Sections 8.1 & 8.2 Systems of Linear Equations in Two Variables

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Learning Objectives:

- Solve Systems of linear equations in two variables by graphing.
- Solve systems of linear equations in two variables by the addition method.
- Solve systems of linear equations in two variables by the substitution method.

Recall

Definition (The Equation of a Line: Standard Form)

Suppose A, B and C represent any real numbers. A **linear equation in two variables** is an equation having the *form*

$$A\,y + B\,x = C,$$

For example, $1\,y - 1\,x = 3$ is a linear equation in the two variables x and y.

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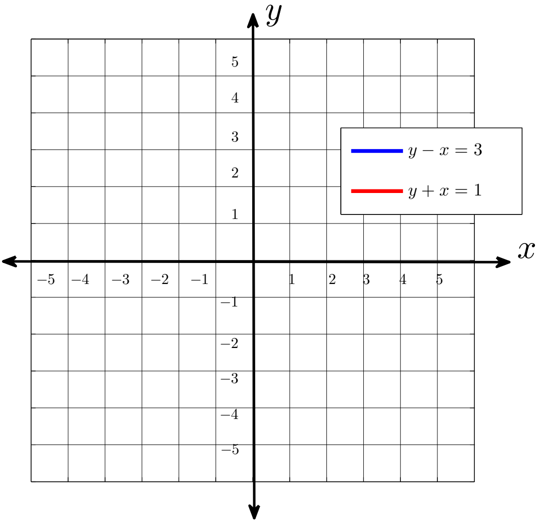
Classroom Example: Confirm whether or not the ordered pair, $(1, 1)$, belongs to the following system of equations:

$$1y - 1x = 3$$
$$1y + 1x = 1$$

Is $(-1, 2)$ an ordered pair that belongs to the solution set?

We can find the solution graphically!

We can also find the solution to the system by sketching the graph of both lines and finding their intersection point.



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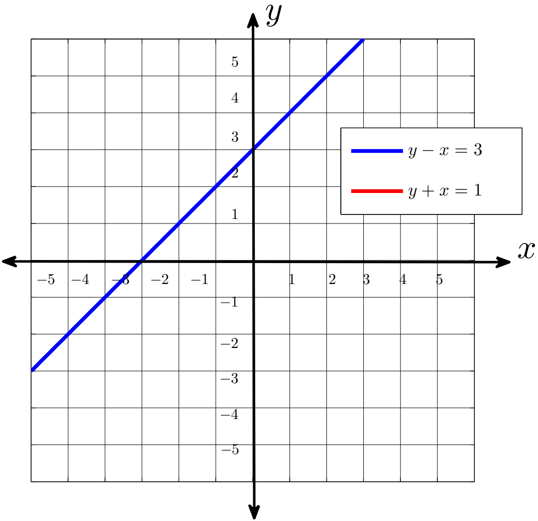

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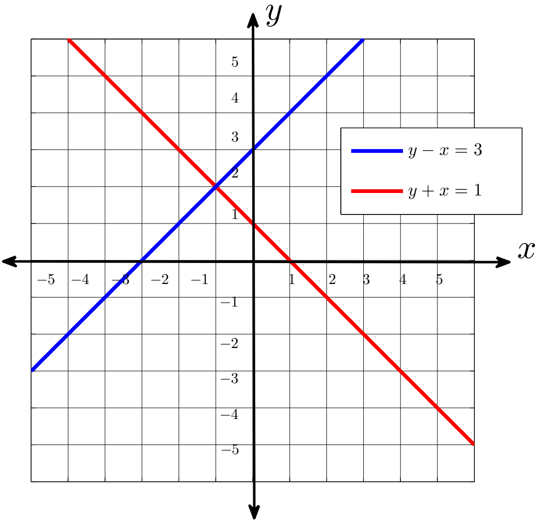

