Paired Data and the Rectangular Coordinate System

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Department of Mathematics

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Paired Data and the Rectangular Coordinate System

Learning Objectives:

- Graph ordered pairs on a rectangular coordinate system.
- Graph linear equations by finding intercepts or by making a table.
- Graph horizontal and vertical lines.

Ordered Pairs

We now turn our attention to equations containing two variables, x and y. Paired data plays an important role in these type of equations.

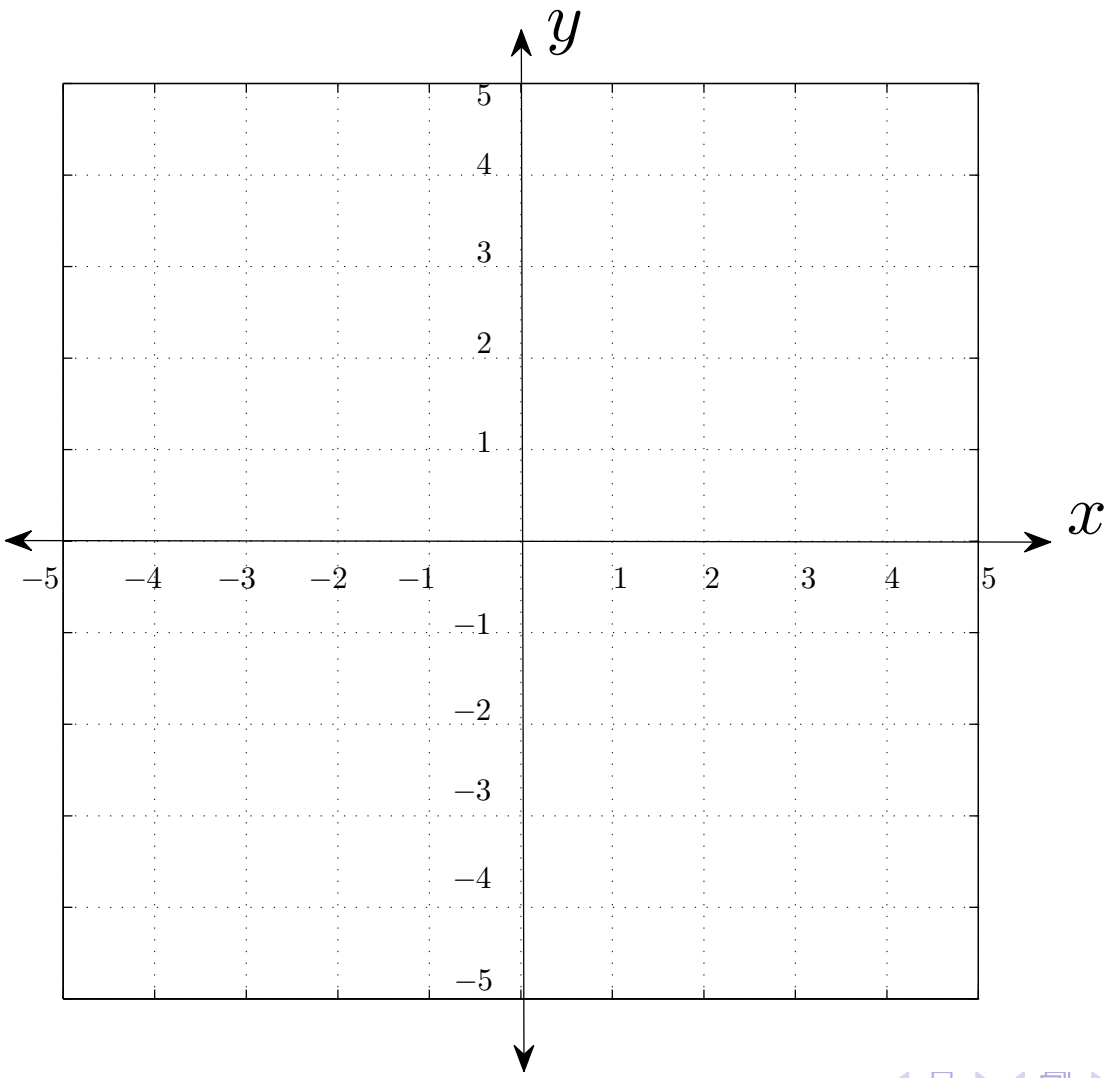
Ordered Pairs

Definition

A pair of numbers enclosed in parenthesis and separated by a comma, such as $(-2, 1)$, is called an ordered pair of numbers. The first number in the pair is called the x-coordinate of the ordered pair; the second number is called the y-coordinate. For the ordered pair $(-2, 1)$, the x-coordinate is -2 and the y-coordinate is 1.

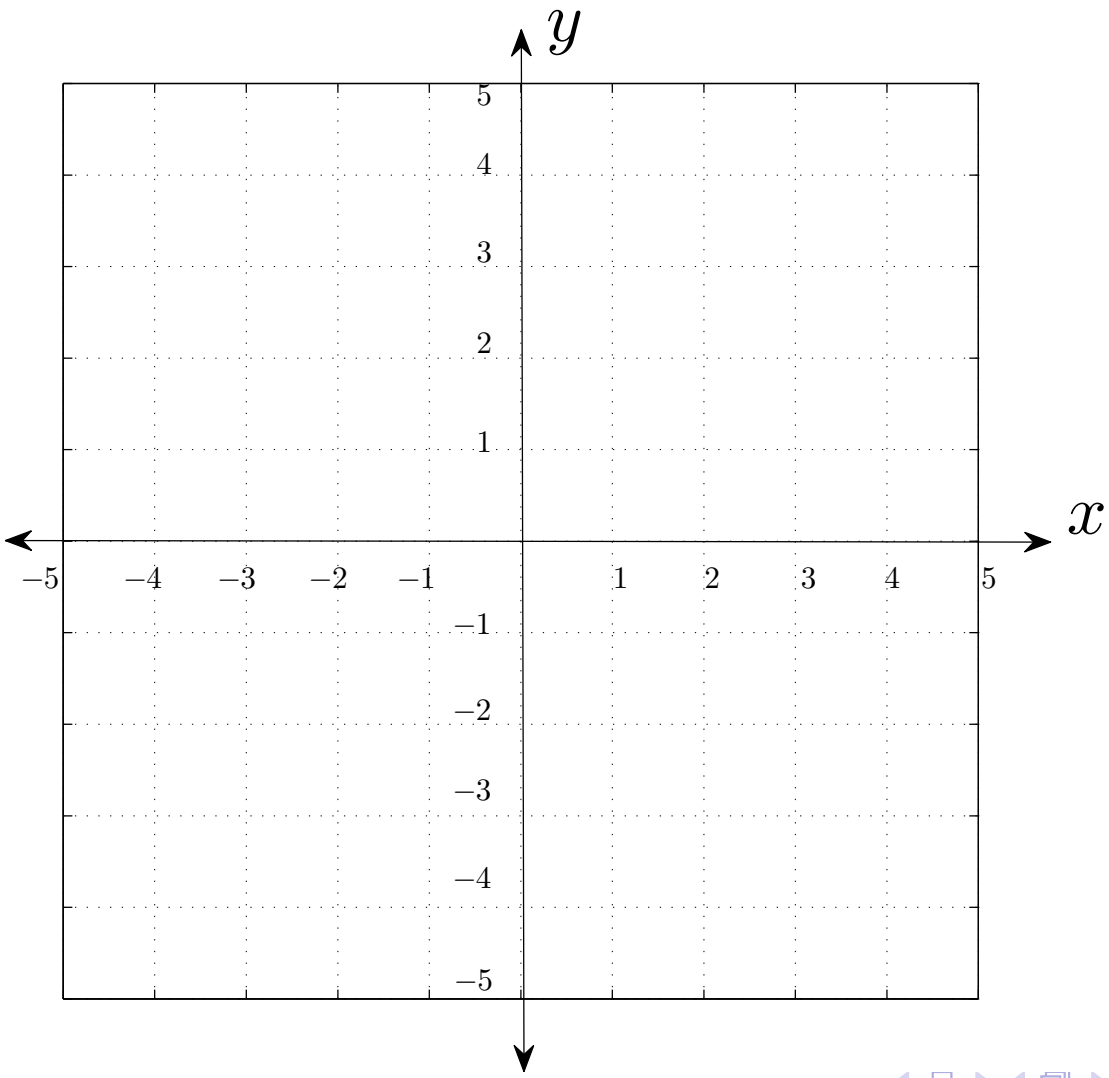
The Rectangular Coordinate System

We use a rectangular coordinate system to visualize ordered pairs.



The Rectangular Coordinate System

A rectangular coordinate system is made by drawing two real number lines at right angles to each other.



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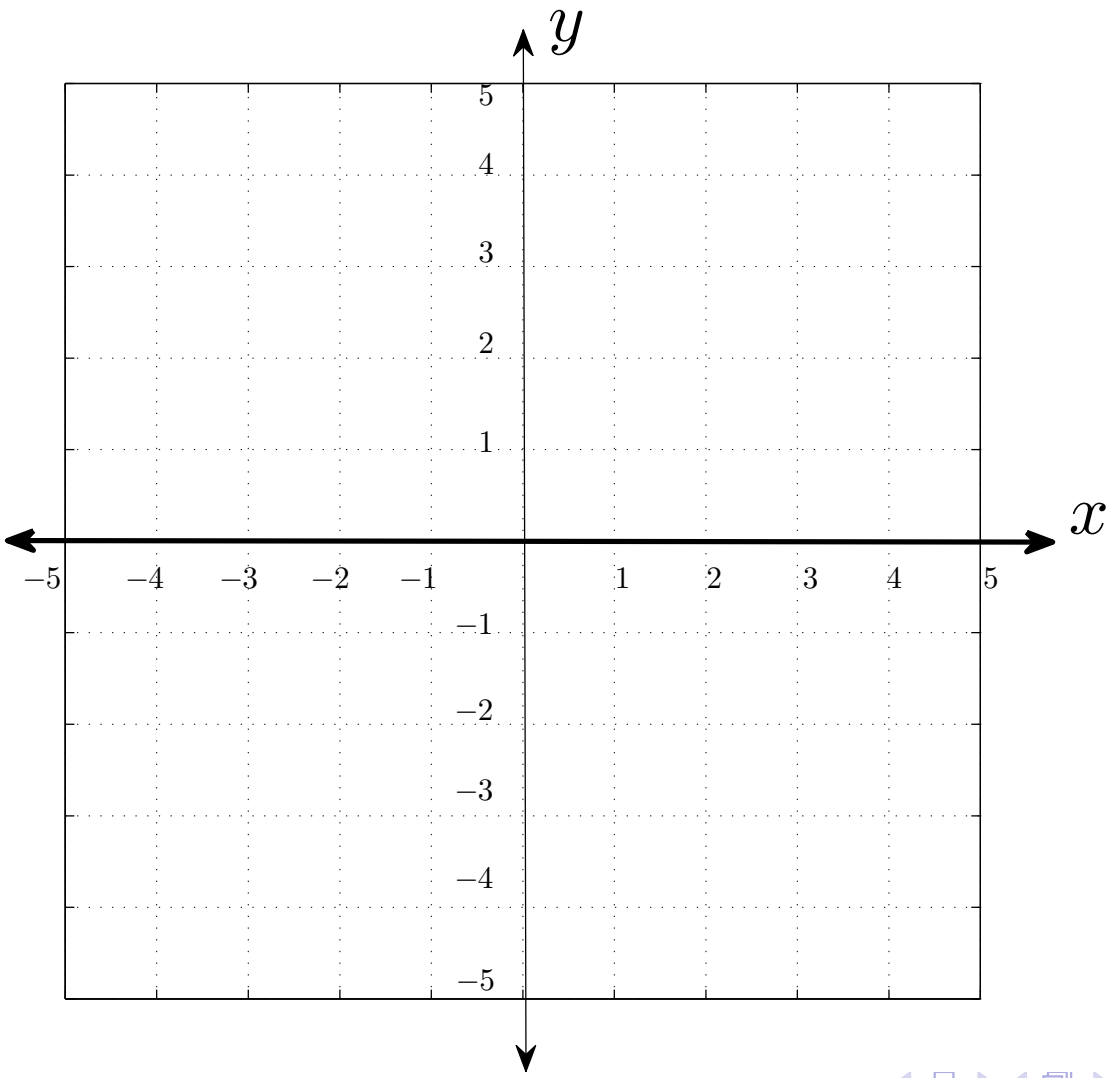

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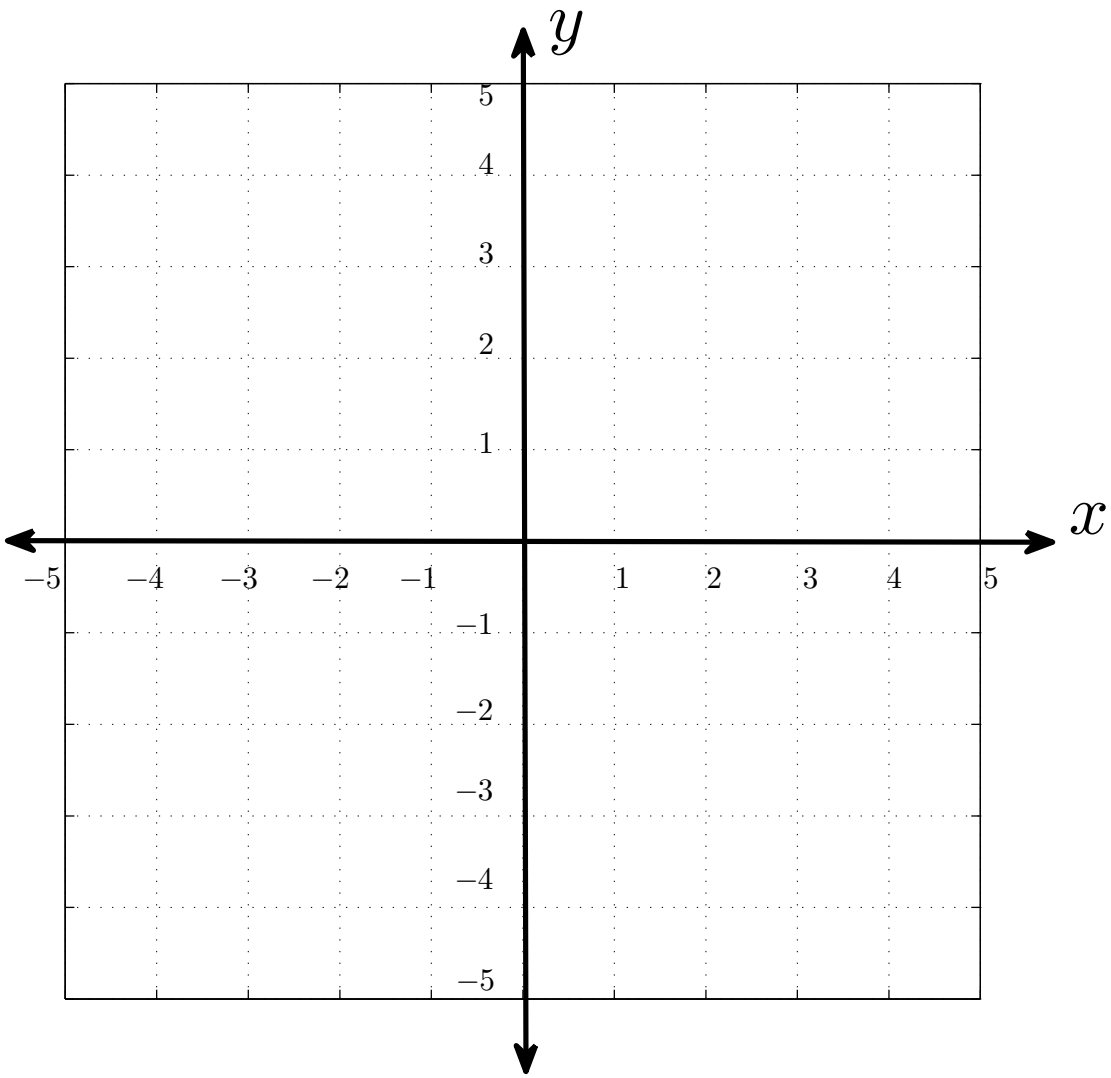
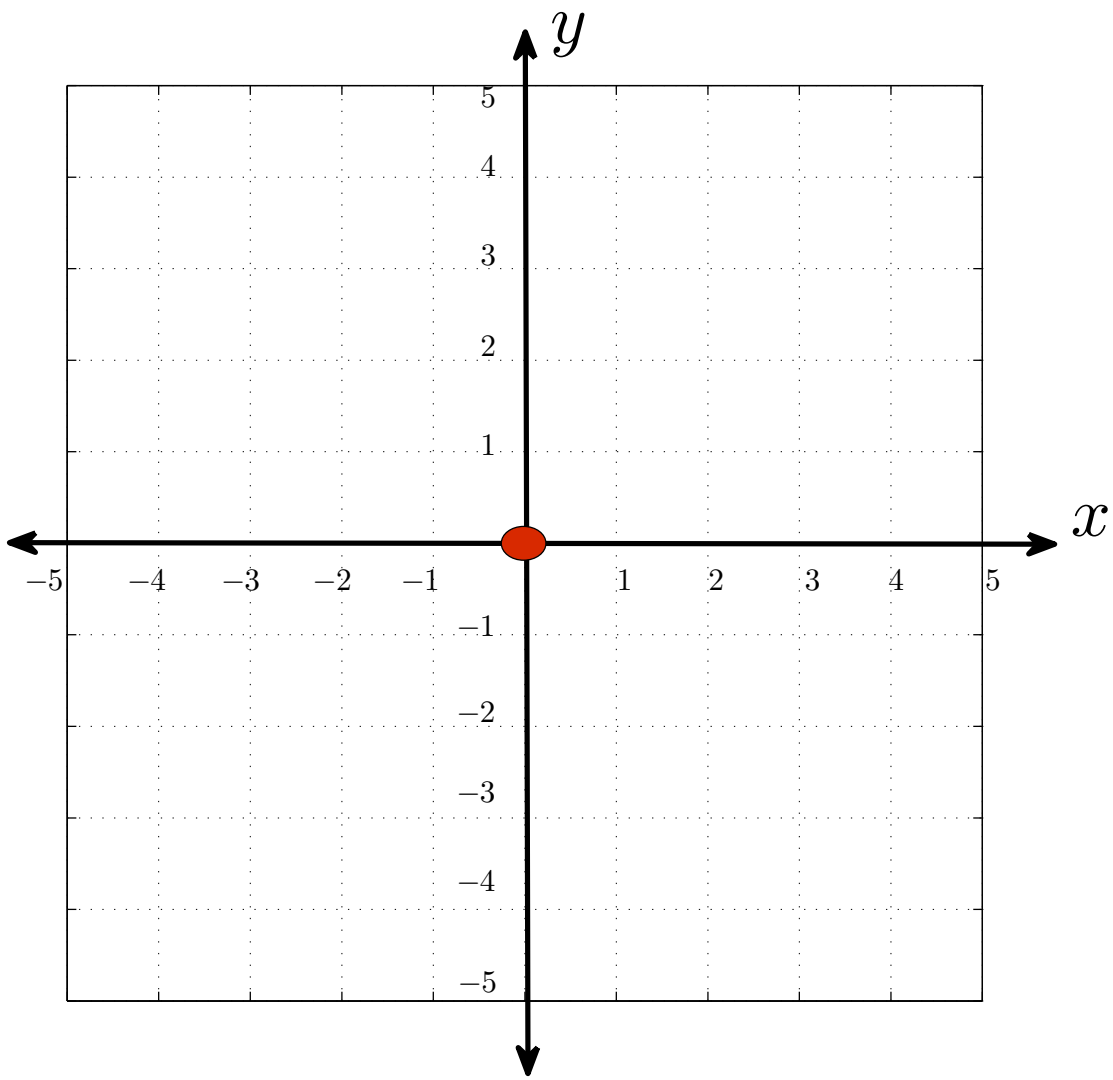


