1.5 A Tale of Two Numbers

Explore

Every day we are surrounded by rates and ratios. Read the definitions and examples in the box below. Then complete #1 and 2 that follow.

look IT UP

Ratio and Rate

A **ratio** is a comparison of two numbers and can be written three different ways.

For example, if the student to faculty ratio at a college is 40 to 1, the ratio can also be written as 40:1 or $\frac{40}{1}$.

When the quantities being compared in the ratio have different units, the ratio is called a **rate**.

For example, the rate 30 mpg (miles per gallon) is a comparison of two quantities, 30 miles and 1 gallon. Since the units (miles and gallons) are different, we refer to this ratio as a rate.

1. Suppose you have set the cruise control on a long car trip to 70 mph. Below are several interpretations of the rate 70 mph. Circle all that are correct.

a. $\frac{70 \text{ miles}}{1 \text{ hour}}$

b. 70 miles every hour

c. 35 miles in 30 minutes

d. 70 miles

e. 140 miles after 2 hours

f. 1 mile in 70 hours

2. Supposed you have been hired for a job that pays \$12 per hour. Below are several interpretations of the rate \$12 per hour. Circle all that are correct.

a. In 12 hours, you earn \$1.

b. $\frac{\$12}{1 \text{ hour}}$

c. In 1 hour, you earn \$12.

d. In 1 minute, you earn 20 cents.

e. In an 8-hour shift, you earn \$96.

f. In a 5-day workweek with 8-hour shifts, you earn \$480.

Discover

Making sense of ratios and rates can be challenging. One way to make the process easier is to write the ratio or rate as a fraction with units in both the numerator and the denominator. This technique is one we will use throughout the book. Including units with the numbers can give you some insight into which operation to perform.

3. In #2d, we had to determine if earning 20 cents per minute is the same as earning \$12 per hour. Write \$12 per hour as a fraction to get started.

Using this fraction as a guide, write "in 1 minute, you earn 20 cents" as a fraction.

Use this fraction to write new, equivalent fractions. For example, 20 cents in 1 minute means 100 cents in 5 minutes. So you will earn $1 in 5 minutes. Continue with this idea until you can write a fraction that has dollars in the numerator and hours in the denominator.

STICKY note

Develop a habit of writing rates and ratios as fractions with units listed in both the numerator and the denominator.

It is common for recipes and mixtures to use the term "part" instead of a specific unit like "cup" or "tablespoon." A part is any unit of volume such as teaspoon, cup, quart, or gallon. The person creating the mixture can choose the unit based on the size of the quantity being mixed. If we are mixing paint for a wall, a part could be a quart. If we are mixing paint for a canvas, a part could be a one-second squirt from a paint tube.

4. A juice drink recipe calls for 2 parts orange juice and 1 part pineapple juice. Below are several statements. Circle all that are correct.

a. For every cup of pineapple juice, you will need 2 cups of orange juice.

b. For every quart of pineapple juice, you will need 2 quarts of orange juice.

c. When the juices are mixed together, the mixture will be $\frac{2}{3}$ orange juice.

d. The ratio of orange juice to pineapple juice is 2:1.

e. If you have 20 ounces of orange juice, you will need to add 10 ounces of pineapple juice.

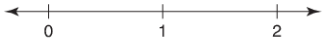
f. If you use 4 cups of pineapple juice, you can make 8 cups of the drink.

Connect

Fractions, rates, ratios, and percents are all related. In the following problem, you will look at the same number from many perspectives.

5. $\frac{3}{4}$ is a fraction as well as a ratio. We will interpret it in several ways.

a. Draw four pictures of $\frac{3}{4}$ using the following interpretations.



b. Below are 3 boxes of equal size. Suppose they represent 3 candy bars and we want to divide them among 4 people equally. Draw lines to indicate this division. What part of a whole candy bar does each person receive? What do you notice?

c. Suppose you scored 75% on your last 100 point exam. Explain how to interpret this percent as a rate. Which two quantities are being compared?

Reflect

lesson

WRAP-UP

What's the point?

Rates and ratios are everywhere, and understanding what is being compared goes a long way when working with them.

What did you learn?

How to interpret rates that are commonly used in daily life

Cycle 1 Question: What can be learned?

The lesson introduces the idea of writing numbers with units, which treats a number as a quantity. To see how this practice can help you, imagine completing #3 from the lesson without units. Why might that be hard?

1.5 Homework

Skills

MyMathLab

- Interpret rates that are commonly used in daily life.

1. Maddy lost 18 pounds in 8 weeks. Find her rate of pounds lost per week.

2. Compare in hours: 27 hours to 3 days. Write the ratio as a fraction in lowest terms.

Concepts and Applications

- Interpret rates that are commonly used in daily life.

3. Suppose the sales tax in your town is 8%. Write at least 3 interpretations of this ratio.

4. Find a rate in a newspaper or an online news article and write 3 interpretations of it. Do not use an example that was used in class.

5. A fifth-grader comments to his friend that the ratio of boys to girls in his class is 5 to 7. The friend replies, "Your class only has 12 students?" Explain the flaw in the second student's reasoning.

6. If the ratio of boys to girls in a student's class is 5 to 7, what fraction of the class is composed of boys? What fraction is composed of girls?

7. Suppose the ratio of desktop to laptop computers in a school is 3 to 4. Give three different scenarios that would result in the school having this ratio of computers. For each scenario, write the ratio of desktop to laptop computers and then simplify it.

8. If a school has 70 computers in the ratio of 3 desktops for every 4 laptops, how many are desktops? Explain how you answered this question. Did you use a picture?

9. A baby panda born at a national zoo weighed 8 ounces at birth and reached 75 pounds at one year of age. What was the panda's growth rate per month? Per week?

1.6 Part and Whole

1. At a particular college, half of all freshmen start in developmental math. Of those who start at this level, $\frac{4}{5}$ of them start in beginning algebra. Knowing this, what is the chance that a freshman at this college starts in beginning algebra? Give your answer as a percent. Do not use a calculator to answer this question!