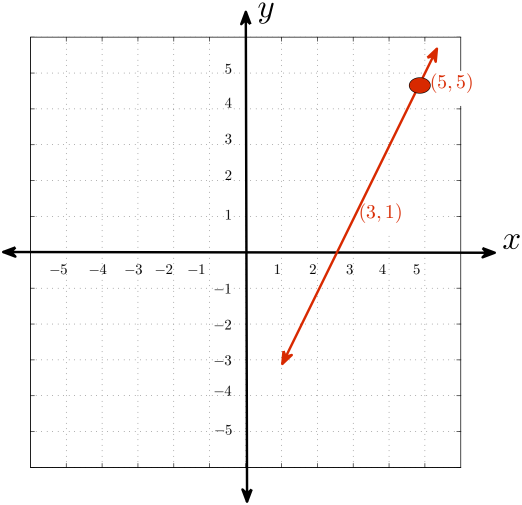

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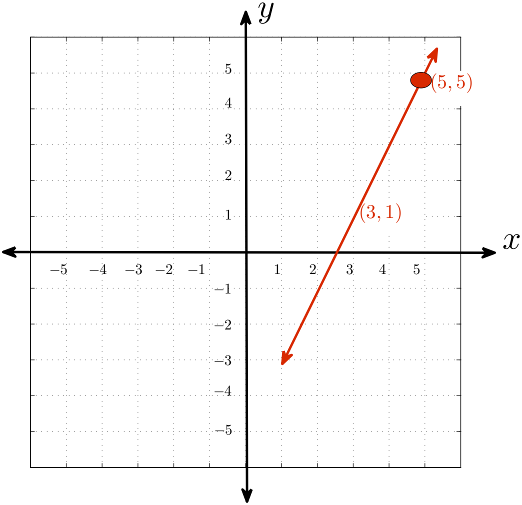

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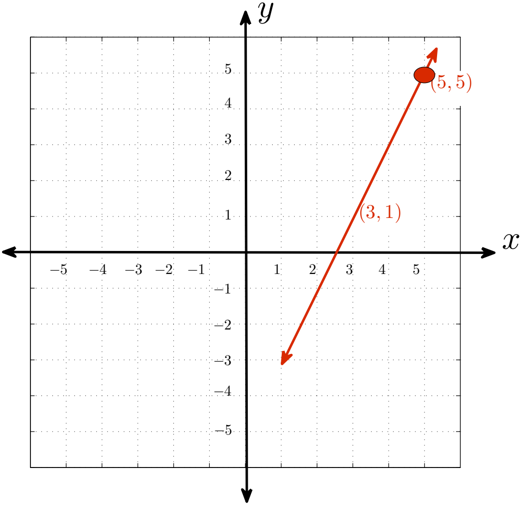

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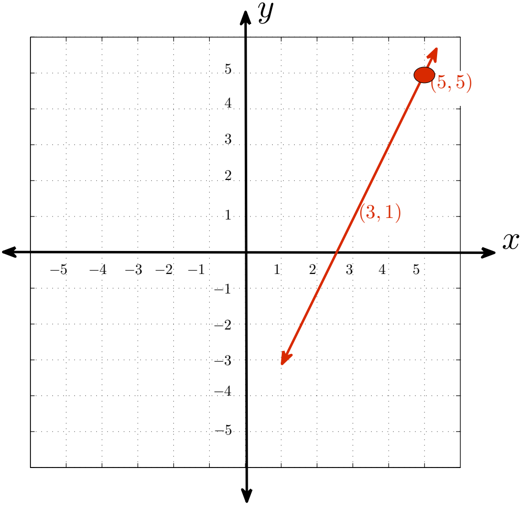

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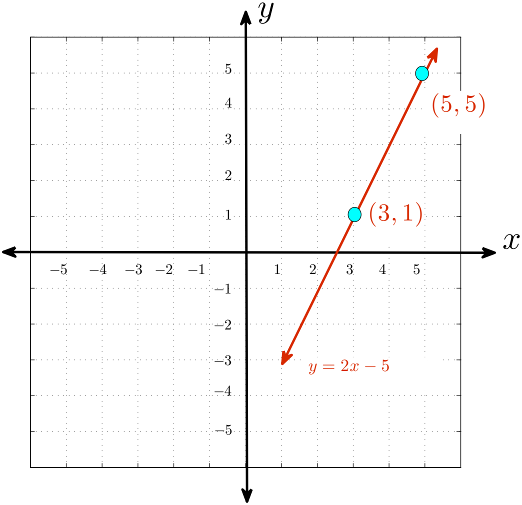

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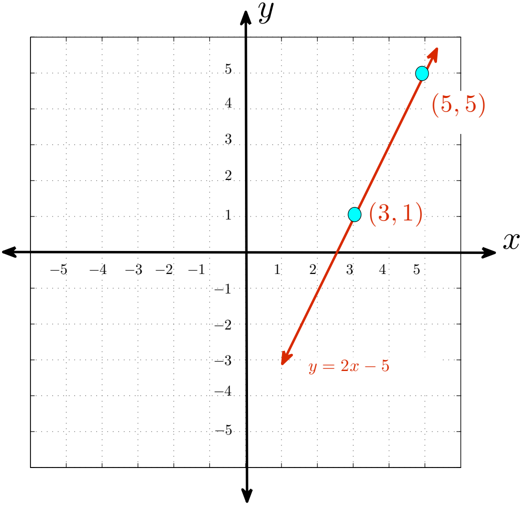