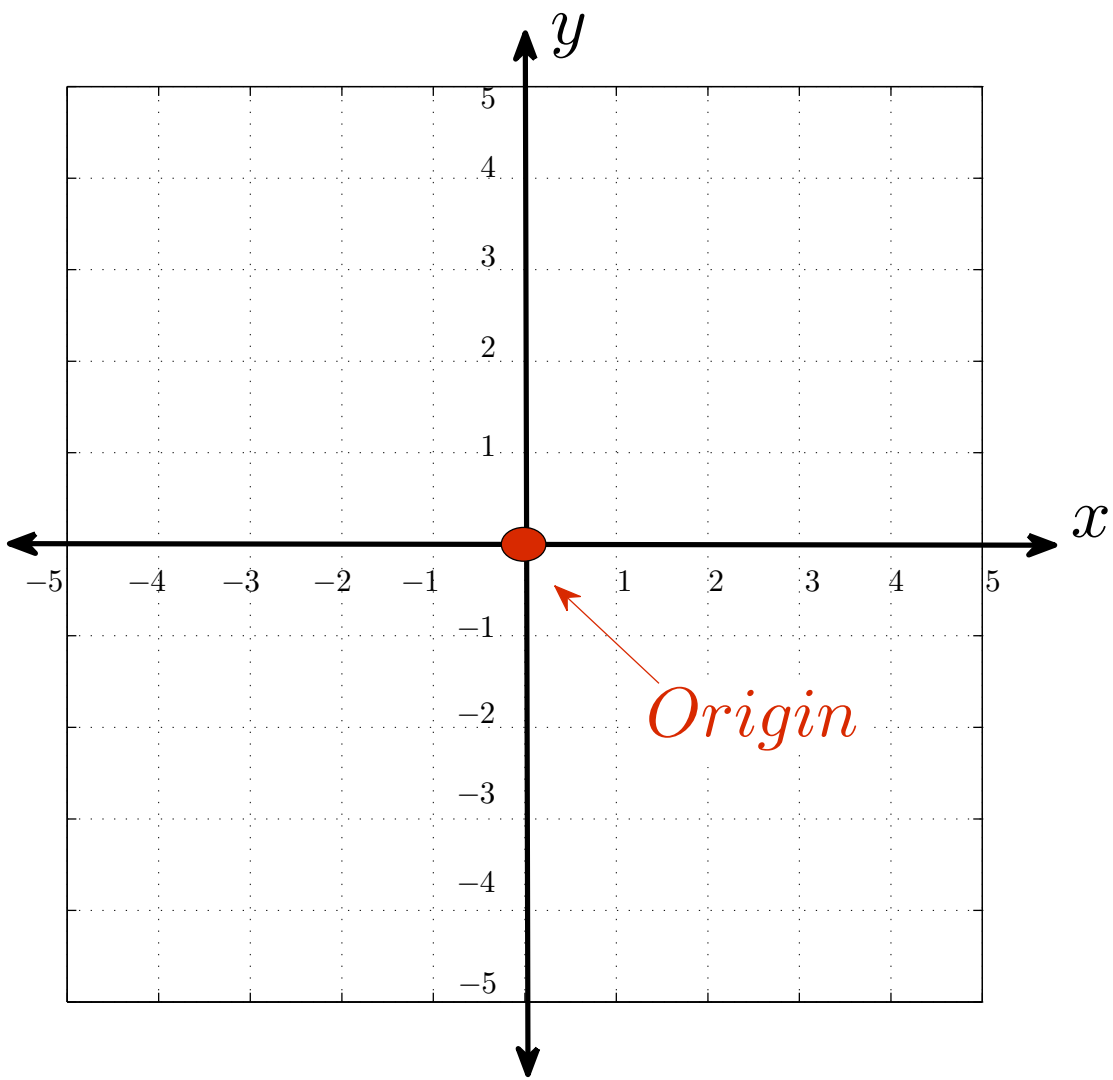
The Rectangular Coordinate System

Two number lines, called axes, cross each other at zero.



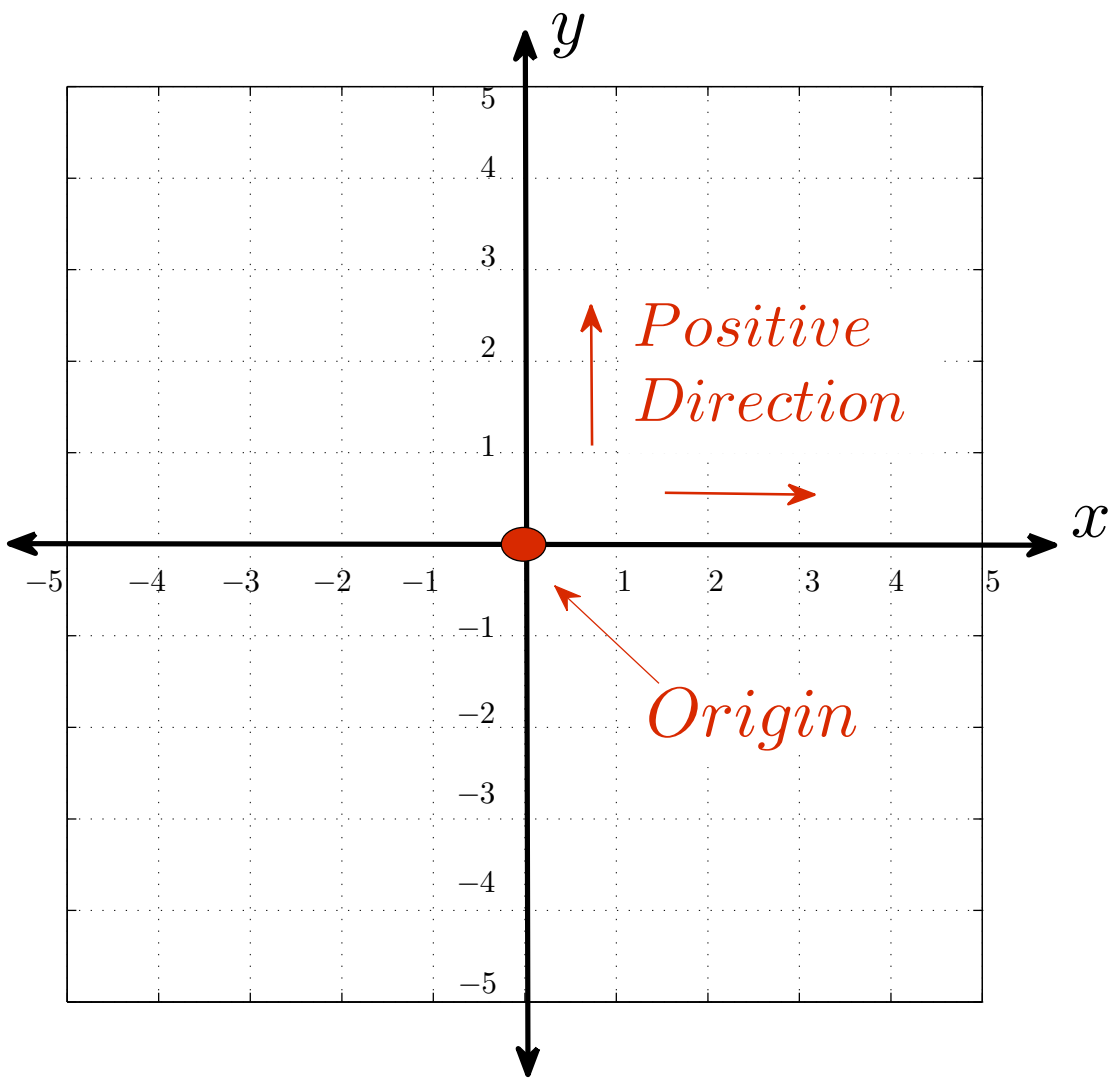
The Rectangular Coordinate System

Two number lines, called axes, cross each other at zero. This point is called the *origin*.



The Rectangular Coordinate System

Relative to the origin, positive directions are to the right and up.



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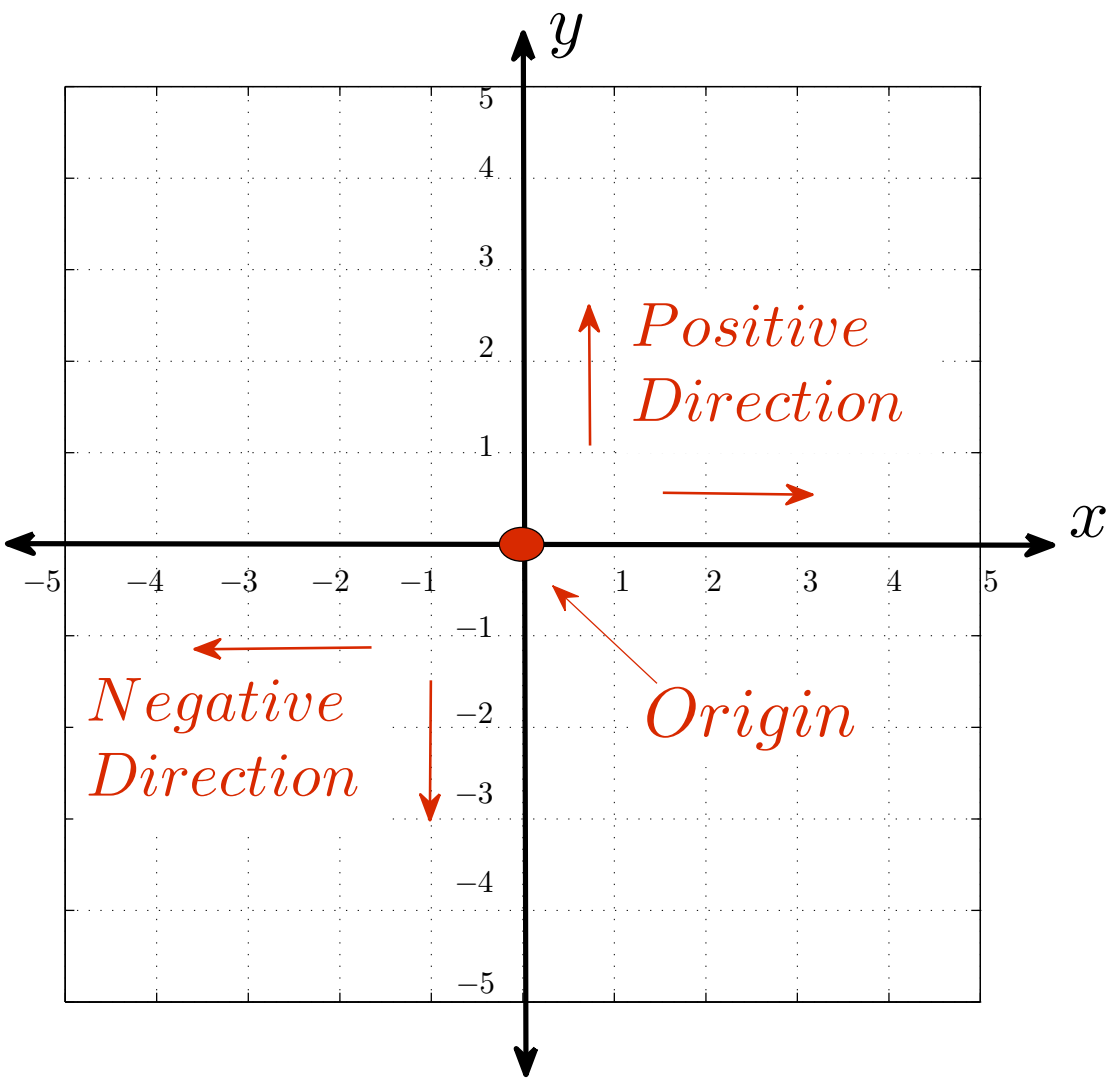
Negative directions are to the left and down.