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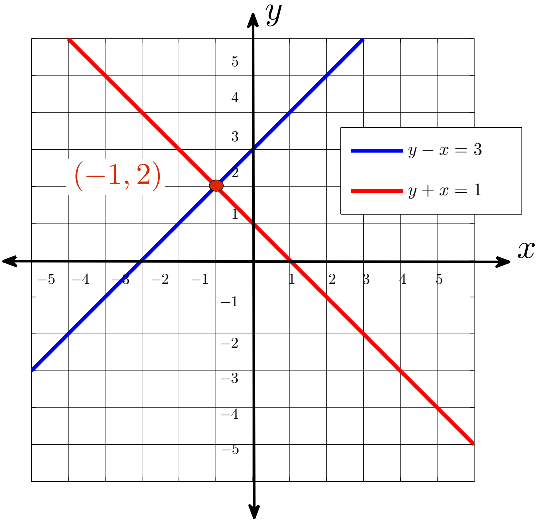

Definition

Generalized Form for a System of Two Linear Equations Suppose $A_1,\ A_2,\ B_1,\ B_2,\ C_1$ and C_2 are real numbers. A **system of linear equations in two variables** has the form

$$A_1 \cdot y + B_1 \cdot x = C_1$$

$$A_2 \cdot y + B_2 \cdot x = C_2$$

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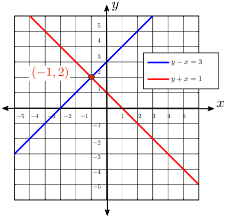
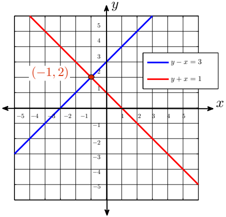


Case 1
One Solution

1. **Case 1** The two lines intersect at one and only one point. The coordinates of the point give the solution to the system. This is what usually happens. In this case, we say the system is consistent.



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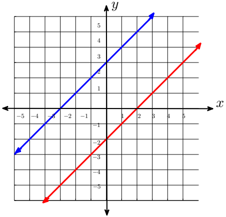
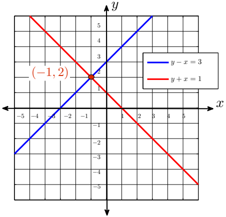


Case 2
No Solution

1. **Case 1** The two lines intersect at one and only one point. The coordinates of the point give the solution to the system. This is what usually happens. In this case, we say the system is consistent.
2. **Case 2** The lines are parallel and therefore have no points in common. the solution set to the system is the empty set, $\varnothing$. In this case, we say the system is inconsistent.



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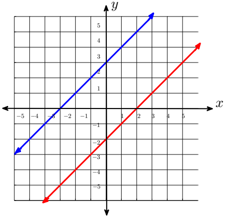
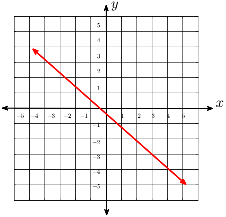


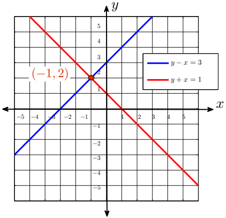
Case 2
No Solution



Case 3
Infinite Solutions

1. **Case 1** The two lines intersect at one and only one point. The coordinates of the point give the solution to the system. This is what usually happens. In this case, we say the system is consistent.
2. **Case 2** The lines are parallel and therefore have no points in common. the solution set to the system is the empty set, $\varnothing$. In this case, we say the system is inconsistent.
3. **Case 3** The lines are coincident, meaning they are on top of each other. That is, their graphs represent the same line. The solution set consists of all ordered pairs that satisfy either equation. In this case, the equations are said to be dependent.

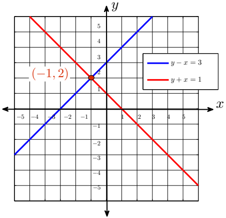
First, write both equations in slope-intercept form ($y = mx + b$).



Case 1
One Solution

First, write both equations in slope-intercept form ($y = mx + b$).