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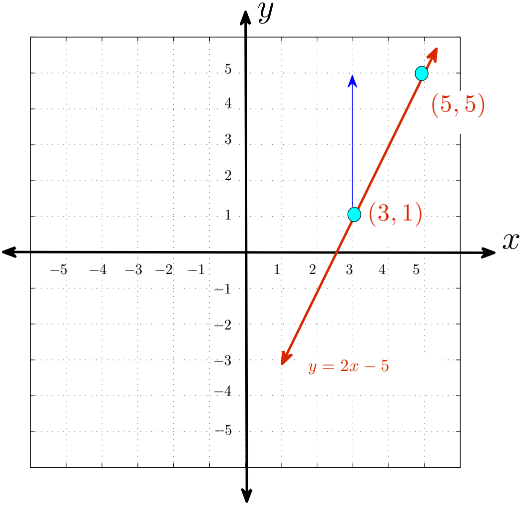

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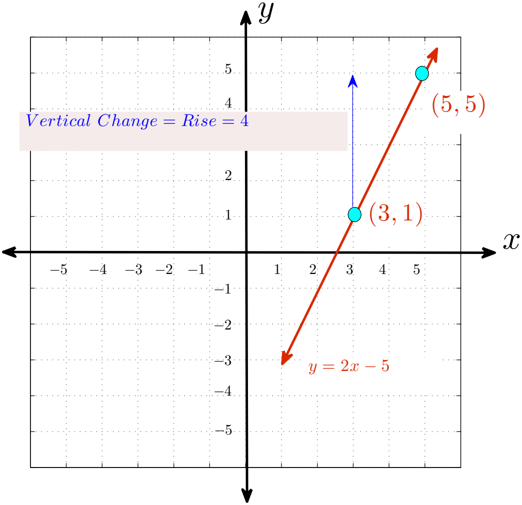

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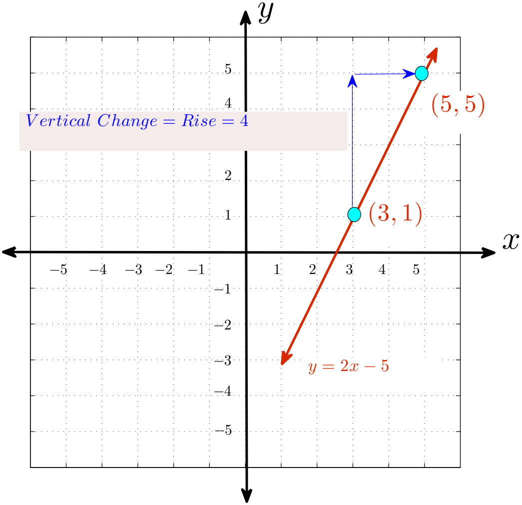

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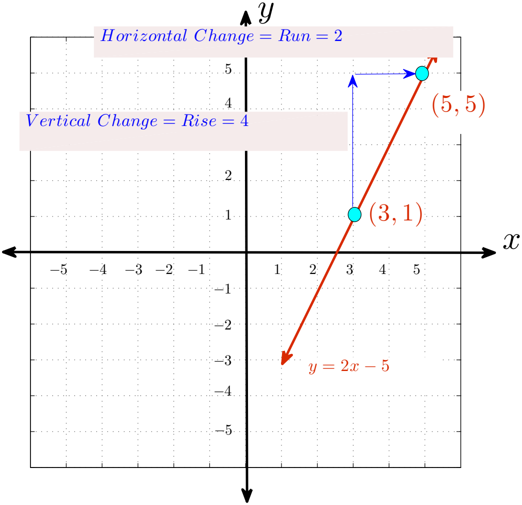
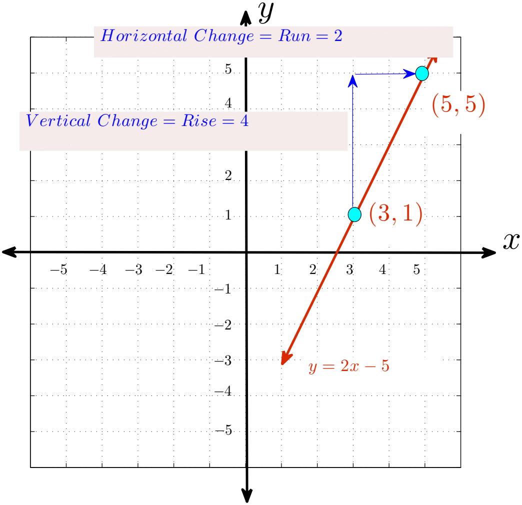
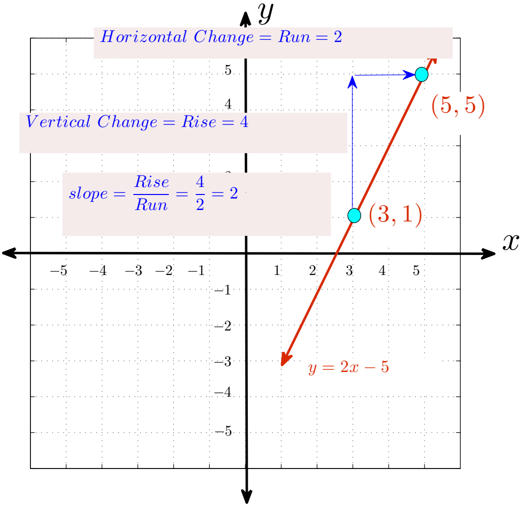


$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{4}{2} = 2$$



Notice that the vertical change is measured by subtracting the y-coordinates of the two points, 5 – 1 $=$ 4.