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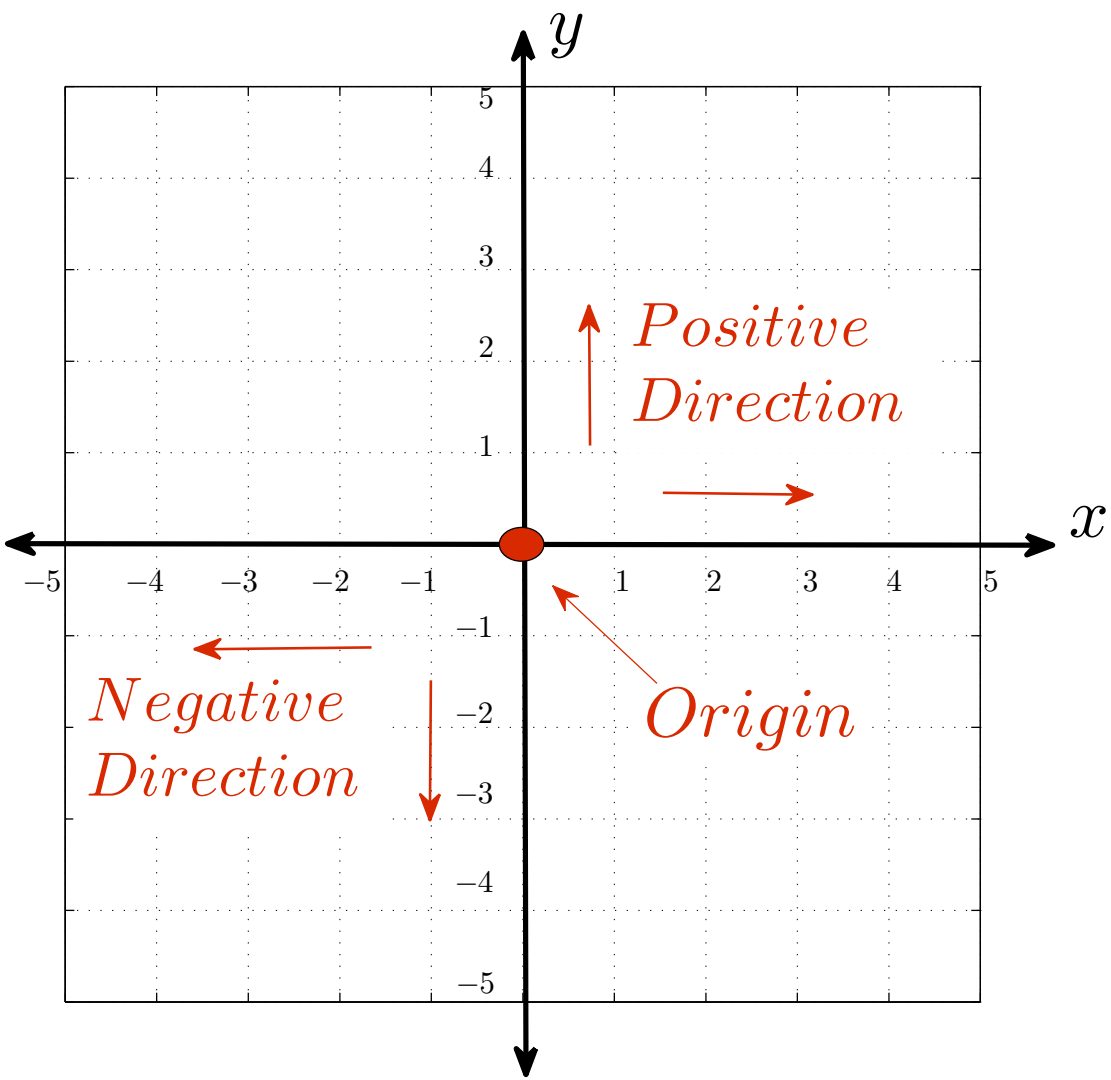
The horizontal number line is called the *x*-axis



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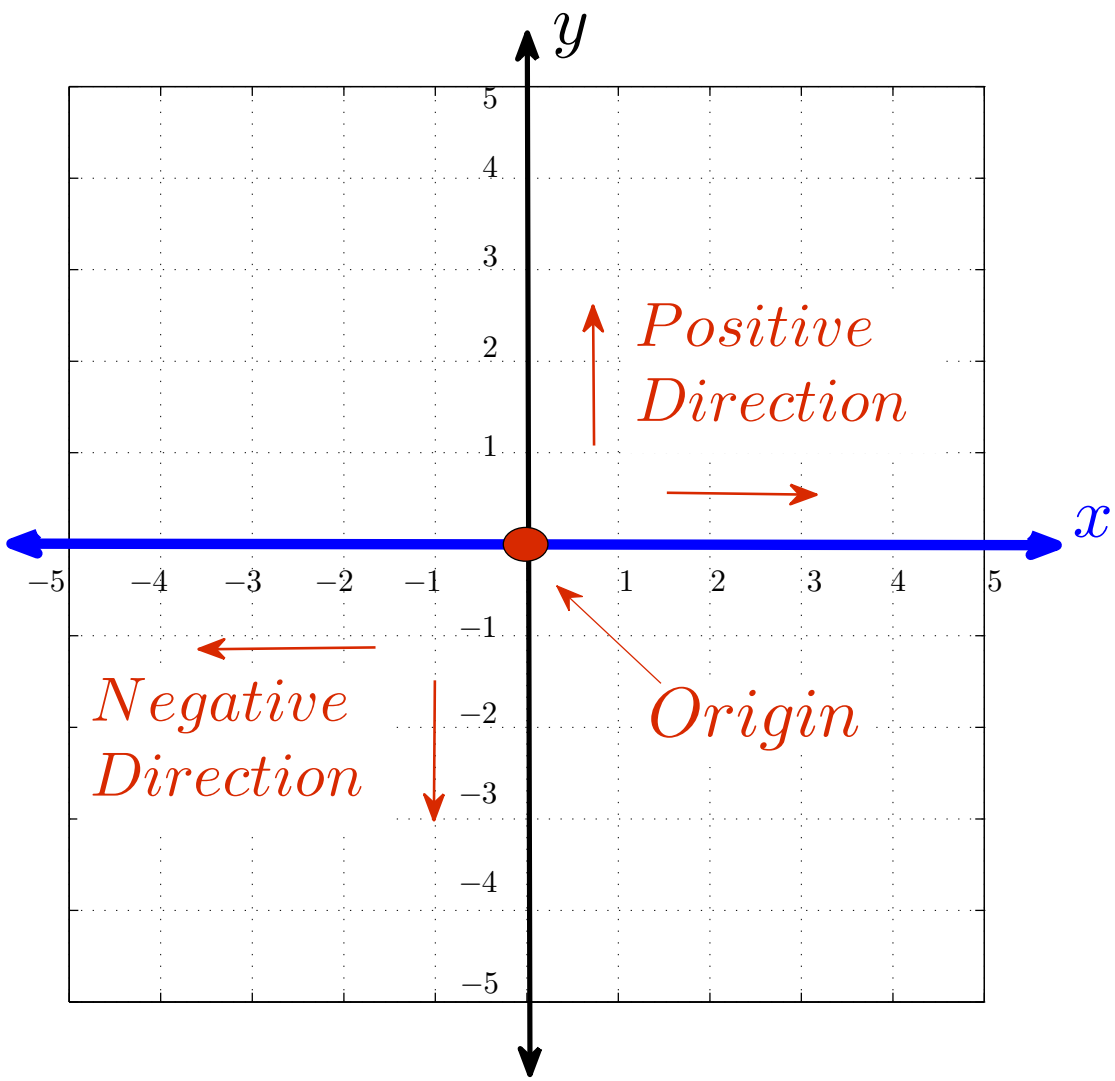

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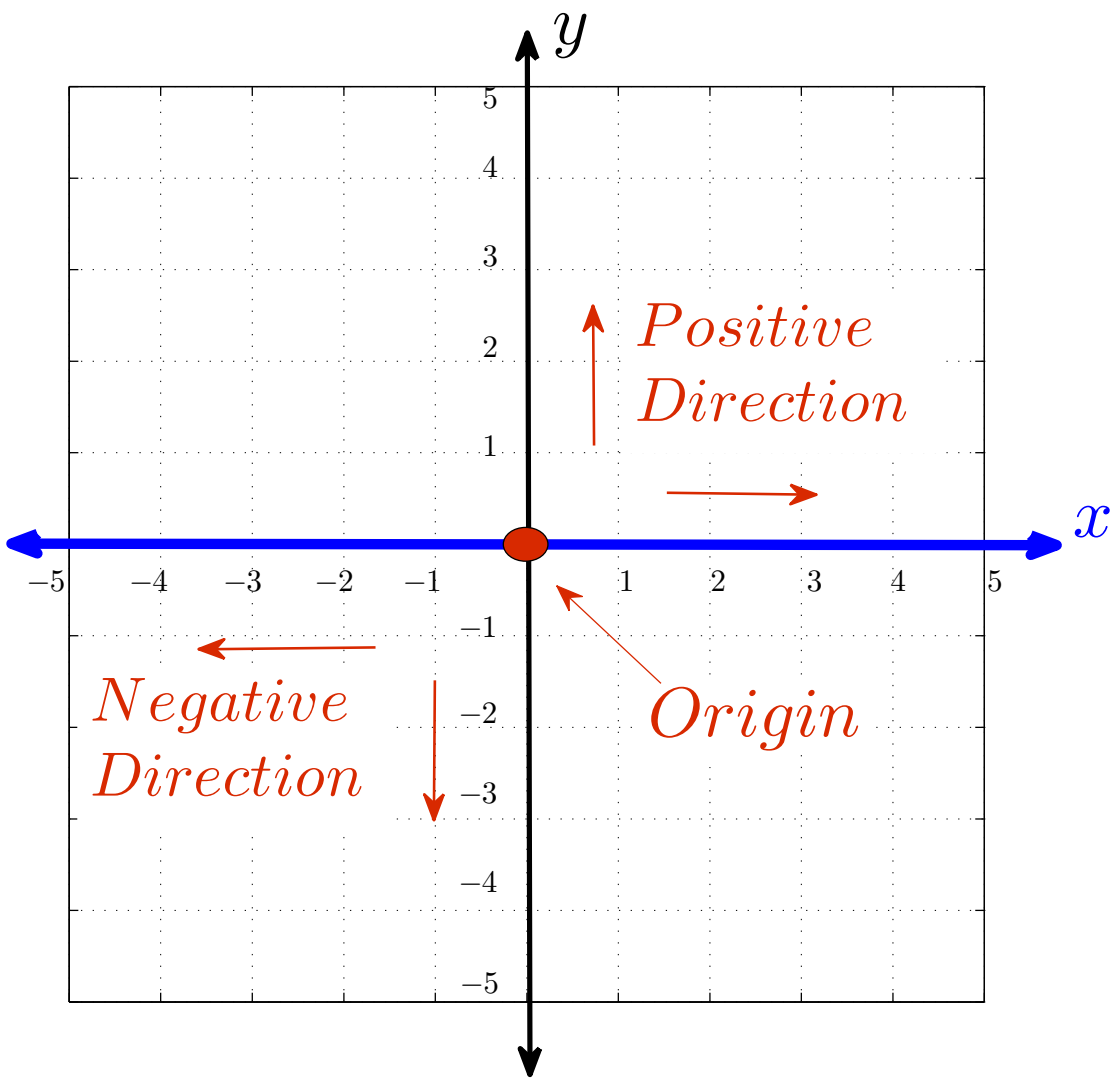
The horizontal number line is called the *x*-axis and the vertical number line is called the *y*-axis.



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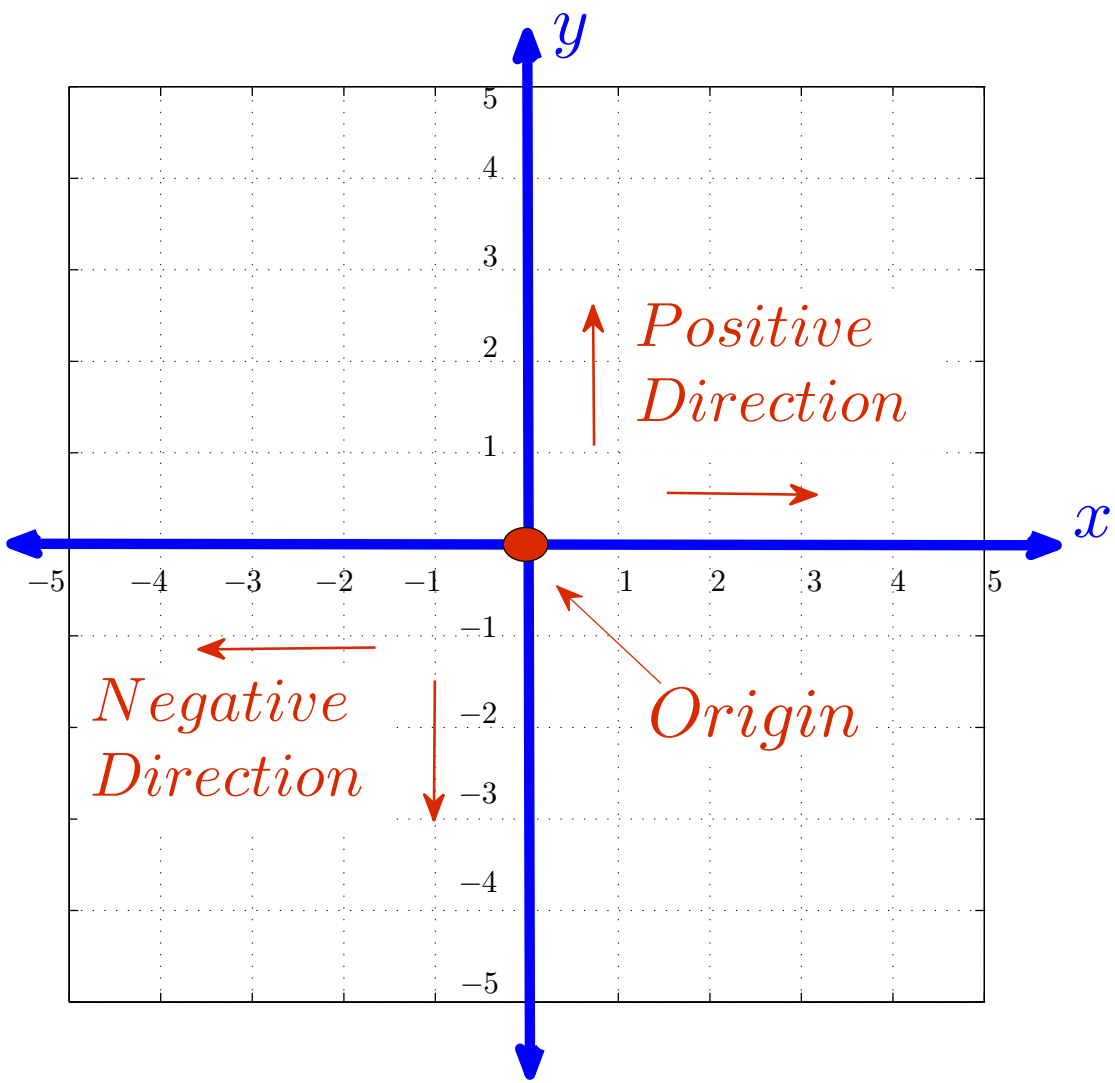
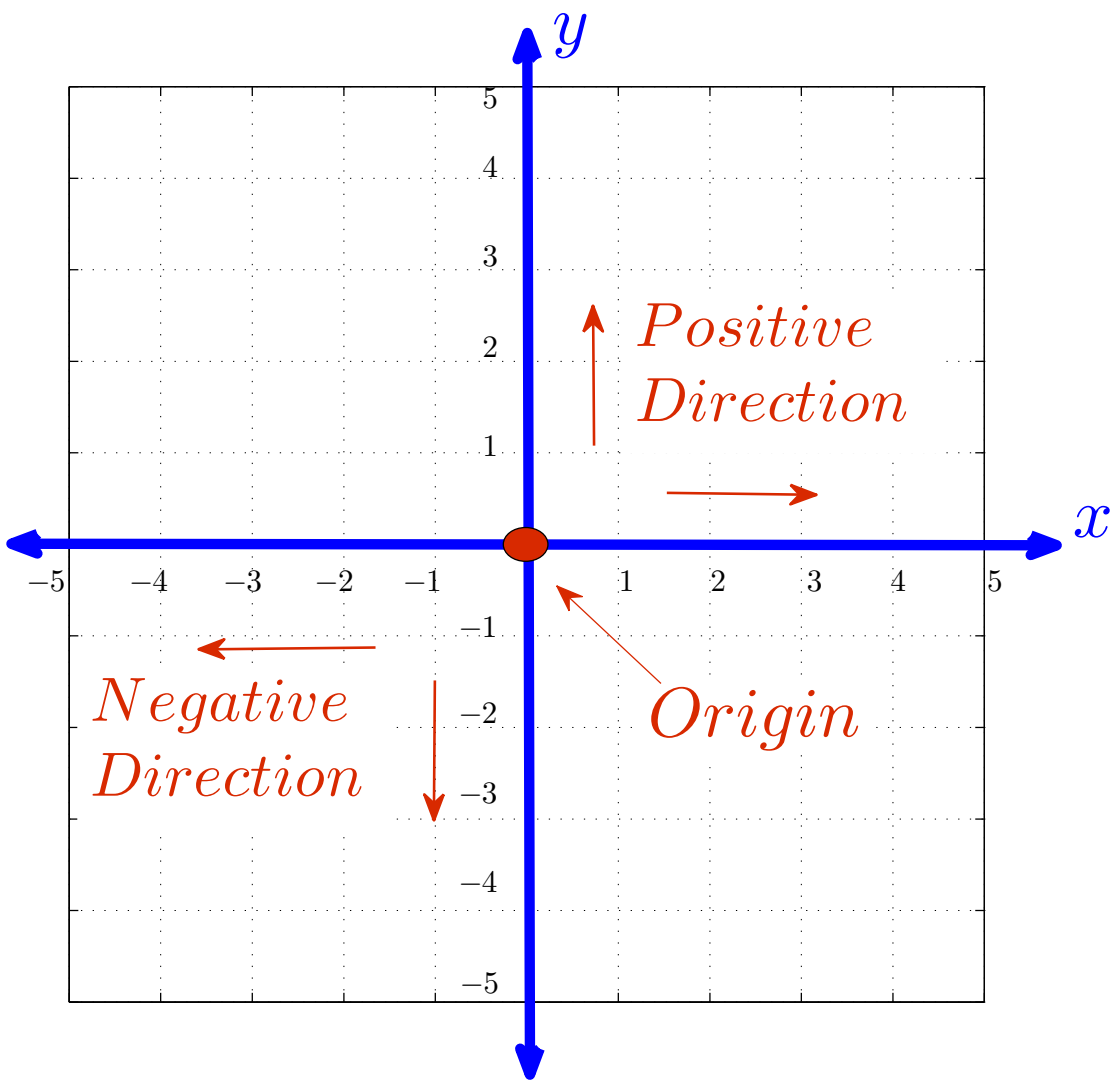


The Rectangular Coordinate System

The two number lines divide the coordinate system into four quadrants, which we number I through IV in a counterclockwise direction.



