1.11 Higher or Lower?

Explore

At your job, there are two proposed pay raise structures being offered by management to the labor union. For Option 2, the increases will be applied in the order they are listed.

Option 1: **5% increase**

Option 2: **3% increase + $1,000**

1. Which option would you prefer? Why?

2. Who benefits the most under each option? Brainstorm ways to answer this question. List your approach below.

Discover

To determine who benefits most from each pay raise option, one approach is to look at many salaries—high, low, and in between—and determine how they will change under each option. In other words, we want to numerically apply the options several times in order to understand the process and see if we can determine a pattern.

3. Let's apply a 5% raise to a salary of $25,000.

This means we take 5% of the salary and then add it to $25,000. You may remember a rule for finding a percent of a number, but if not, we can use a scaling process to find the result.

$25,000 is the whole, so ________%.	$25,000	is	100%
To scale down to 1%, ________ by 100.	$250	is	1%
To scale up to 5%, ________ by 5.	$1,250	is	5%

We scaled down to 1% and then up to 5% because the calculations are easy.

We could have also scaled down to 5% in one step by dividing by 20.

To summarize this process in one calculation, we actually multiplied $25,000 by what fraction?

This is the same as multiplying $25,000 by what decimal?

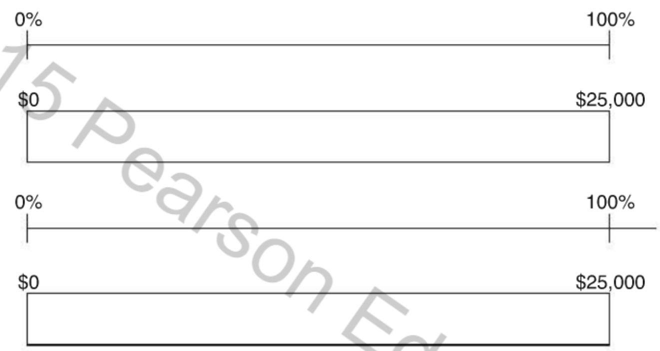
4. So 5% of $25,000 is ________. Let's use a picture to verify that our answer is reasonable. The whole rectangle (100%) represents $25,000. A number line is drawn as a guide.

Mark where 5% would be on the number line and shade from $0 to the 5% line in the rectangle. Does that amount look like $1,250?

What is the new salary?

Increasing a salary by 5% means finding 5% of the salary and then adding this 5% to the salary. Do this visually by drawing the same 5% rectangle at the end of the $25,000 rectangle.

What percent of the original is it?



We just saw that we can increase an amount by a certain percent in two steps. First, take the percent of the original amount (by multiplying the original amount by the percent in decimal form). Then add this amount to the original amount. However, this computation can also be accomplished in just one step. Ask yourself what percent, or decimal, we can multiply by that will account for our original amount plus the percent we want to add to it.

5. a. What decimal should we multiply $25,000 by to increase it by 5%?

b. Verify this works with $25,000.

c. What is the advantage of this method over the original one of finding 5% and adding it on?

d. State a shortcut for increasing a number by a percent.

How it works

To find the percent of a number:

1. Change the percent to a decimal by moving the decimal point two places to the left.
2. Multiply the decimal by the number.

For example, 80% of an instructor's students typically pass her introductory biology course. She has 240 students enrolled this semester. How many can she expect to pass?

We need to find 80% of 240. 80% is the same as 0.80. Multiply this by 240: $0.80(240) = 192$. This answer makes sense since 80% is much more than half but not quite the whole 240 students.

To increase a number by a percent:

Option 1: 1. Find the percent of the number. 2. Add this amount to the number.

Option 2: Multiply the number by ____________________

For example, a company's executives expect a 2% increase in sales in the upcoming year. If the company had \$62,000 in sales last year, what can the executives expect in the following year?

We can find 2% of \$62,000 and then add it: $(0.02)(\$62{,}000) = \$1{,}240$; $\$62{,}000 + \$1{,}240 = \$63{,}240$

Alternatively, we can find 102% of \$62,000: $(1.02)(\$62{,}000) = \$63{,}240$

When you are asked to increase an amount by a percent, choose the method (one- or two-step) that makes sense to you.

6. To begin this lesson, we had two pay structures, a 5% raise or a 3% raise with an additional increase of \$1,000. Suppose an employee makes \$15,000. Find his or her new salary under each option. Which option benefits this person the most?

Option 1 (5% increase)

Option 2 (3% increase + \$1,000)

7. Let's explore which option is best for a variety of salaries. These salaries have been rounded to the nearest thousand dollars. Complete the table.

Name	Current Salary (\$)	New Salary (\$), Option 1 (5% increase)	New Salary (\$), Option 2 (3% increase + \$1,000)	Difference (\$) (Option 1 − Option 2)
John	15,000			
Marissa	18,000			
Jay	20,000			
Carrie	25,000			
Bob	38,000			
Paul	40,000			
Leah	46,000			
Juan	55,000			
Evelyn	60,000			
Emily	80,000			

8. a. List the names of the employees who benefit from each option.

b. How was the table constructed to make analyzing the information easier?

c. What does the result in the difference column mean for John? For Juan?

d. In general, which level of salaries benefits most under each option?

Connect

9. These calculations can be done on Excel. To use it, we need to generalize the process used to calculate each option. To do this, let's start with a specific case and then move to a general one.

a. For a salary of $10,000, write your calculation to find the new salary under each option. Show all work.