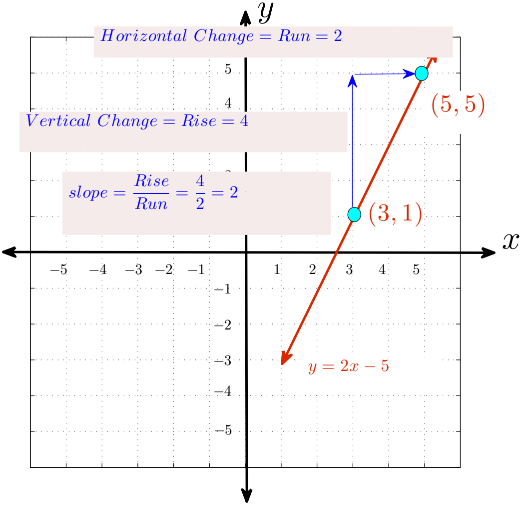


$Horizontal\ Change = Run = 2$

$Vertical\ Change = Rise = 4$

$slope = \dfrac{Rise}{Run} = \dfrac{4}{2} = 2$

Notice that the vertical change is measured by subtracting the *y*-coordinates of the two points, $5 - 1 = 4$. The horizontal change is the difference between the *x*-coordinates, $5 - 3 = 2$.



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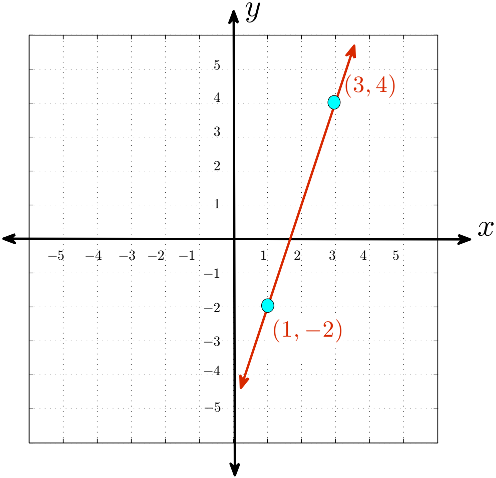
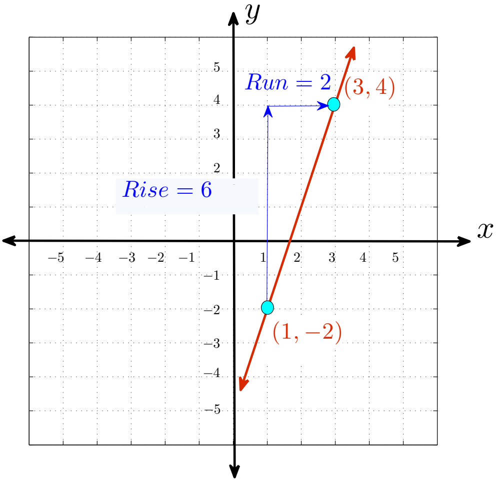


Ex. Find the slope of a line through $(3, 4)$ and $(1, -2)$.

$$slope = \frac{Rise}{Run} = \frac{6}{2} = 3$$



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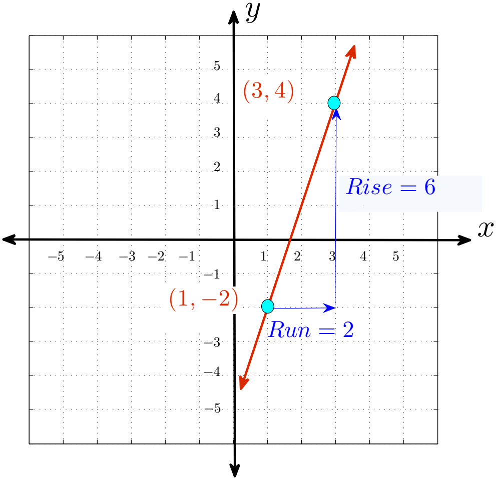

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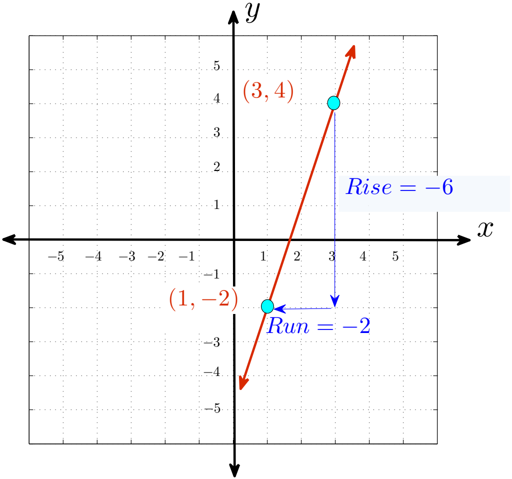

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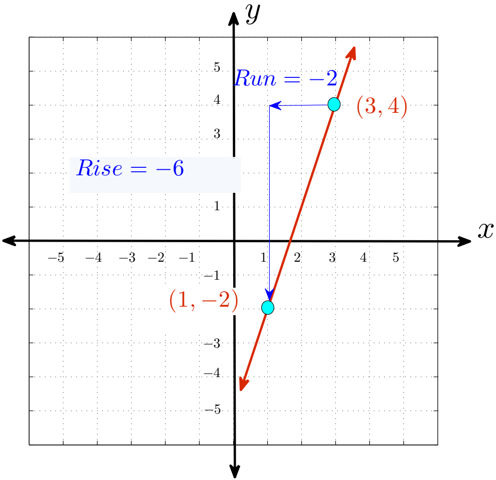