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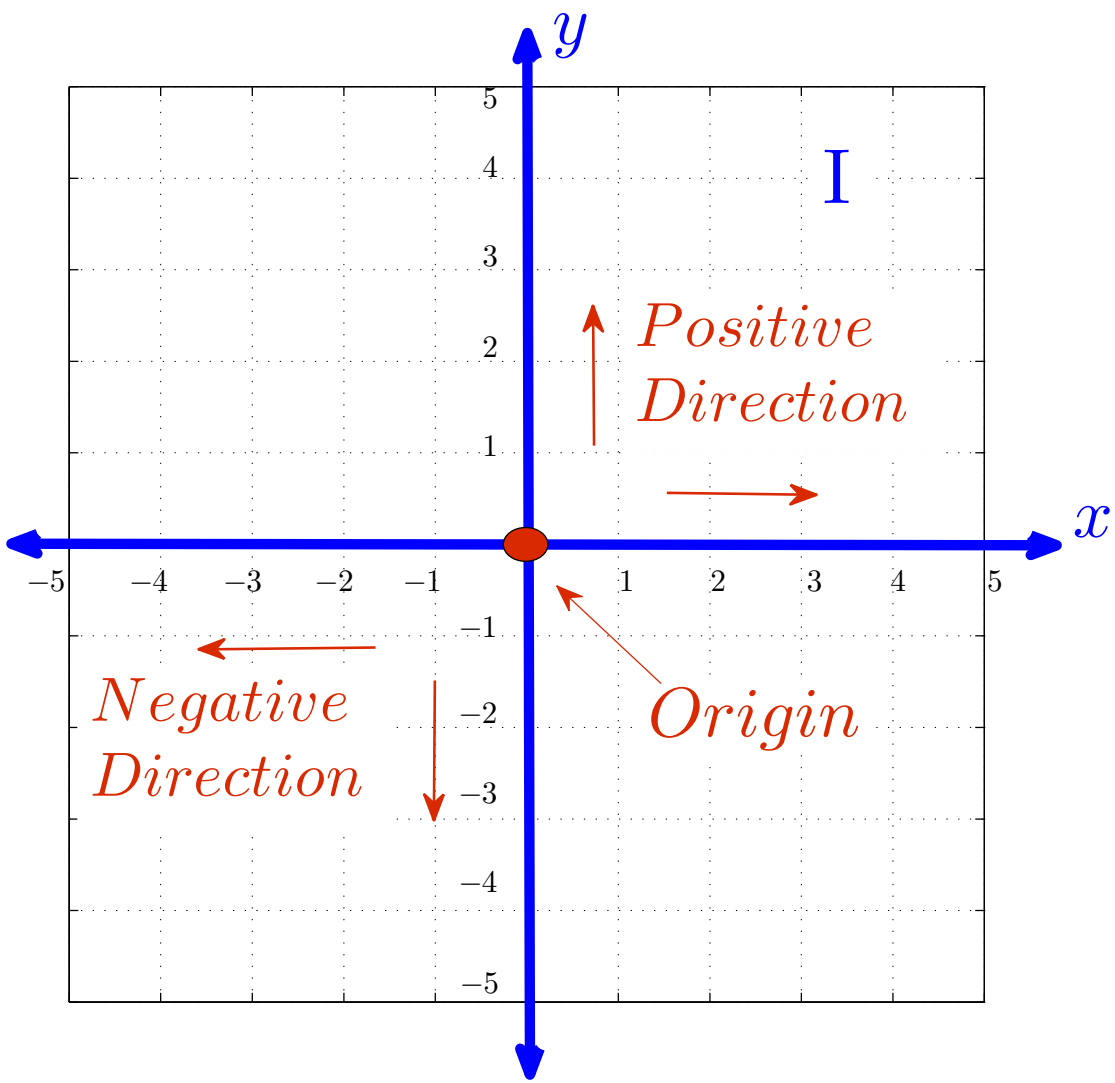

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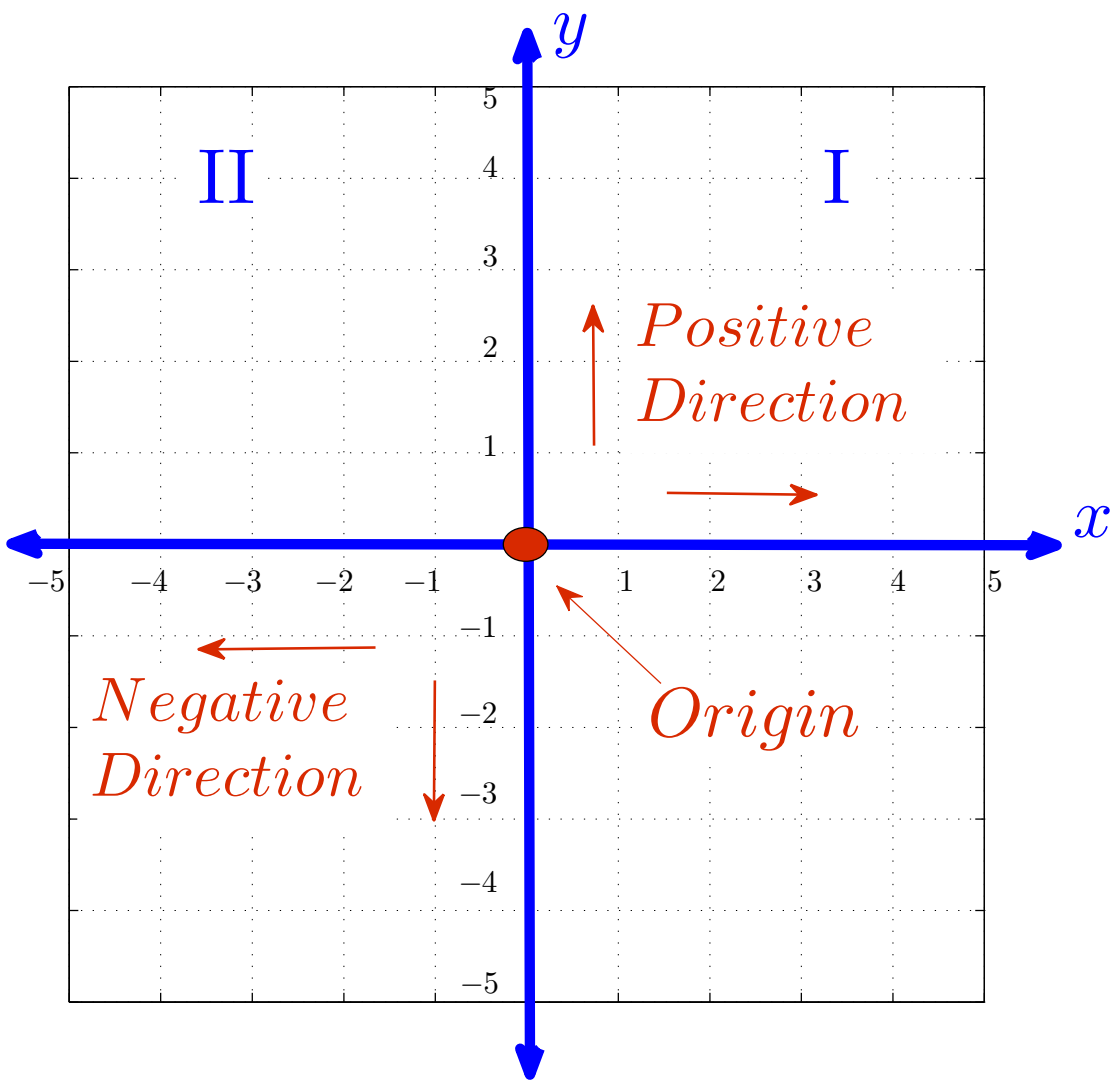

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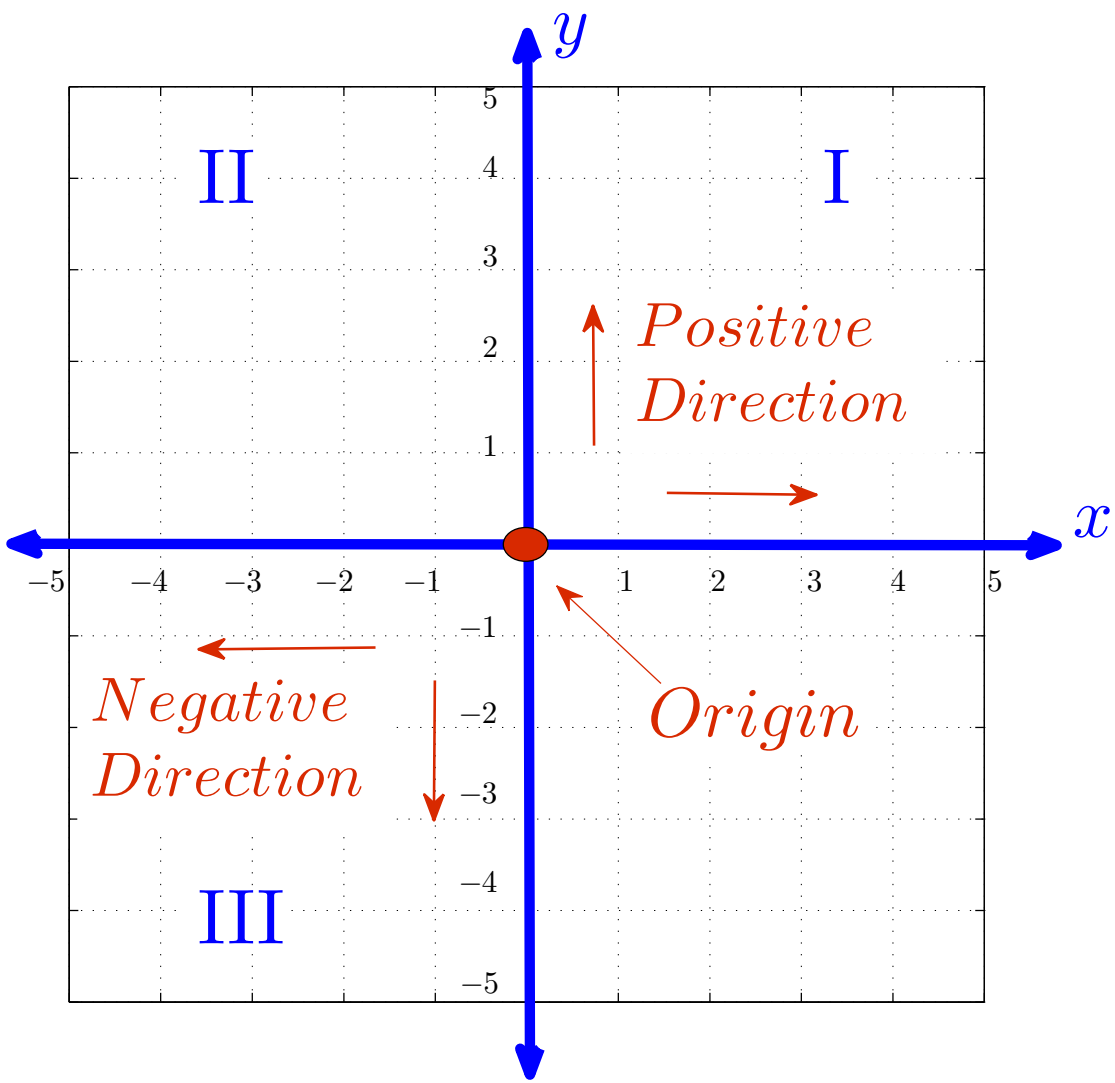

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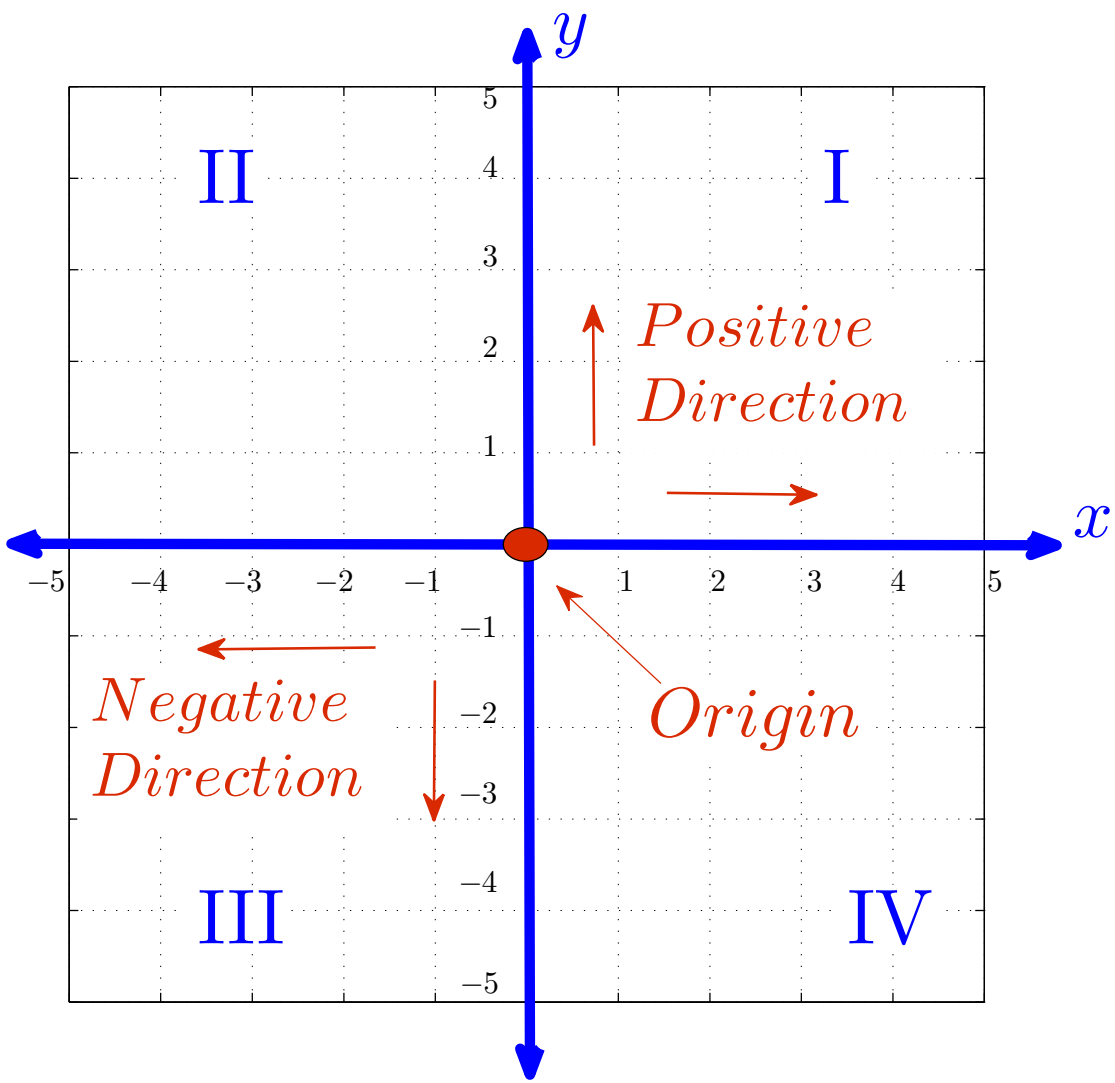