Option	Calculation	Result
1: Increase by 5%		
2: Increase by 3% + $1,000		

b. For a salary of *S* dollars, write your calculation to find the new salary under each option. To do this, use your calculation above, replacing $10,000 with *S*.

Option	Calculation
1: Increase by 5%	
2: Increase by 3% + $1,000	

10. Is there any salary for which the two options would be the same? If so, what is that salary? If not, explain why not. We can use a spreadsheet to answer the question.

11. How could we find (without using a spreadsheet or table) the exact salary at which each option will give the *same* new salary? Start by listing both formulas below. Then state mathematically that the two formulas are equal.

Salary under Option 1 =

Salary under Option 2 =

We will not solve this equation algebraically now. In future cycles, we will learn many techniques to do so.

These formulas are examples of a special type of mathematical relationship known as a **function**. We will encounter many functions in many different formats throughout the book.

look IT UP

Function

A **function** is a rule that assigns each input (starting value) to exactly one output (ending value). Functions can be displayed as graphs, tables, and equations.

For example, the formula *Salary for Option 1* $= 1.05S$ is a function. Each old salary, S, will pair with only one new salary using this formula. The old salaries are the inputs, and the new salaries are the outputs from the formula. This can also be seen in the table of salaries. Each old salary has exactly one new salary using this option's formula.

In this case, the new salary depends on the old salary, S. So S is the independent variable. We can also say that the new salary is **a function of** the old salary.

tech TIP

For more detailed instructions on using Excel, consult the Excel appendix.

12. List the salary range that would benefit the most from each pay raise option.

Option 1:

Option 2:

13. Are the differences between options significant? For whom does it matter the most?

Reflect

lesson WRAP-UP

What's the point?

We can use numerical methods, including tables and even Excel, to make comparisons.

What did you learn?

How to calculate the percent of a number
How to increase a number by a percent
How to use a table to make comparisons

Cycle 1 Question: What can be learned?

Which method for increasing a number by a percent makes the most sense to you? Why?

1.11 Homework

Skills MyMathLab

- Calculate the percent of a number.
- Increase a number by a percent.

1. For a tax rate of 8.25%, find the amount of tax that will be paid on a pair of jeans costing $72. Find the total price of the jeans with tax.

2. At dinner, the meal's total bill comes to $42. Find the amount of the dinner with a 20% tip added to the total.

Concepts and Applications

- Use a table to make comparisons.

We began the lesson by exploring two pay structures. It was stated that in Option 2, the order should be that the 3% increase is applied first and then the $1000 is added. Does the order of these two increases matter?

3. a. Pick five salaries and apply the increase both ways.

Salary	Add $1,000, Then Increase by 3%	Increase by 3%, Then Add $1,000

b. Do the results in your table indicate that the order of the two increases matters? Defend your position.

c. Write a formula for computing the new salary under each order using S for the old salary.

New salary =

New salary =

d. Use the formulas from part c to confirm your conclusion from part b.

e. If these are possible pay structures for a company, which order would the management prefer if they are concerned primarily with keeping employee salaries low? Which would the employees prefer?

4. In this lesson we developed a shortcut for increasing a number by a percent. What is the similar shortcut for decreasing a number by a percent?

a. Decrease $100 by 5% by first finding 5% of $100 and then subtracting.

b. What would you need to multiply $100 by in order to calculate this percent decrease in one step?

c. In general, how do you decrease a number by a given percent?

5. Suppose an online electronics retailer is running two different specials. One deal will get you 10% off your total purchase, and the other deal is worth $20 off a purchase of $100 or more. Assuming you spend at least $100, if you are allowed to use both deals, in which order do you want to apply them? Try the calculation both ways with a few different prices or write a formula to represent each option.

6. Consider the following data on years of experience and salaries for a department.

Experience (years)	Salary (dollars)
0	48,000
1	67,000
3	54,000
4	58,000
5	62,000
5	64,000
11	74,000
15	81,000
16	85,000
23	92,000

Is salary a function of experience? Explain.

1.12 The X Factor

Discover

Representing a quantity with a letter, word, or name is commonplace in real life and has already been seen in the previous lesson. When we move from a specific numeric situation to a generalized situation using variables, we move from arithmetic to algebra. Here is some key vocabulary we will use often.

look IT UP

Algebra

Algebra: Algebra is a branch of mathematics in which letters are used to represent numbers and numeric operations are generalized.

Constant: A constant is a value that does not change. For example, numbers are constants.

Term: A term is a constant or variable or the product or quotient of constants and/or variables. For example, 5, M, and $5M$ are all terms.

Expression: An expression is a mathematical phrase containing one or more terms separated by plus or minus signs. For example, $1.03S + 1{,}000$ is an expression with two terms.

Equation: An equation is a mathematical statement that two expressions are equal. For example, $1.05S = 1.03S + 1{,}000$ is an equation.

It is helpful to think of an expression as a phrase and an equation as a complete sentence.

STICKY note

Pi is approximately 3.14159.... Its decimal expansion continues forever and never forms a pattern that repeats. We have a symbol, π, to allow us to write the number exactly and succinctly. Even though the number π is represented by a symbol, it is still a constant and not a variable.

One way to make sense of this terminology is to compare and contrast the terminology using a Venn diagram.

1. Make a Venn diagram with one circle labeled "Constant" and the other labeled "Variable."

   a. Decide if the circles will overlap. If you can think of some characteristic that constants and variables share, then the circles should overlap.