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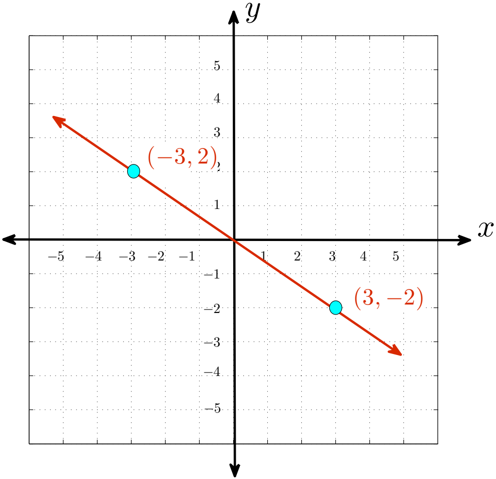
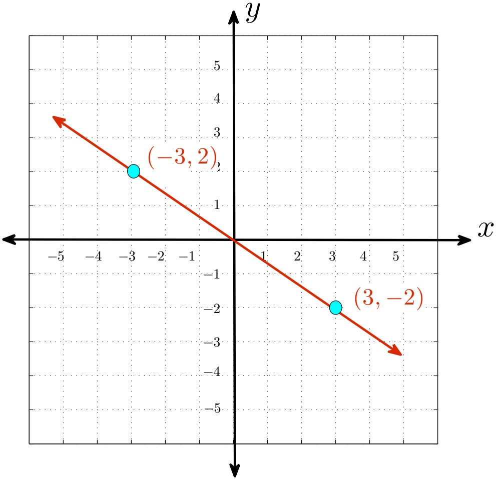


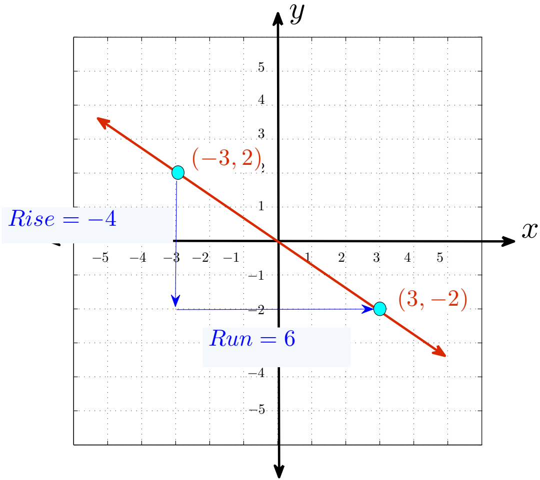
Ex. Find the slope of a line through $(-3, 2)$ and $(3, -2)$

We expect a negative slope for the solution because the graph of the line <u>falls</u> from left to right.



Ex. Find the slope of a line through $(-3, 2)$ and $(3, -2)$.

$$slope = \frac{Rise}{Run} = \frac{-4}{6} = \frac{2\cdot(-2)}{2\cdot\ \ 3} = \frac{\cancel{2}\cdot(-2)}{\cancel{2}\cdot\ \ 3} = -\frac{2}{3}$$



Review Topics

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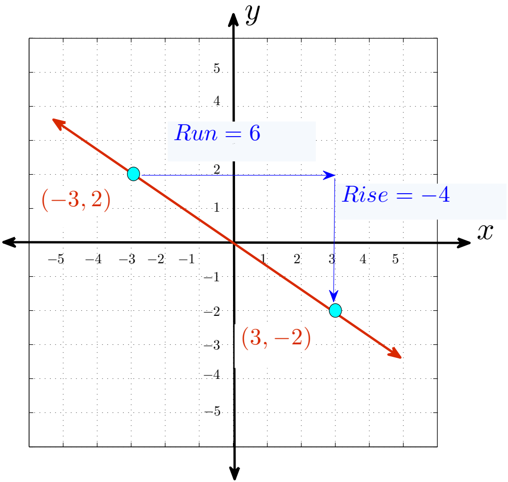
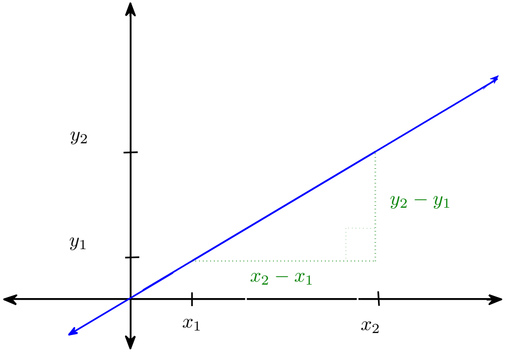


Definition

Let (x_1, y_1) and (x_2, y_2) be any two points on the rectangular coordinate plane. The **SLOPE** of a line which passes through the points (x_1, y_1) and (x_2, y_2) is m, where m is given by the formula:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$



Classroom Examples: Take some time out to work these 2 problems.

Use the slope formula, $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$, to find the slope of a line containing the given points.

- $(7, -4)$ and $(4, 2)$
- $(2, -3)$ and $(-1, -3)$

The Equation of a Line

Suppose, now, that line l has slope m and y-intercept b, then what is the equation for l? Because the y-intercept is b, we know that point $(0, b)$ is on the line.

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$$\frac{y - b}{x - 0} = m$$

Multiplying both sides of this equation by x, then adding b to both sides gives

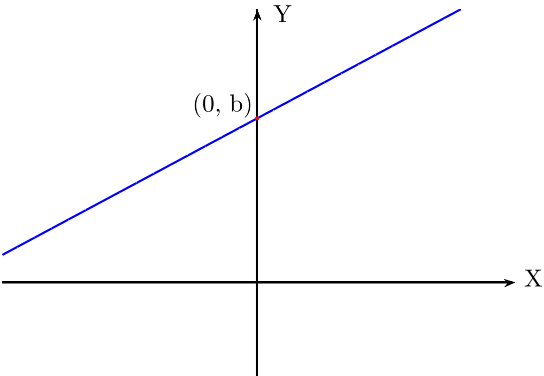
$$y = mx + b$$

Definition (The Equation of a Line: The Slope-Intercept Form)

Suppose m and b are real numbers (constants). Then another form for the equation of a line is

$$y = mx + b$$

where m represents the slope of the line and b represents the y-intercept.