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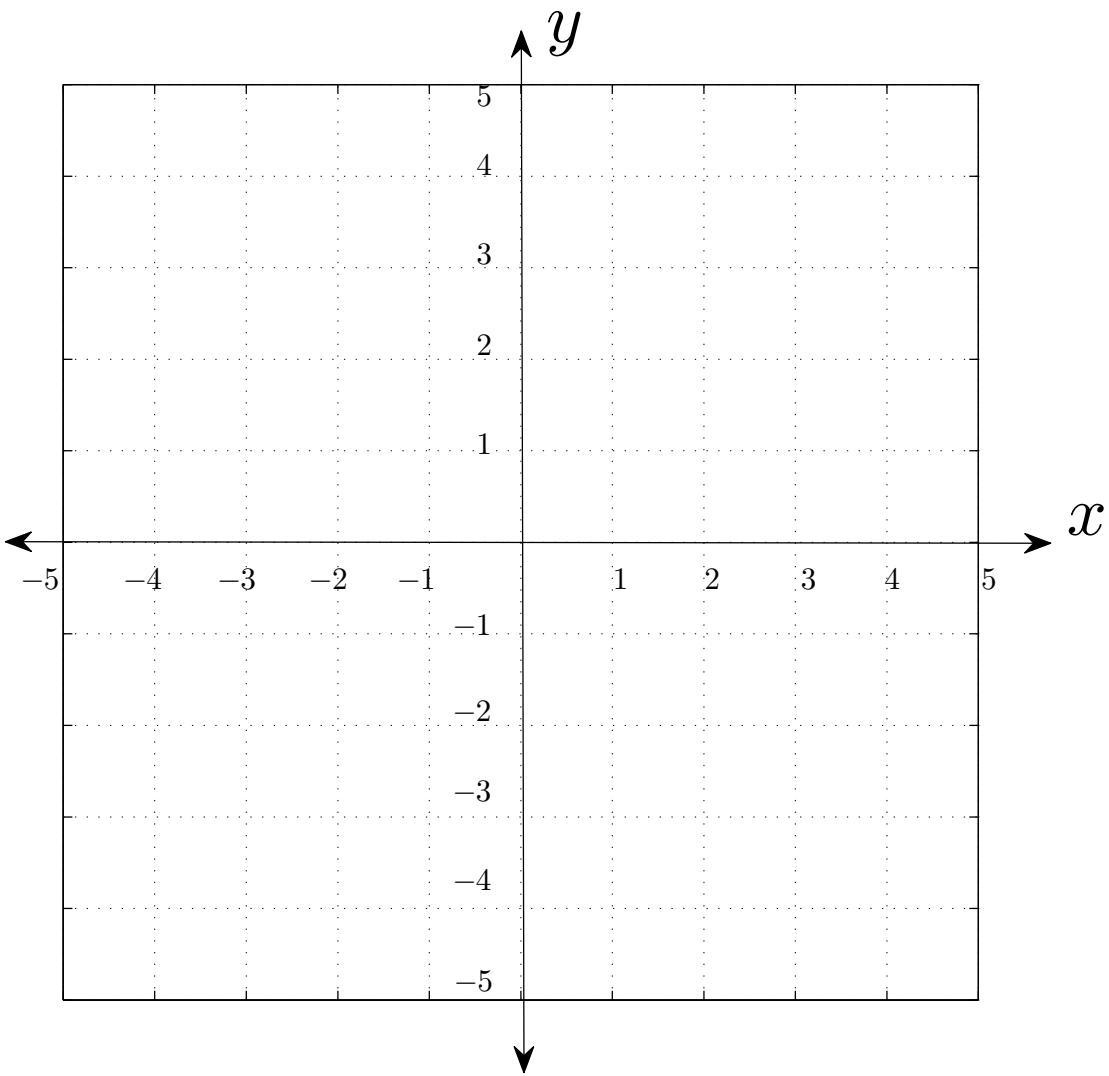
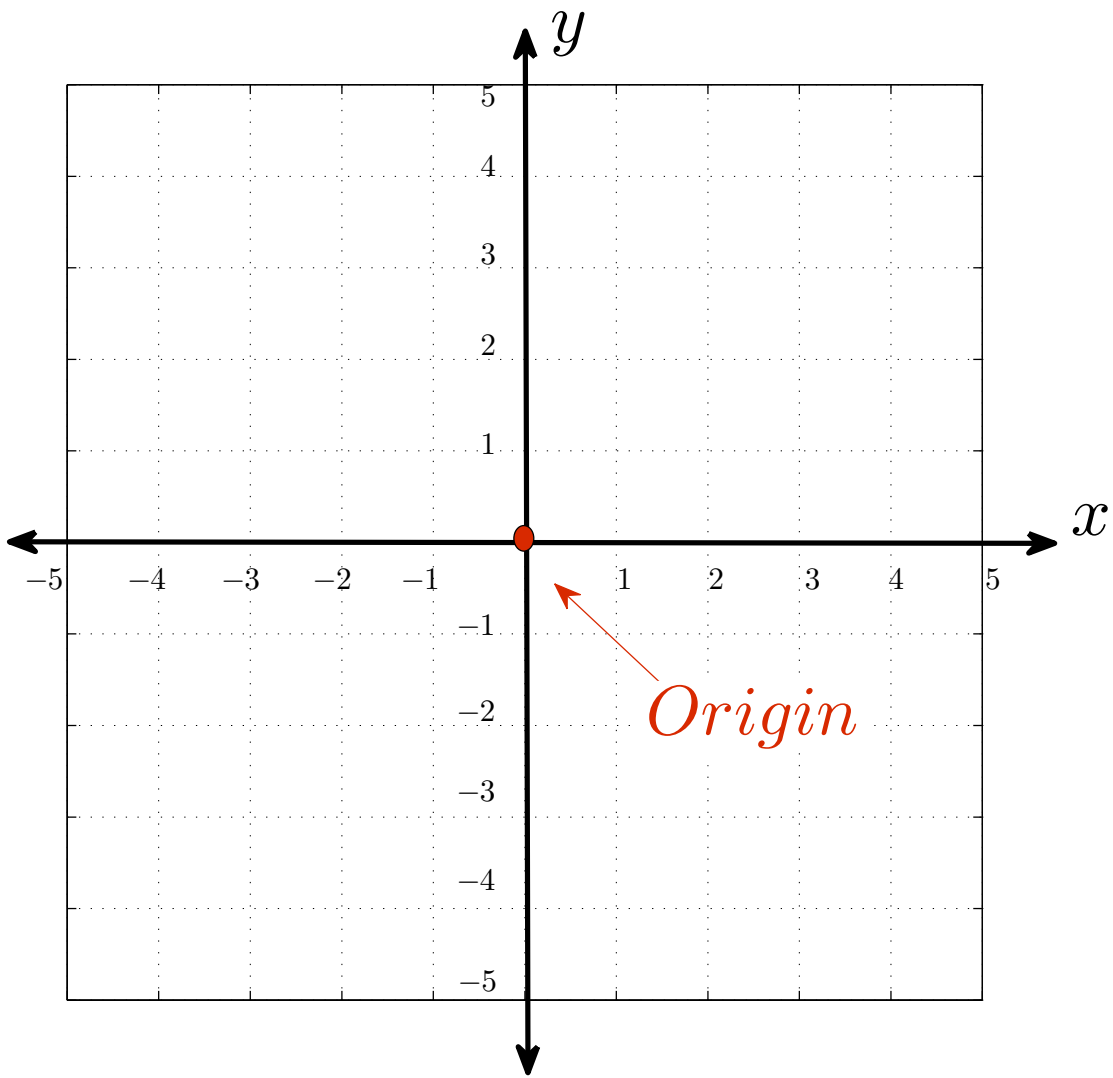
To graph the ordered pair (a, b) on the rectangular coordinate system, we:

1. begin at the origin and move along the x-axis a units right or a units left (right if a is positive and left if a is negative).
2. From that point we move b units up or down (up if b is positive and down if b is negative).
3. The point where we end up is the graph of the ordered pair.

Example 1: Plot (graph) the following ordered pairs: $(2,3),\ (-2,3),\ (-2,-3),\ (2,-3),$



To plot $(2,3)$, begin at the origin.



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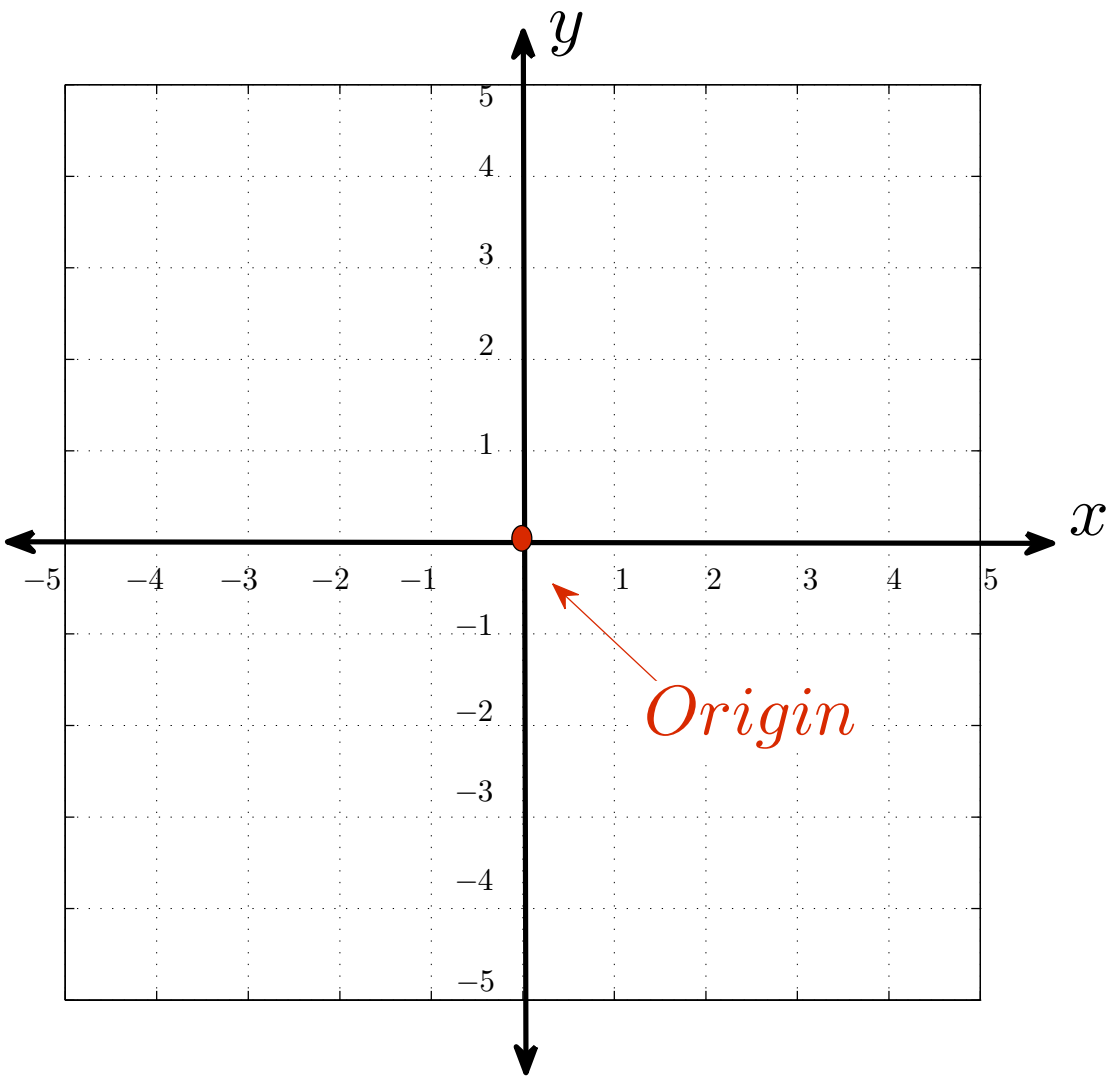

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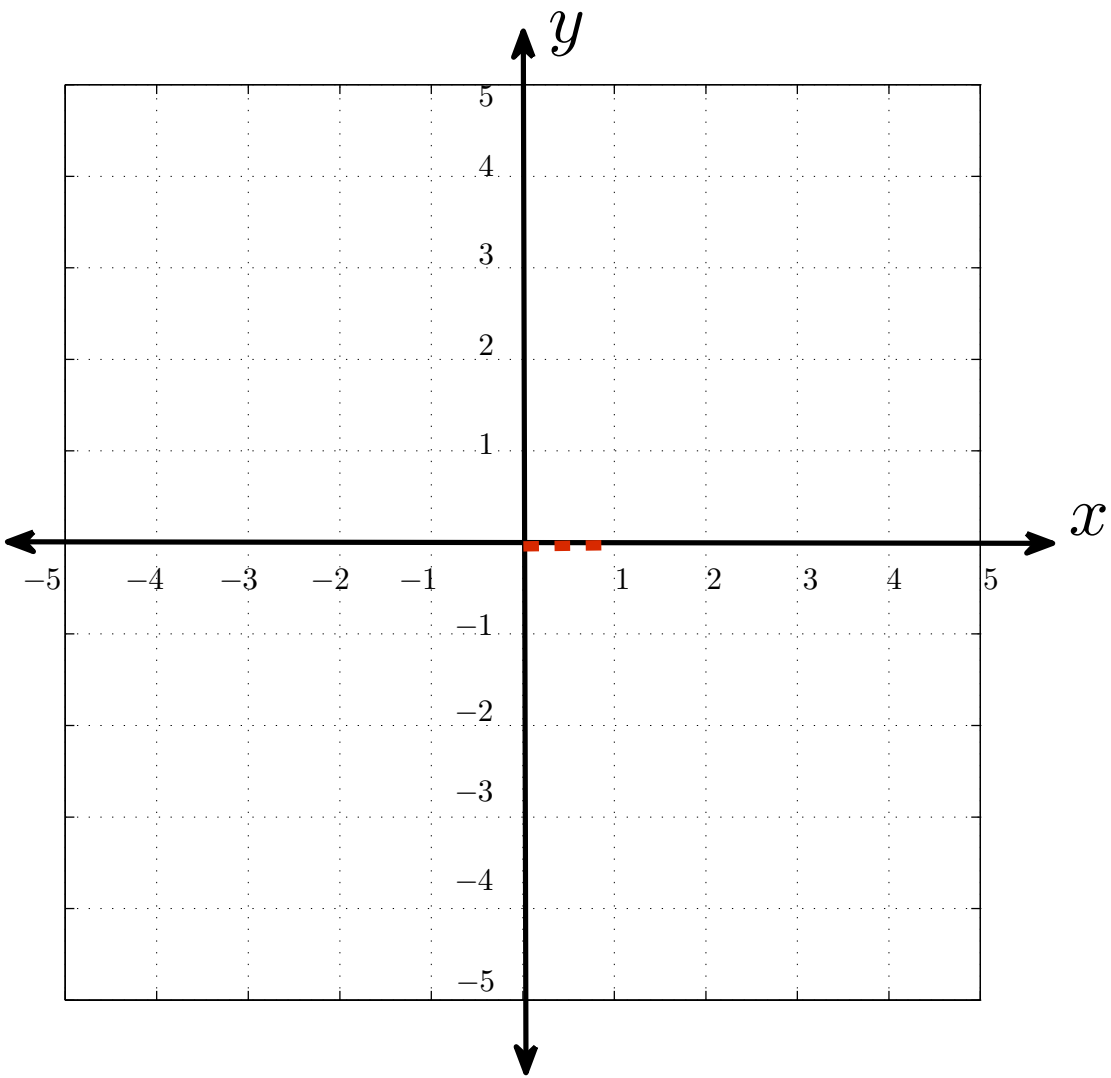