- **Case 1** If the slopes of both lines *are not* equivalent, the system is consistent.



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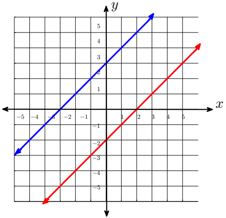
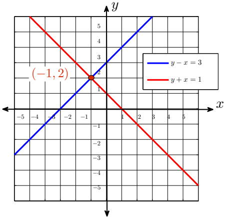


Case 2
No Solution

First, write both equations in slope-intercept form ($y = mx + b$).

- **Case 1** If the slopes of both lines *are not* equivalent, the system is consistent.
- **Case 2** If the slopes of both lines *are* equivalent, and the intercepts are not equal, then the lines are parallel, and the system is inconsistent.

How to tell, without graphing, which of the 3 cases you have:



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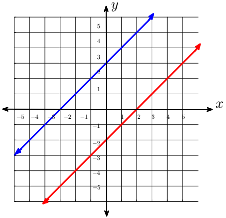
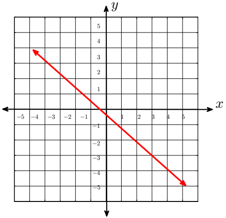


Case 2
No Solution



Case 3
Infinite Solutions

First, write both equations in slope-intercept form ($y = mx + b$).

- **Case 1** If the slopes of both lines *are not* equivalent, the system is <u>consistent</u>.
- **Case 2** If the slopes of both lines *are* equivalent, and the intercepts are not equal, then the lines are parallel, and the system is <u>inconsistent</u>.
- **Case 3** If the slopes of both lines *are* equivalent, and the intercepts are equal, then the lines are coincident (on top of each other), and the system is <u>dependent</u>.

Classroom Example: Determine, without graphing, which of the 3 cases exist for the following system of linear equations:

$$2y - x = -6$$
$$2y - x = -4$$

Classroom Example: Solve the system:

$$3y + 4x = 10$$
$$y + 2x = 4$$

Solving a System of Linear Equations by the Elimination Method

Step 1: Decide which variable to eliminate. (In some cases, one variable will be easier to eliminate than the other. With some practice, you will notice which one it is.)

Step 2: Use the multiplication property of equality on each equation separately to make the coefficients of the variable that is to be eliminated opposites.

Step 3: Add the respective left and right sides of the system together.

Step 4: Solve for the remaining variable.

Step 5: Substitute the value of the variable from step 4 into an equation containing both variables and solve for the other variable. (Or repeat steps 2–4 to eliminate the other variable.)

Step 6: Check your solution in both equations, if necessary.

Try this on your own: Solve the system:

$$3y - 9x = -18$$
$$y - 7x = 42$$

Try this on your own: Solve the system:

$$3x - 5y = 2$$
$$2x + 4y = 1$$

Classroom Example (Special Case): Solve the system:

$$2x + 7y = 3$$
$$4x + 14y = 1$$

Classroom Example (Special Case): Solve the system:

$$\begin{aligned} 2x + 7y &= 3 \\ 4x + 14y &= 6 \end{aligned}$$

What happens:	Geometric Interpretation	Classification
Both variables are eliminated and the resulting statement is *false*.	The lines are parallel, and there is not a solution to the system.	The system is inconsistent.
Both variables are eliminated and the resulting statement is *true*.	The lines coincide, and there is an infinite number of solutions.	The system is dependent.

Classroom Example: Solve the following system using the Substitution Method:

$$2x - 3y = -6$$
$$y = 3x - 5$$

Solving a System of Equations by the Substitution Method

Step 1: Solve either one of the equations for x or y. (This step is not necessary if one of the equations is already in the correct form, as in the last example.

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Step 2: Substitute the expression for the variable obtained in step 1 into the other equation and solve it.

Step 3: Substitute the solution for step 2 into any equation in the system that contains both variables and solve it.

Step 4: Check your results, if necessary.