Classroom Examples: Take some time out to work these 2 problems.

5. Determine the slope and y-intercept for the line $4x - 5y = 7$.

6. Graph the line $-3x + 2y = -6$ using the slope-intercept form ($y = mx + b$).

Suppose m and b represent particular, but arbitrarily chosen real numbers (fixed constants). A **linear function**, $f(x)$, has the form

$$f(x) = m \cdot x + b$$

where m represents the slope of the line and b represents the y intercept.

Example: Suppose $f(x) = 2x - 1$
What is the slope of f and what is it's y intercept.
Explain the geometric interpretation of $f(3)$.

Theorem (Point-Slope Equation of a line)

The line with slope m that passes through the point (x_1, y_1) is given by the equation:

$$y - y_1 = m(x - x_1)$$

We call this the ***Point-Slope Equation of a line****.*

Classroom Examples

7. Find the equation of the line with slope 3 that contains the point $(-1, 2)$. Use the Point-Slope Form, $y - y_1 = m(x - x_1)$.

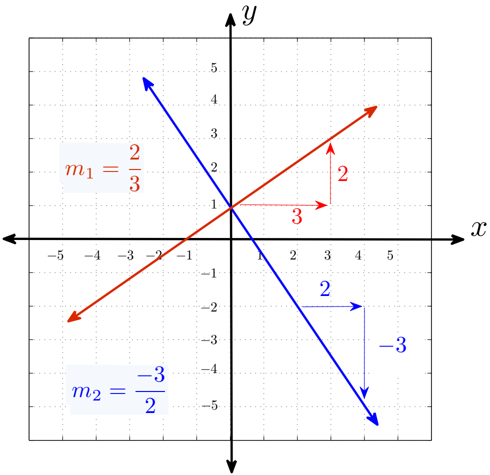
8. Find the equation of the line through $(2, 5)$ and $(6, -3)$.

9. Find the equation of the line that goes through the point (3,2), and is perpendicular to the graph of $3y - y = 2$.

Theorem

If line L_1 has slope m_1 and line L_2 has slope m_2, then

$$L_1 \textit{ is perpendicular to } L_2 \textit{ if and only if } m_1 \cdot m_2 = -1 \left(\textit{or } m_1 = -\frac{1}{m_2} \right.$$

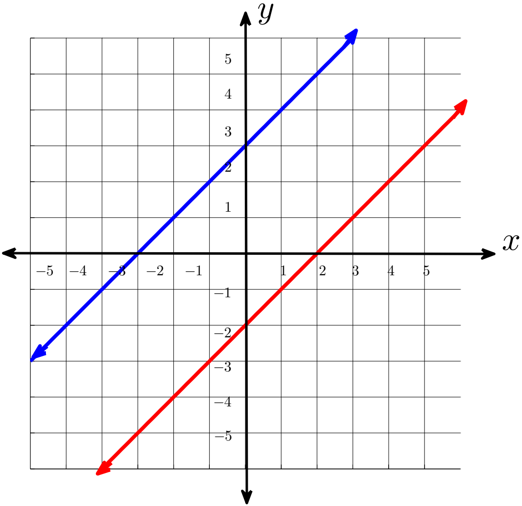


Classroom Example

9. Find the equation of the line that goes through the point (3,2), and is perpendicular to the graph of $3y - y = 2$.

Theorem

Two lines have the same slope if and only if they are parallel.



Classroom Example

10. Find the equation of the line that goes through the point (3,2), and is parallel to the graph of $3y - y = 2$.

Definition (The Equation of a Line: Standard Form)

Suppose A, B and C represent any real numbers. A **linear equation in two variables** is an equation having the *form*

$$A\,x + B\,y = C,$$

For example, $2\,x + 3\,y = 1$ is a linear equation in the two variables x and y which is written in standard form.

Determine whether or not the following equations are **linear equation in two variables**. If not, why?

11. $y = 2x - 1$
12. $2y - x = 4$
13. $2x^2 + y = 1$

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- To **evaluate an algebraic expression**, substitute a numerical value for each variable into the expression and simplify the result by applying the order of operations in a left to right fashion.

Definition

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Definition

A **constant** is a single number, such as 8 or 9.

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$$ax^n,$$

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For instance,

$$3, \qquad 5x, \qquad 7x^4, \qquad \text{and} \qquad 9x^{200}$$

are all examples of monomial functions.

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3	3	0
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- $p(x) = 4x^{-3}$ is not a monomial because the exponent of the variable, x, is -3 and -3 is not a whole number.

- $p(x) = 2x^{1/3}$ is not a monomial because the exponent of the variable is $1/3$, and $1/3$ is not a whole number.

A polynomial of degree n is an expression of the form:

$$a_n\,x^n + a_{n-1}\,x^{n-1} + \cdots + a_3\,x^3 + a_2\,x^2 + a_1\,x + a_0$$

where n is a non-negative integer and $a_n \neq 0$.

- The numbers $a_n, a_{n-1}, \ldots, a_3, a_2, a_1, a_0$ are the *COEFFICIENTS* of the polynomial.
- a_0 is called the *CONSTANT TERM*.
- $a_n\,x^n$ is called the *LEADING TERM* of the polynomial.
- a_n is called the *LEADING COEFFICIENT* of the polynomial.
- n is called the *DEGREE* of the polynomial.

Definition

A **polynomial** is a monomial or a sum of monomials.

11	monomial
$3x^4$	monomial
$2x^2 + 1$	binomial
$5x^3 + x - 1$	trinomial
$x^{1/2} + 5$	is not a polynomial
$\sqrt[5]{x + 5}$	is not a polynomial
$\dfrac{1}{x - 1}$	is not a polynomial

Definition

The **degree of polynomial expression** is the degree of the leading term (the term which has x raised to the largest power).