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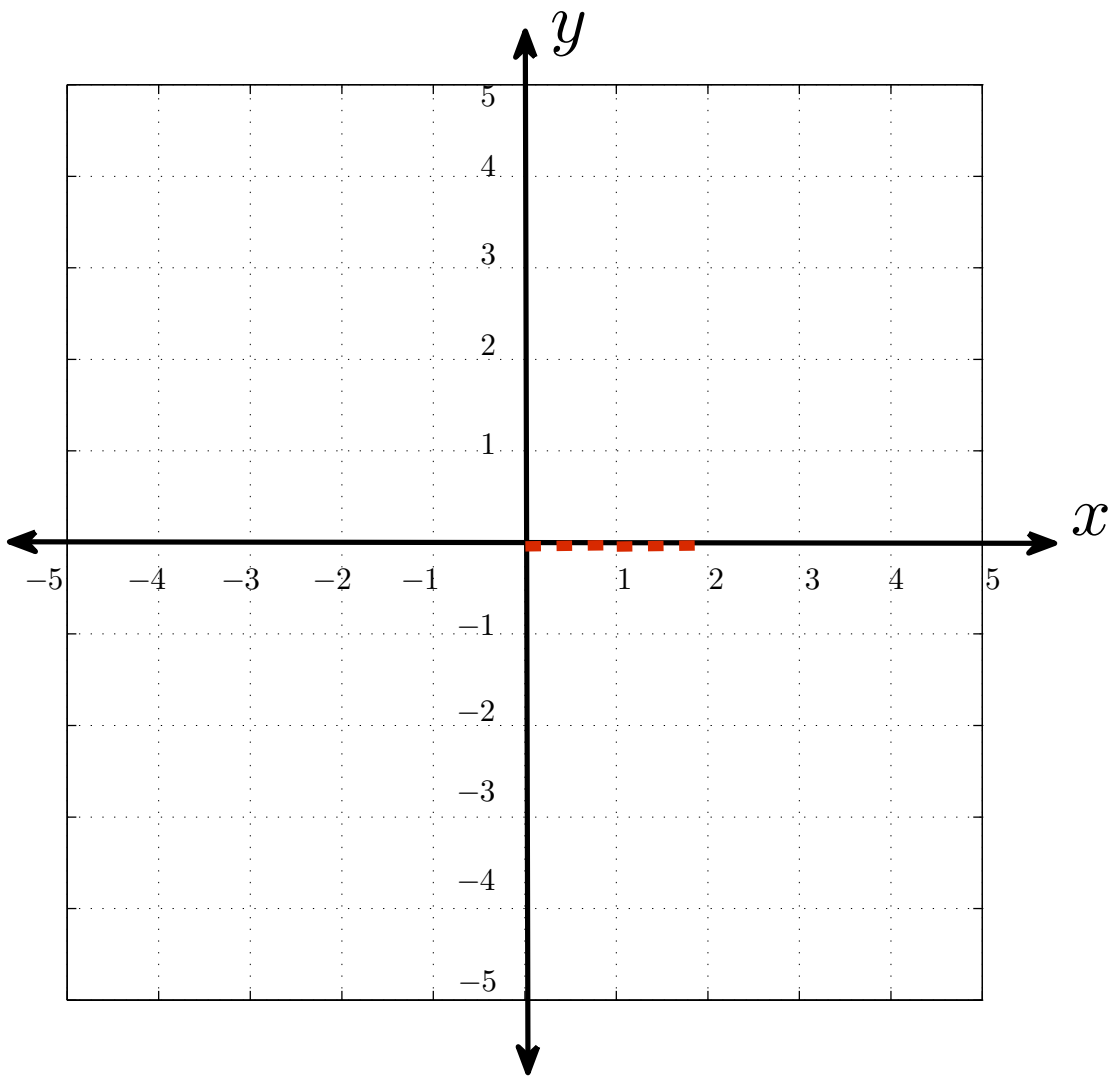
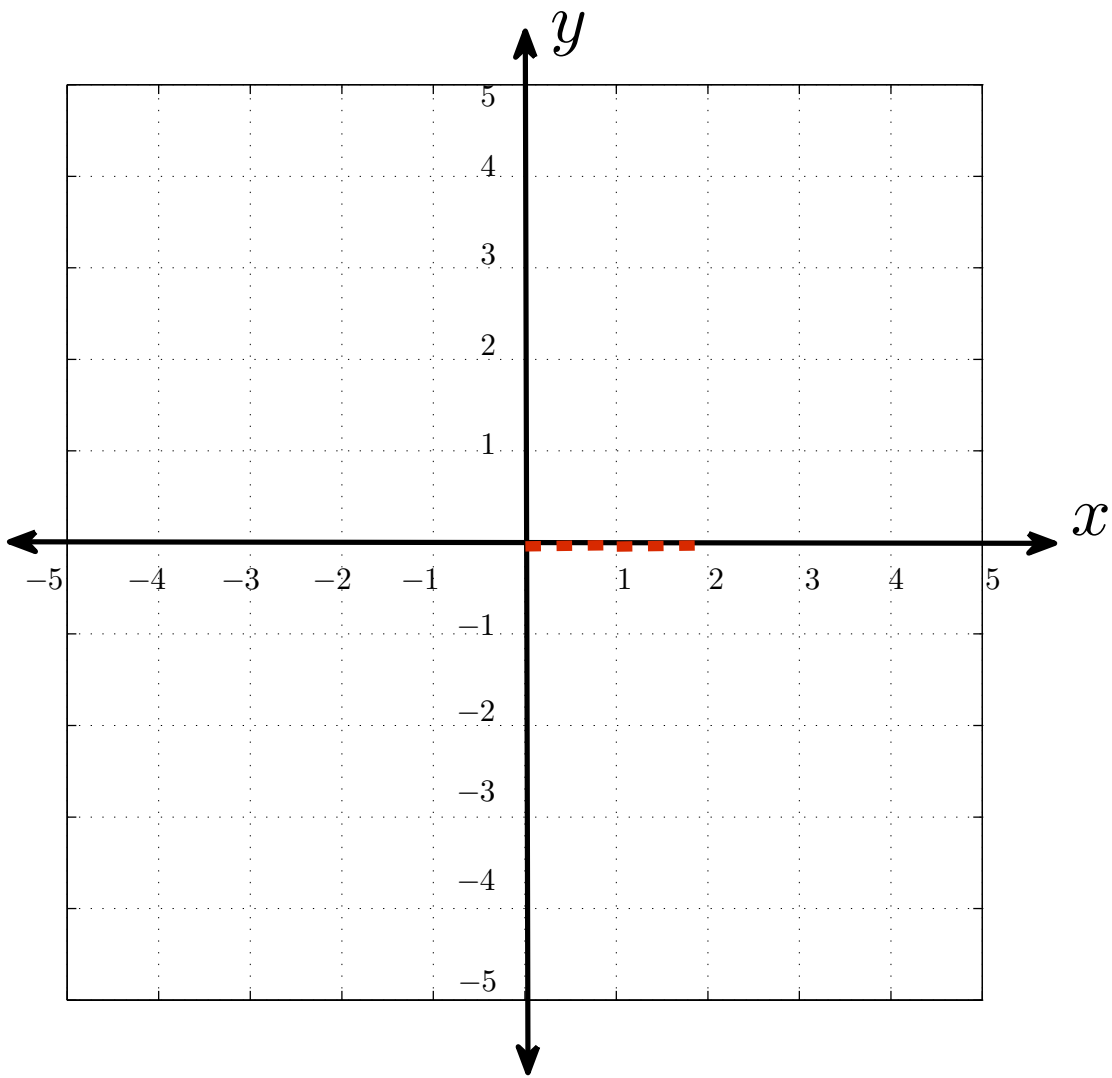


From that point, move in the upwards (positive y) direction 3 units.



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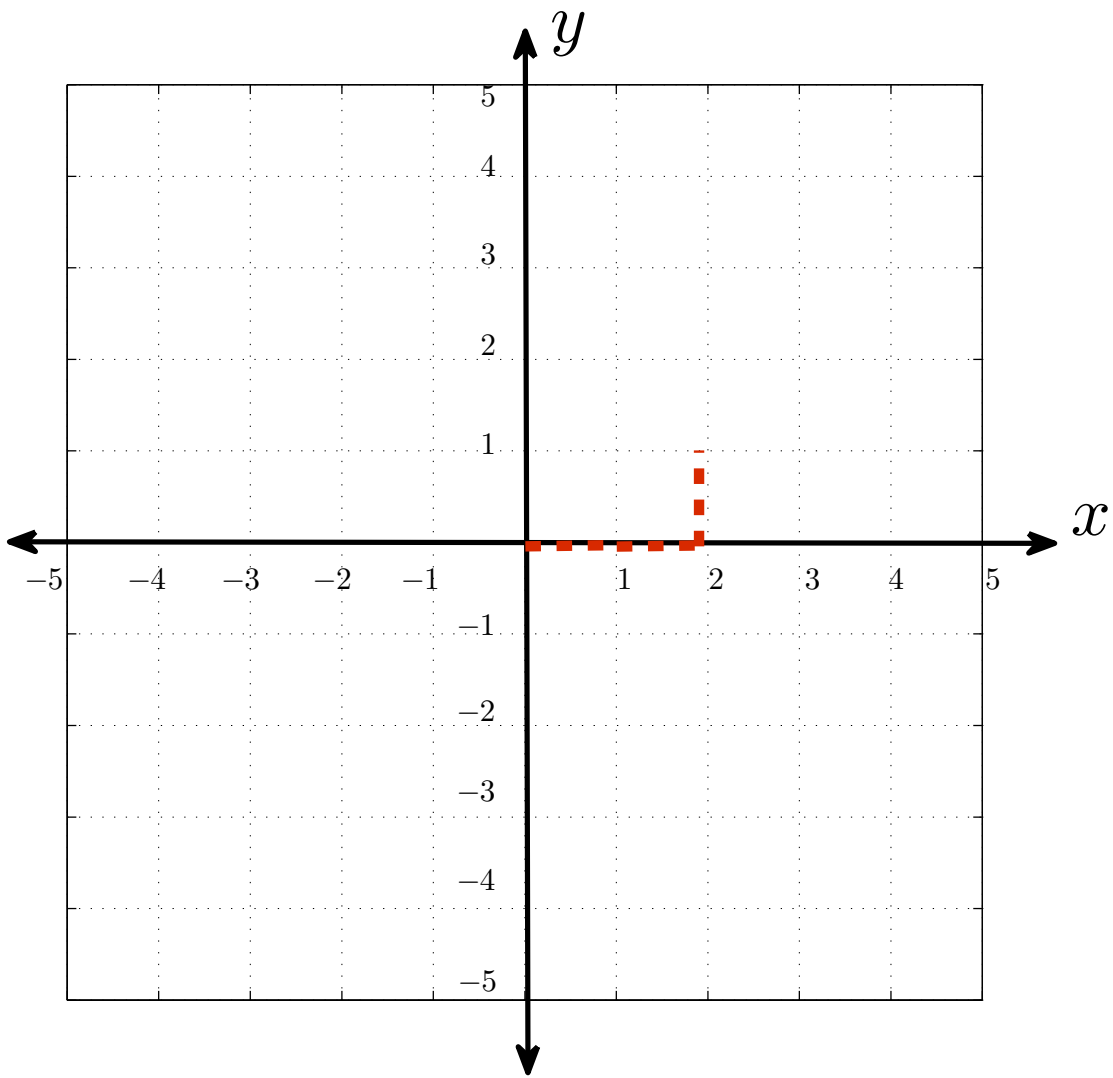

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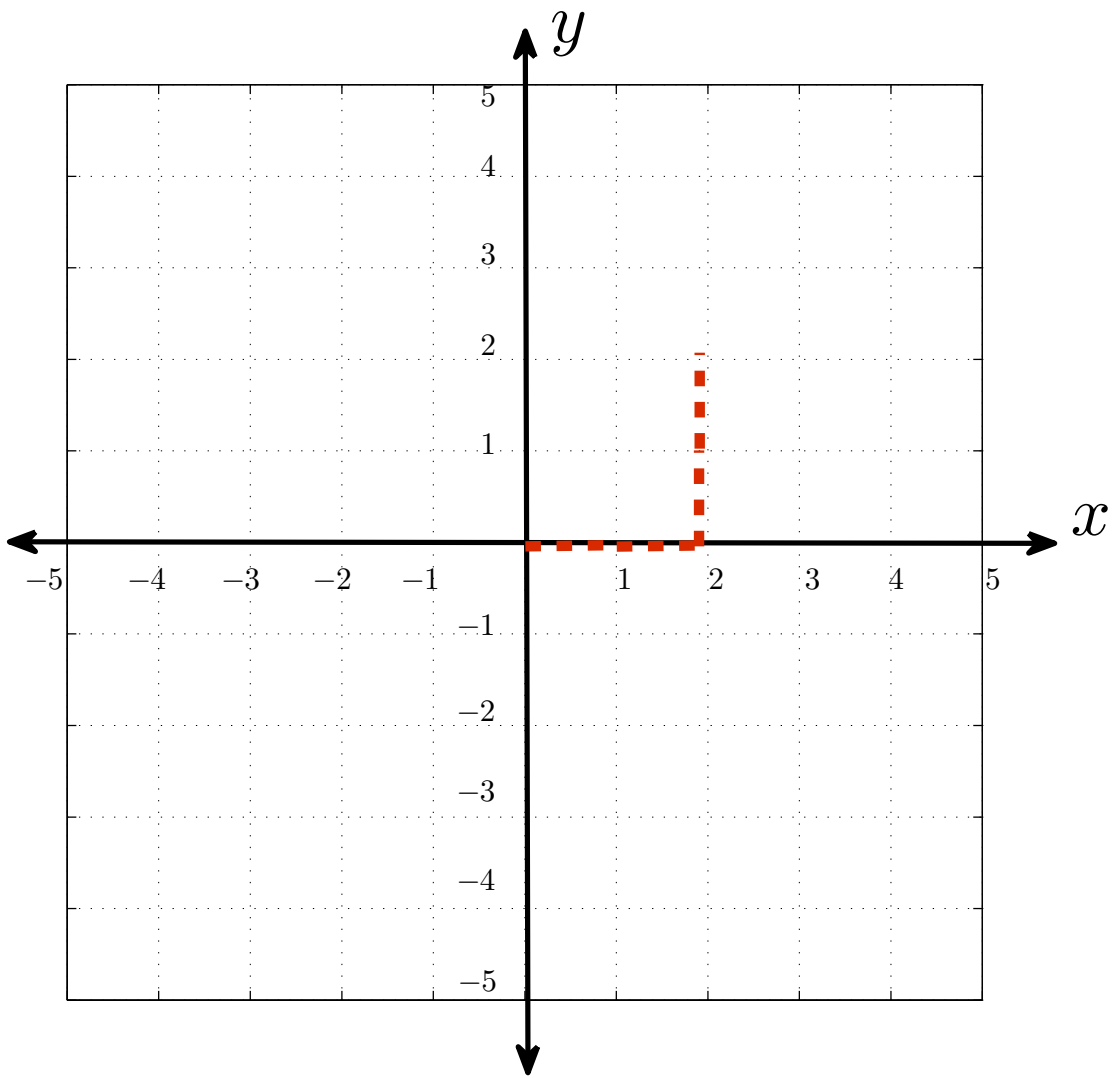

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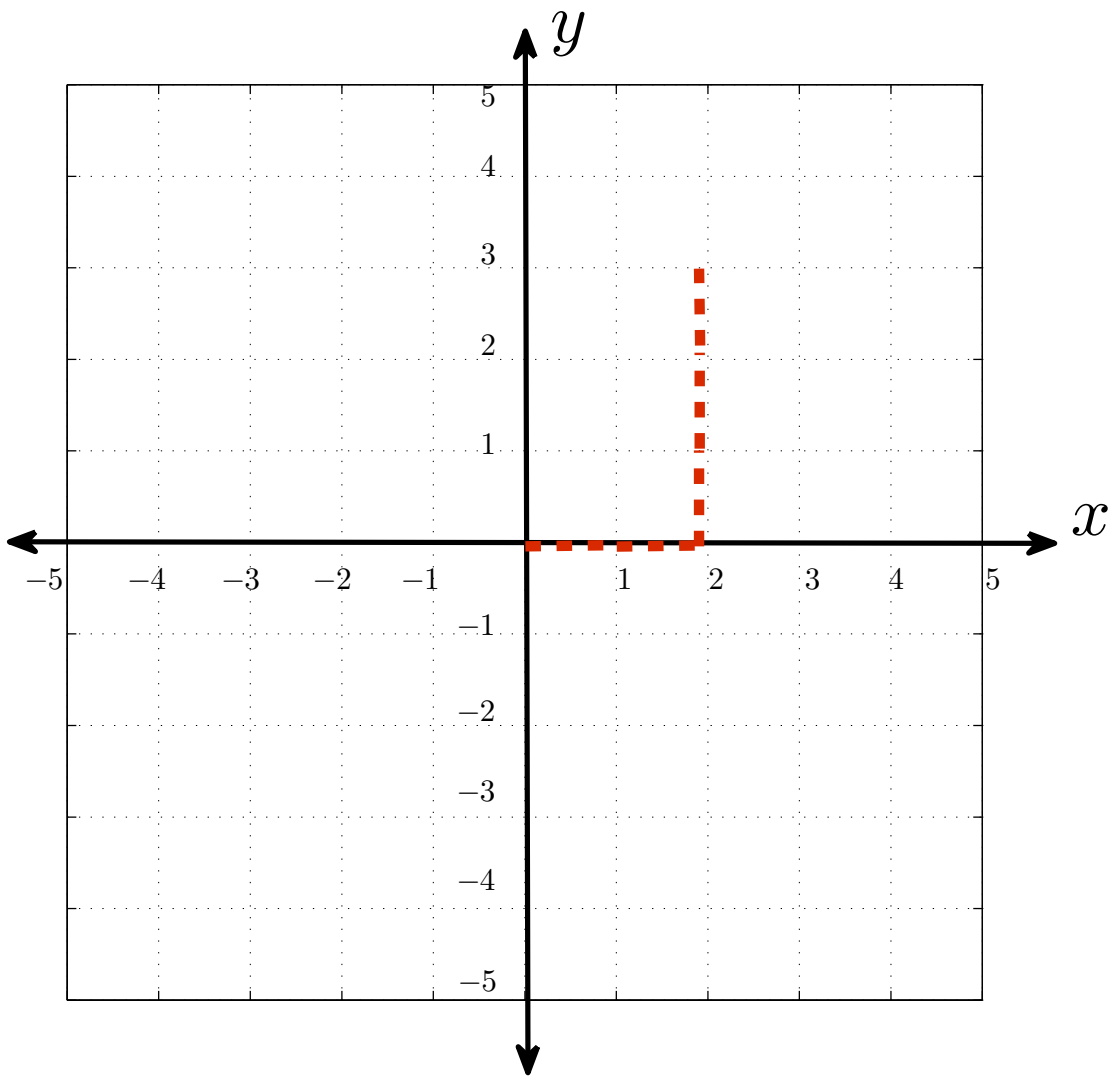

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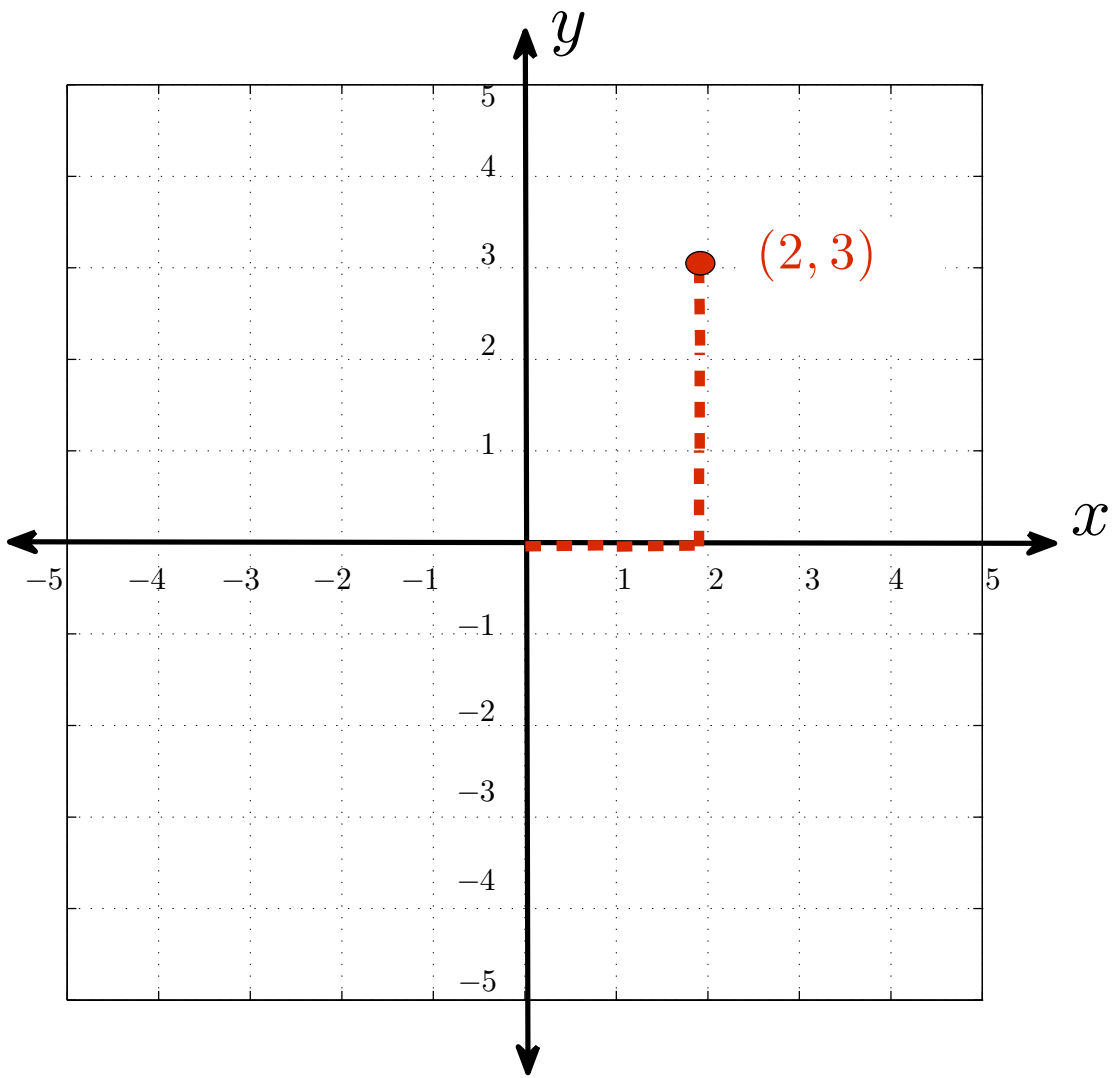

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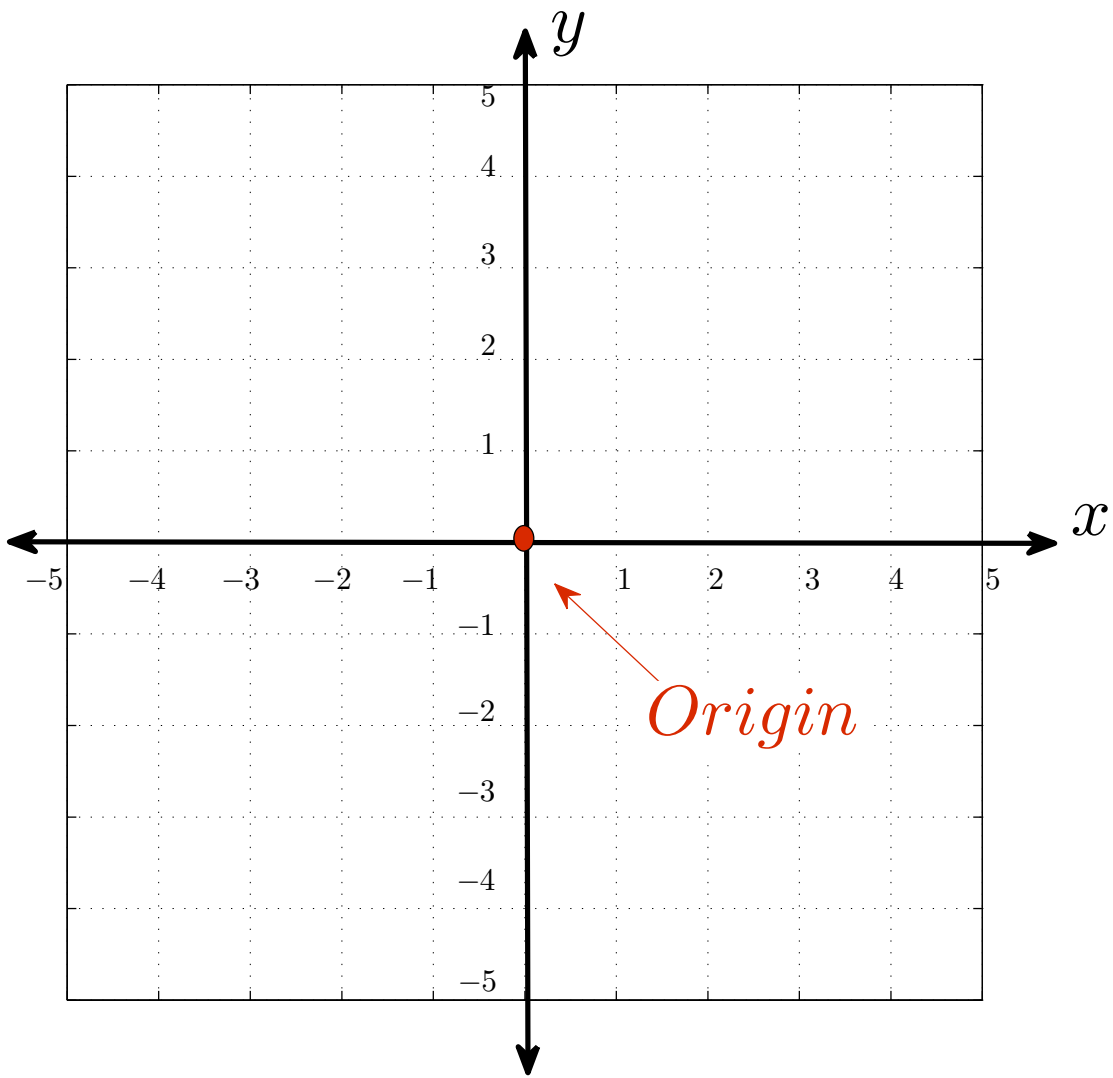

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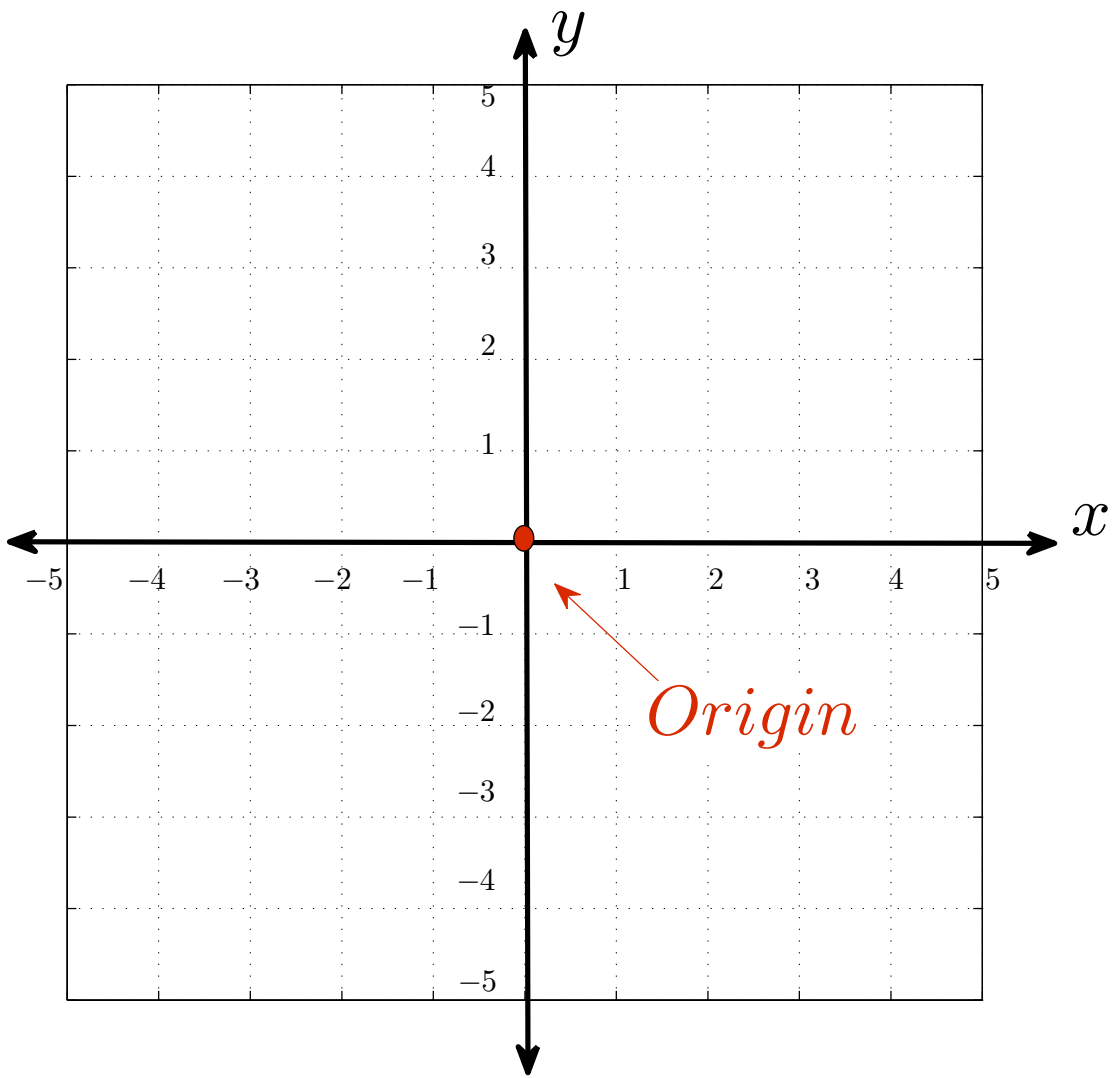

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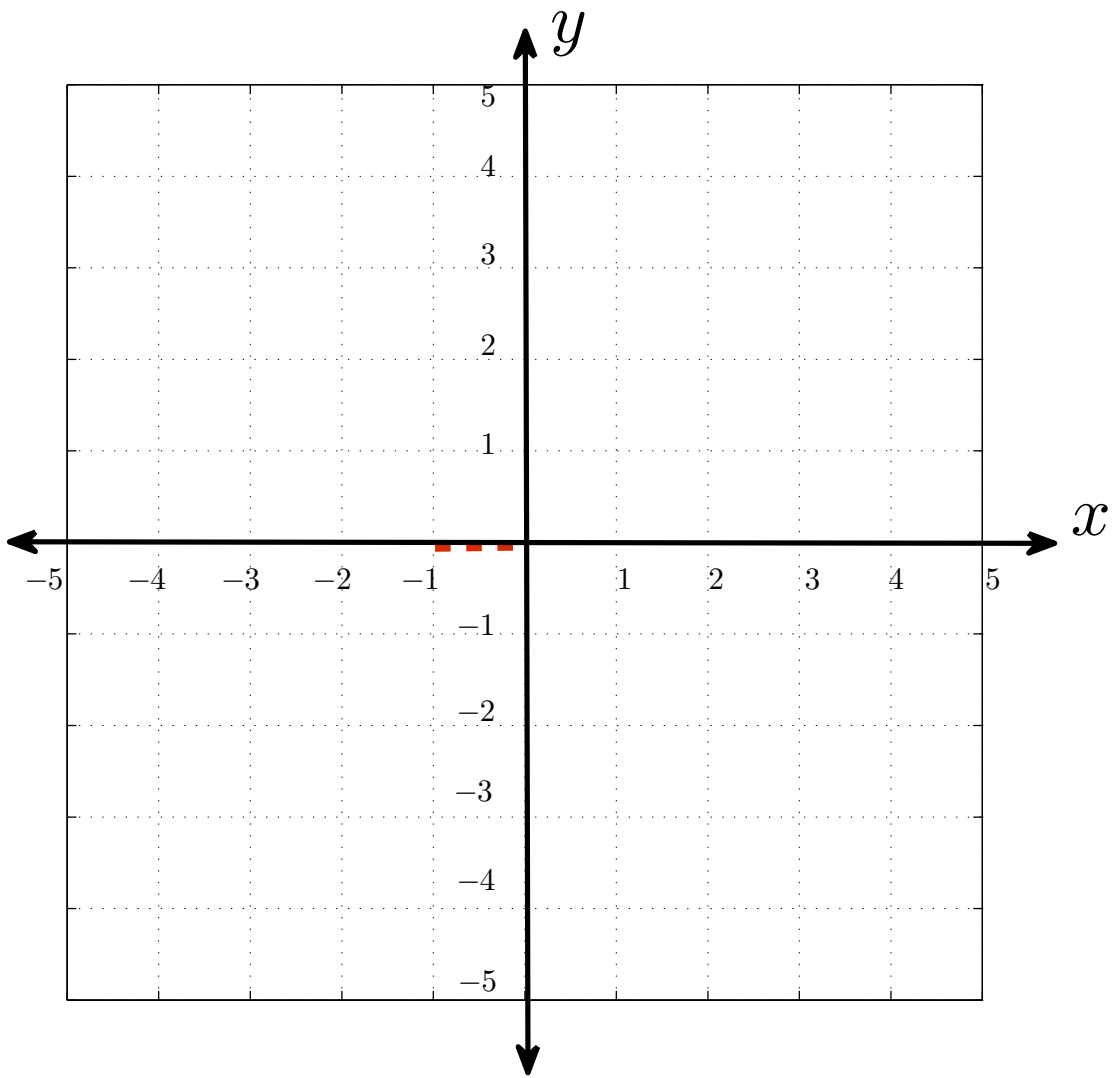

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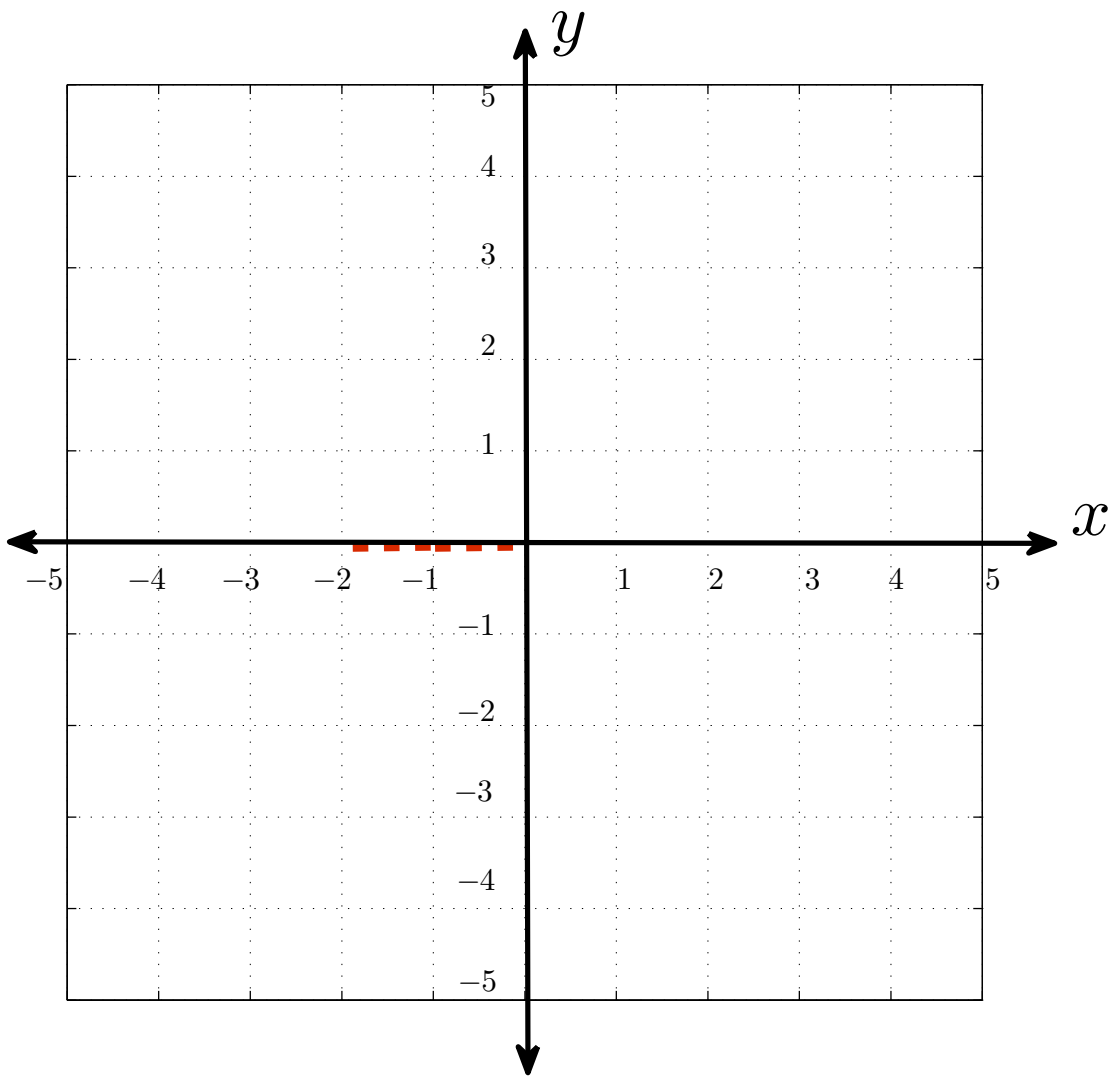