Polynomial Function	Degree	Leading Term	Leading Coefficient	Constant term
$-2x^4 - 3x - 5$	4	$-2x^4$	-2	-5
$x^5 - 3x^6 - 10x - 4$	6	$-3x^6$	-3	-4
$5x^{10} - 8x^3 - 10x + 5$	10	$5x^{10}$	5	5
$17x + 4$	1	$17x$	17	4
24	0	24	24	24

Polynomial addition and subtraction is as one would expect.

Example Compute the difference $(x^2 - 5x) - (3x^2 - 4x - 1)$

$$(x^2 - 5x) - (3x^2 - 4x - 1) =$$

$$
\begin{aligned}
&= (x^2 - 5x) - 1(3x^2 - 4x - 1) && \text{since } -a = (-1) \cdot a \\
&= (x^2 - 5x) + (-1)(3x^2 - 4x - 1) && \text{since } a - b = a + (-b) \\
&= (x^2 - 5x) - 3x^2 + 4x + 1 && \text{distr. prop} \\
&= x^2 - 5x - 3x^2 + 4x + 1 && \text{assoc. prop} \\
&= (x^2 - 3x^2) + (-5x + 4x) + 1 && \text{comm. and assoc. props} \\
&= \boxed{-2x^2 - x + 1} && \text{addn closure prop}
\end{aligned}
$$

Definition (Like Terms)

Like terms are terms that contain the same variable(s) raised to the same power(s). Like terms can be combined or collected together.

Example Identify the like terms in $4x^3 + 5x - 7x^2 + 2x^3 + x^2$

Solution:

like terms: $4x^3$ and $2x^3$ same variable and exponent

like terms: $-7x^2$ and x^2 same variable and exponent

Example Identify the like terms in $8x^2y^2 + 4x - 6x^5 + 2x^2y^2$

Solution:

like terms: $8x^2y^2$ and $2x^2y^2$ same variables and exponents

Multiplying Polynomials

The product of two binomials results in four terms before the like terms are combined. The acronym "Foil" stands for *FIRST, OUTER, INNER, LAST*, and should remind you how to compute the product of two binomials. Consider the following product:

$$(a+b)(c+d) = a(c+d)+b(c+d) = \overbrace{ac}^{F} + \overbrace{ad}^{O} + \overbrace{bc}^{I} + \overbrace{bd}^{L}$$

The product of the two binomials consists of four terms:

- the product of the *FIRST* term of each (ac),
- the product of the *OUTER* term of each (ad),
- the product of the *INNER* term of each (bc), and
- the product of the *LAST* term of each (bd).

2 Examples:

a.) $(x+4)\cdot(2x-3)$ b.) $(3\sqrt{6}-2\sqrt{5})^2$

Example: Multiply $(x^2 - 3x + 4) \cdot (2x - 3)$

Solution

$$
\begin{aligned}
&(x^2 - 3x + 4) \cdot (2x - 3) = && \\
&= (2x - 3) \cdot (x^2 - 3x + 4) && \text{comm prop } \times \\
&= 2x \cdot (x^2 - 3x + 4) + (-3) \cdot (x^2 - 3x + 4) && \text{distr. prop } \times \\
&= 2x^3 - 6x^2 + 8x - 3x^2 + 9x - 12 && \text{distr. prop } \times \\
&= 2x^3 + (-6x^2 - 3x^2) + (8x + 9x) - 12 && \text{comm., assoc. } + \\
&= \boxed{2x^3 - 9x^2 + 17x - 12} && \text{addn closure prop}
\end{aligned}
$$

#1 RULE: FACTOR OUT THE GCF

Factoring reverses multiplication. Consider the polynomial expression $6x^2 - 3x$, whose two terms have a greatest common factor, $3x$.

$$6x^2 - 3x = (3x) \cdot (2x) - (3x) \cdot (1)$$

$$= 3x \cdot (2x - 1) \qquad \text{since } a \cdot b - a \cdot c = a \cdot (b - c)$$

We can rewrite $6x^2 - 3x$ as a difference of two products. Afterwards, we can rewrite an equivalent expression using the distributive property. We call this process **factoring out the gcf**.

Definition (The #1 Rule of Factoring)

The first step to factoring any algebraic expression is to factor out the gcf (if there is one).

Definition

The **greatest common factor (GCF)** for a polynomial is the largest monomial that divides (is a factor of) each term of the polynomial.

Example: The greatest common factor for $25x^5 + 20x^4 - 30x^3$ is $5x^3$ since it is the largest monomial that is a factor of each term.

$$\begin{aligned} 25x^5 + 20x^4 - 30x^3 &= 5x^3 \cdot (5x^2) + 5x^3 \cdot (4x) - 5x^3 \cdot (6) \\ &= 5x^3 \cdot (5x^2 + 4x - 6) \end{aligned}$$

How to find the GCF of a polynomial

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Factor the greatest common factor from each of the following.

- $8x^3 - 8x^2 - 48x$
- $15a^7 - 25a^5 + 30a^3$
- $12x^4y^5 - 9x^3y^4 - 15x^5y^3$
- $4(a+b)^4 + 6(a+b)^3 + 16(a+b)^2$
- $x(x+7) + 2(x+7)$

Factoring Trinomials with a Leading Coefficient of 1

Earlier in the chapter, we multiplied binomials.

$$(x+2)\cdot(x+8) = x^2 + 10x + 16$$

$$(x+6)(x+3) = x^2 + 9x + 18$$

In each case, the product of the two binomials is a trinomial.

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$$= x^2 + (a+b)x + a\cdot b$$

Factoring Trinomials with a Leading Coefficient of 1

In general,

$$\begin{aligned}(x+a)\cdot(x+b) &= x^2+ax+bx+a\cdot b\\ &= x^2+(a+b)x+a\cdot b\end{aligned}$$

We can view this generalization as a factoring problem

$$x^2+(a+b)x+a\cdot b=(x+a)\cdot(x+b)$$

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$$\begin{aligned}(x+a)\cdot(x+b) &= x^2+ax+bx+a\cdot b\\ &= x^2+(a+b)x+a\cdot b\end{aligned}$$

We can view this generalization as a factoring problem

$$x^2+(a+b)x+a\cdot b=(x+a)\cdot(x+b)$$

To factor a trinomial with a leading coefficient of 1, we simply find the two numbers *a* and *b* whose sum is the coefficient of the middle term, and whose product is the constant term.

Factoring Trinomials with a Leading Coefficient of 1

Factor.

- $x^2 + 5x + 4$
- $x^2 + 7x + 6$
- $x^2 + 9x + 14$
- $x^2 + 11x + 24$
- $x^2 + 19x + 34$
- $x^2 + 12x + 27$
- $x^2 + 20x + 64$
- $x^2 + 18x + 65$
- $x^2 - x + 5$
- $x^2 + 5xy + 4y^2$
- $x^2 + 5xy + 6y^2$
- $x^2 + 12xy + 27y^2$
- $m^2 + 19mn + 60n^2$
- $x^2 + 2x - 15$
- $x^2 - 7x - 18$
- $x^2 + x - 20$
- $x^2 + 10x - 24$

Polynomials with four terms can sometimes be **factored by grouping**.

Example Factor $x^4 - 2x^3 - 8x + 16$

Solution:

$$x^4 - 2x^3 - 8x + 16 = (x^4 - 2x^3) + (-8x + 16) \quad \text{(assoc. prop +)}$$

$$= \Big[x \cdot (x^3) + (-2) \cdot (x^3)\Big] + \Big[(-8) \cdot x + (-8) \cdot (-2)\Big]$$

$$= x^3(x-2) - 8(x-2) \quad \text{(distr. prop.)}$$

$$= x^3(x-2) - 8(x-2) \quad \text{(identify common factor)}$$

$$= \boxed{(x-2)(x^3-8)} \quad \text{(distr. prop)}$$

**Technically we are not done. This is not the prime factorization of the given polynomial since $x^3 - 8$ can be factored with the difference of cubes formula.

Problem: Factor ax^2+bx+c	Problem: Factor $10x^2-11x-6$
(1) Multiply *a* times *c*.	(1) $a=10,\ b=-11,\ c=-6,$ so clearly $a\cdot c=-60$
(2) List all possible pairs of numbers whose product is *ac*	$-60 \qquad -60$ ↙↘ ↙↘ $-6\cdot10 \qquad -1\cdot60$ $6\cdot(-10) \qquad 1\cdot(-60)$ $3\cdot(-20) \qquad -5\cdot12$ $-3\cdot20 \qquad 5\cdot(-12)$ $\boxed{-15\cdot4} \qquad 15\cdot(-4)$
(3) Box the pair whose sum is *b* ↗	(3) $b=-11$, and $-15+4=-11$

Problem: Factor $ax^2 + bx + c$

Problem: Factor $10x^2 - 11x - 6$

(4) Replace b with the sum of the circled pair. Distribute x into this quantity

(4) $10x^2 - 11x - 6$

$= 10x^2 + (-15x + 4x) - 6$

$= 10x^2 - 15x + 4x - 6$

(5) Now factor by grouping: Use parenthesis to group the first two terms, and another () to group the second two terms.

(5) $(10x^2 - 15x) + (4x - 6)$

$= 5x(2x - 3) + 2(2x - 3)$

$= 5x(2x - 3) + 2(2x - 3)$

$= (2x - 3) \cdot (5x + 2)$

Recall: The Number Types

Definition

The set of whole numbers,

$$\mathbb{W} = \{0, 1, 2, 3, 4, \ldots\}$$

is the set of natural numbers unioned with zero, written $\mathbb{W} = \mathbb{N} \cup \{0\}$.

Recall: The Number Types

Definition

The set of integers,

$$\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

is also known as all the positive and negative whole numbers.

Recall: The Number Types

Definition

A rational number is any number that can be expressed as the ratio of two integers. The set of rational numbers is written symbolically as

$$\mathbb{Q} = \left\{ \frac{a}{b} \,\middle|\, \text{a and b are any integers, and } b \neq 0 \right\}$$

Note that any integer "a" is a rational number since $a = \frac{a}{1}$.

Rational expressions

A rational expression is defined similarly as any expression that can be written as the ratio of two polynomials.

Definition (Rational Expressions)

$$\text{rational expressions} = \left\{ \frac{p}{q} \;\middle|\; \text{p and q are polynomials,} \;\; q \neq 0 \right\}$$

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Definition (Rational Expressions)

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Some examples of rational expressions are

$$\frac{1}{x}, \qquad \frac{2m-3}{6n-7}, \qquad \frac{x^2-3x-1}{x^2-3x-5}, \qquad \frac{x-y}{y-x}$$

Rational expressions

Basic Properties

Multiplying (or dividing) the numerator and denominator by the same nonzero expression may change the form of the rational expression, but it will always produce an expression equivalent to the original one.

Rational expressions

Basic Properties

Multiplying (or dividing) the numerator and denominator by the same nonzero expression may change the form of the rational expression, but it will always produce an expression equivalent to the original one.

We use this property to reduce fractions to lowest terms. For example,

$$\frac{6}{8} = \frac{3 \cdot \cancel{2}}{4 \cdot \cancel{2}} = \frac{3}{4}$$

Using Basic Properties

In a similar fashion, we reduce rational expressions to lowest terms by

1. first factoring the numerator and denominator,
2. and then dividing both numerator and denominator by any factors they have in common.

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Example: Reduce $\dfrac{x^2 - 25}{x - 5}$ to lowest terms.

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2. and then dividing both numerator and denominator by any factors they have in common.

Example: Reduce $\frac{x^2 - 25}{x - 5}$ to lowest terms.

Solution:

$$\frac{x^2 - 25}{x - 5} = \frac{(x - 5) \cdot (x + 5)}{x - 5} = \frac{\cancel{(x - 5)} \cdot (x + 5)}{\cancel{(x - 5)}} = x + 5$$

Using Basic Properties

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Try This One! Reduce $\dfrac{x-5}{x^2-10x+25}$ to lowest terms.

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Solution:

$$\frac{x-5}{x^2-10x+25}=\frac{x-5}{(x-5)^2}=\frac{1\cdot(x-5)}{(x-5)\cdot(x-5)}=\frac{1\cdot\cancel{(x-5)}}{(x-5)\cancel{(x-5)}}=\frac{1}{x-5}$$

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Try This One! Reduce $\dfrac{-3+5x}{25x^2-9}$ to lowest terms.

Using Basic Properties

We reduce rational expressions to lowest terms by

1. first factoring the numerator and denominator,
2. and then dividing both numerator and denominator by any factors they have in common.

Try This One! Reduce $\frac{-3+5x}{25x^2-9}$ to lowest terms.

Solution:

$$\frac{-3+5x}{25x^2-9} = \frac{5x-3}{(5x)^2-3^2} = \frac{1\cdot(5x-3)}{(5x+3)\cdot(5x-3)} = \frac{1\cdot\cancel{(5x-3)}}{(5x+3)\cancel{(5x-3)}} = \frac{1}{5x+3}$$

Try This One! Reduce $\frac{5x-3}{3-2x}$ to lowest terms.

Try This One! Reduce $\frac{5x-3}{3-2x}$ to lowest terms.

Solution:

First degree polynomials have form $ax + b$ for real numbers a and b with a not equal to zero. First degree polynomials are always prime, unless the numbers a and b have a greatest common factor. So, the given expression is prime (not factorable), since both first degree polynomials do not have a common constant that can be divided out of both numerator and denominator. Therefore, the given rational expression is in lowest terms.

Try This One! Reduce $\dfrac{16y^3 - 250}{12y^2 - 26y - 10}$ to lowest terms.

Review Topics
Set Definitions
Number Types
Opposites
Absolute Value
Operations
Properties of Real Numbers
Distributive Property
Exponential Notation
The Order of Operations
Linear Equations
Linear Equations
Linear Inequalities
Coordinate System
Lines
slope
Perpendicular & Parallel Lines
Algebraic Expressions
Polynomials
Poly Add/Subtract
Combining Like Terms
Multiplying Poly's
Trinomial times Binomial
Factoring Polynomials
GCF
Factor By Grouping
ac-grouping method
Rational Expressions

Try This One! Reduce $\dfrac{16y^3 - 250}{12y^2 - 26y - 10}$ to lowest terms.

Solution: $$\frac{16y^3 - 250}{12y^2 - 26y - 10} = \frac{\cancel{2} \cdot (8y^3 - 125)}{\cancel{2} \cdot (6y^2 - 13y - 5)} = \frac{(2y)^3 - 5^3}{6y^2 + 2y - 15y - 5}$$

Try This One! Reduce $\dfrac{16y^3-250}{12y^2-26y-10}$ to lowest terms.

Solution: $\dfrac{16y^3-250}{12y^2-26y-10}=\dfrac{\cancel{2}\cdot(8y^3-125)}{\cancel{2}\cdot(6y^2-13y-5)}=\dfrac{(2y)^3-5^3}{6y^2+2y-15y-5}$

$$=\frac{(2y-5)(4y^2+10y+25)}{(6y^2+2y)+(-15y-5)}=\frac{(2y-5)(4y^2+10y+25)}{2y\cdot(3y+2)+(-5)\cdot(3y+2)}=\frac{\cancel{(2y-5)}(4y^2+10y+25)}{(3y+2)\cdot\cancel{(2y-5)}}$$

$$=\frac{(4y^2+10y+25)}{(3y+2)}$$

Try This One! Reduce $\dfrac{3a^3+3}{6a^2-6a+6}$ to lowest terms.

Try This One! Reduce $\frac{3a^3+3}{6a^2-6a+6}$ to lowest terms.

Solution: $\frac{3a^3+3}{6a^2-6a+6} = \frac{3(a^3+1)}{6(a^2-a+1)} = \frac{3(a+1)\cancel{(a^2-a+1)}}{6\cancel{(a^2-a+1)}}$

$= \frac{3(a+1)}{6} = \frac{3(a+1)}{3\cdot 2} = \frac{\cancel{3}(a+1)}{\cancel{3}\cdot 2} = \frac{(a+1)}{2}$

Try This One! Reduce $\dfrac{x^2 - 3x + ax - 3a}{x^2 - ax - 3x + 3a}$ to lowest terms.

Review Topics

Try This One! Reduce $\dfrac{x^2-3x+ax-3a}{x^2-ax-3x+3a}$ to lowest terms.

Solution:

$$\frac{x^2-3x+ax-3a}{x^2-ax-3x+3a}=\frac{(x^2-3x)+(ax-3a)}{(x^2-ax)+(-3x+3a)}=\frac{x(x-3)+a(x-3)}{x(x-a)+(-3)(x-a)}$$

$$=\frac{(x+a)(x-3)}{(x-a)(x-3)}=\frac{(x+a)\cancel{(x-3)}}{(x-a)\cancel{(x-3)}}=\frac{(x+a)}{(x-a)}$$

Try This One! Reduce $\dfrac{a-b}{b-a}$ to lowest terms.

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