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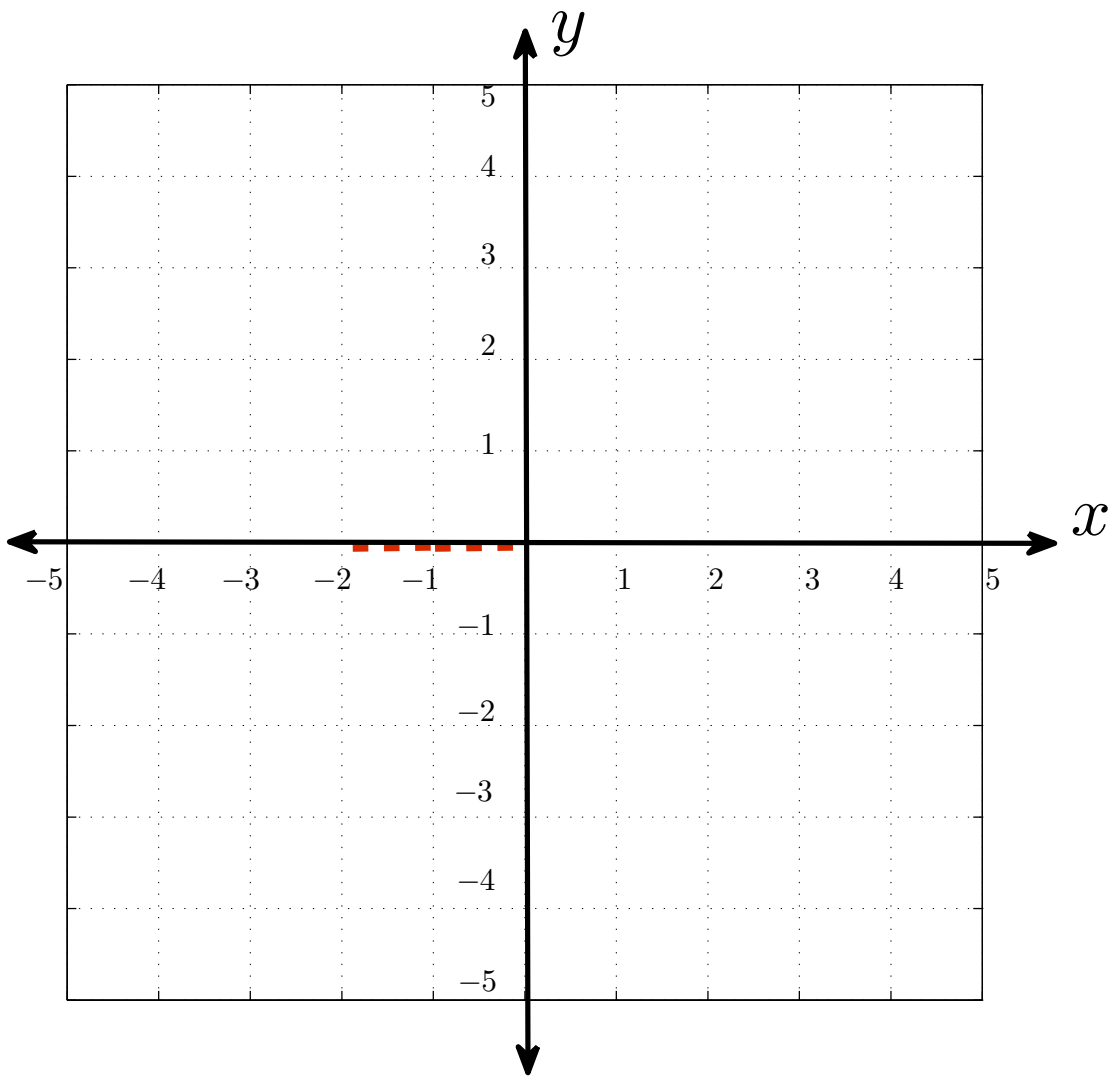

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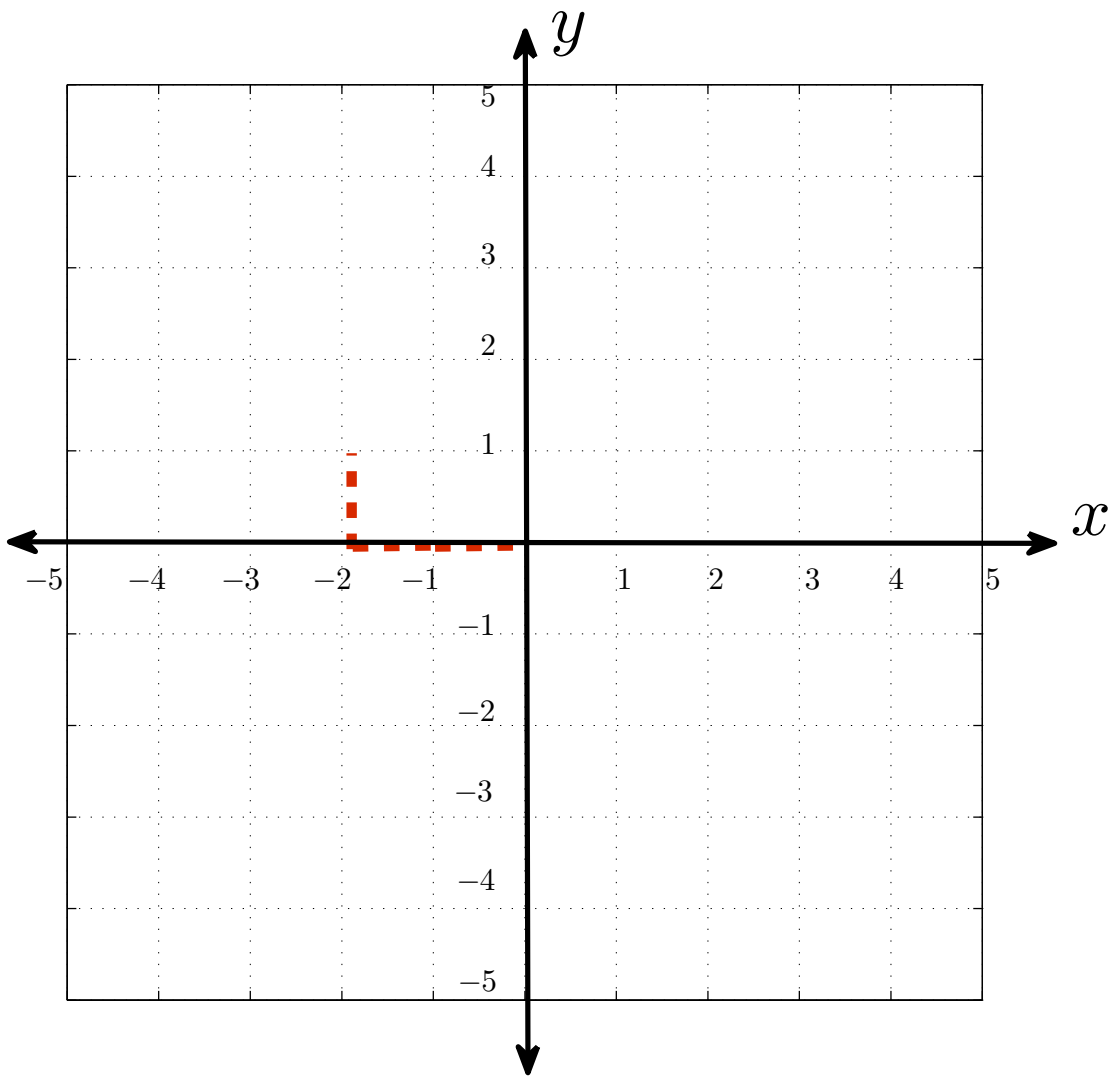

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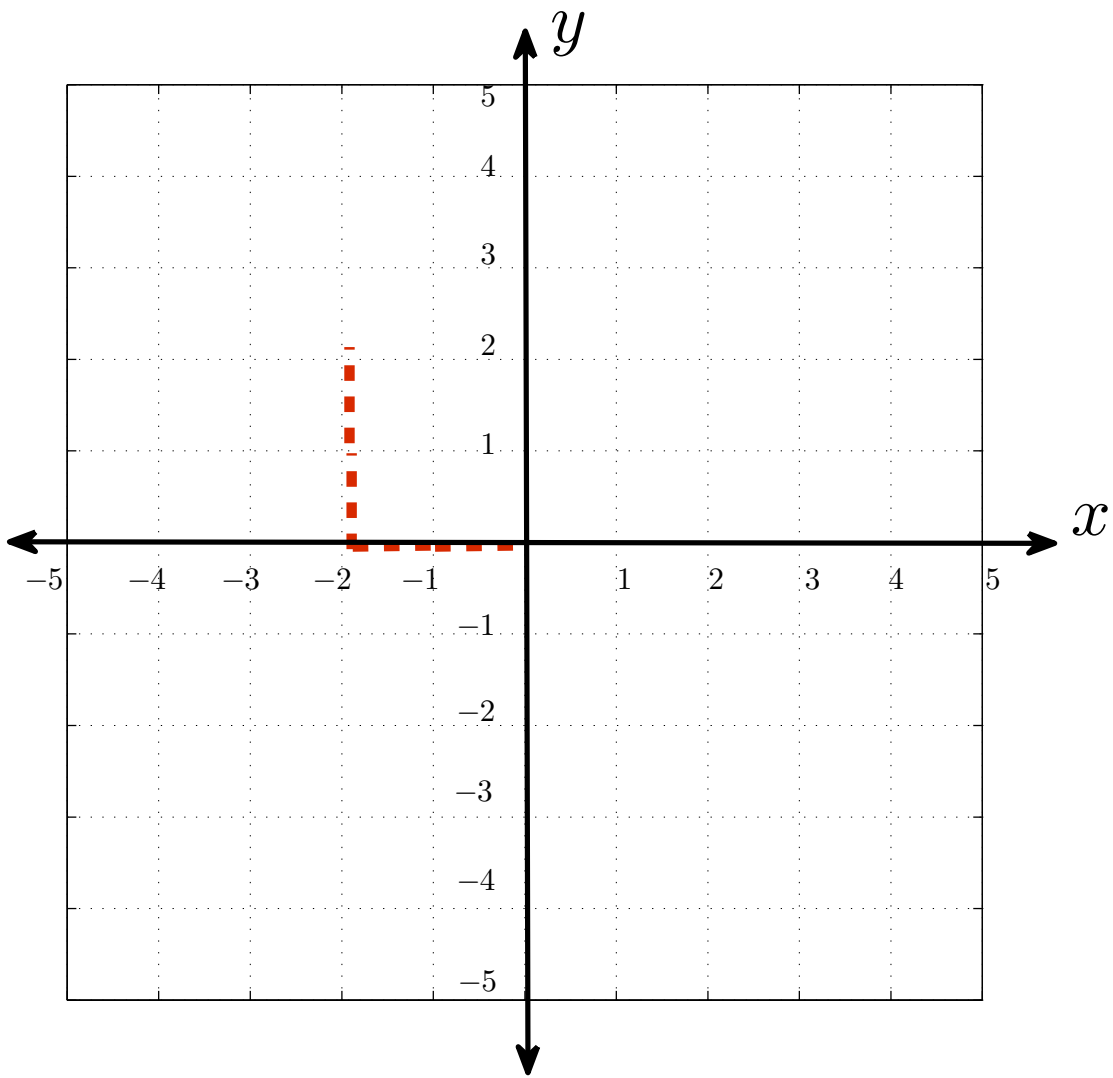

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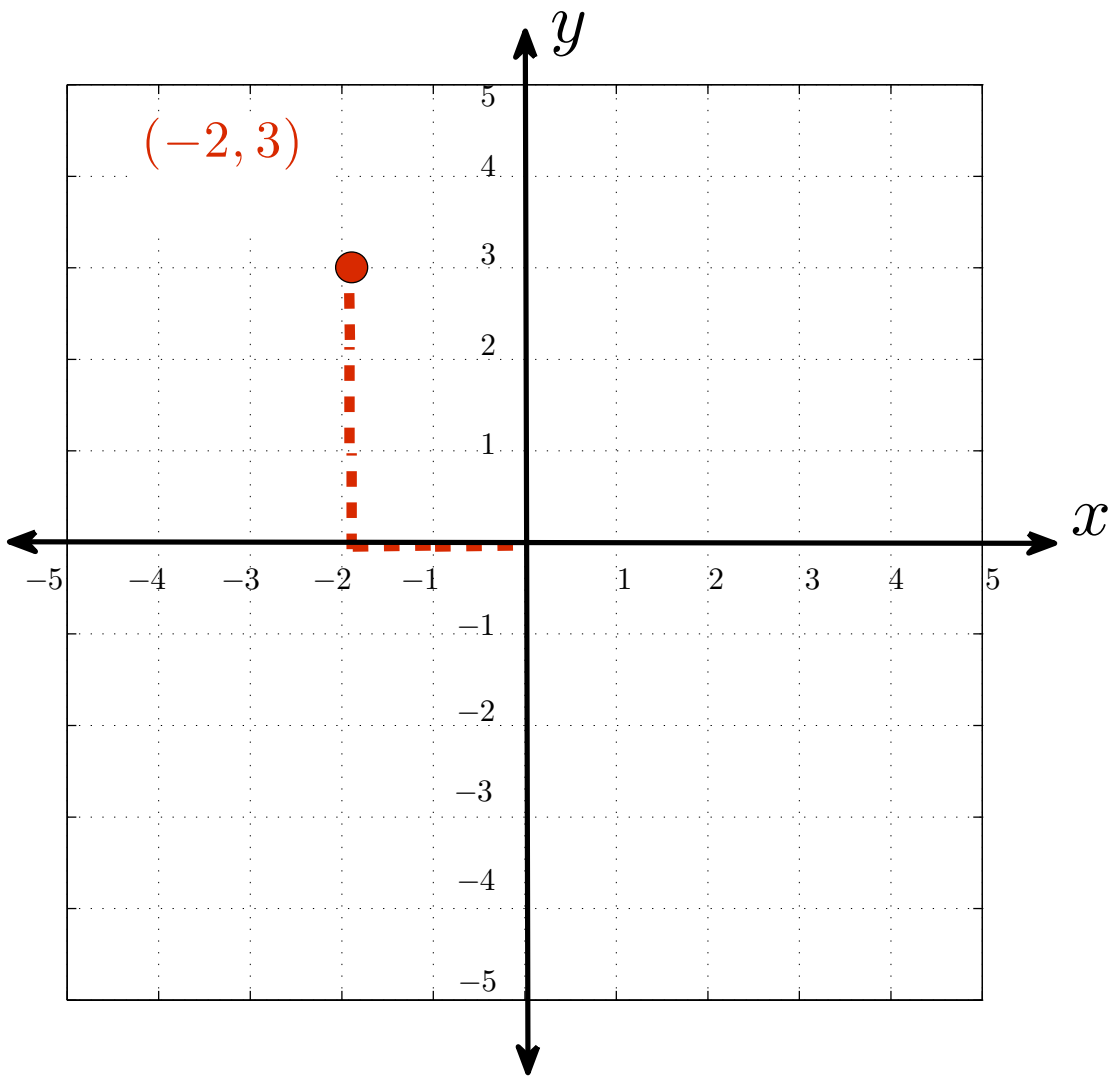

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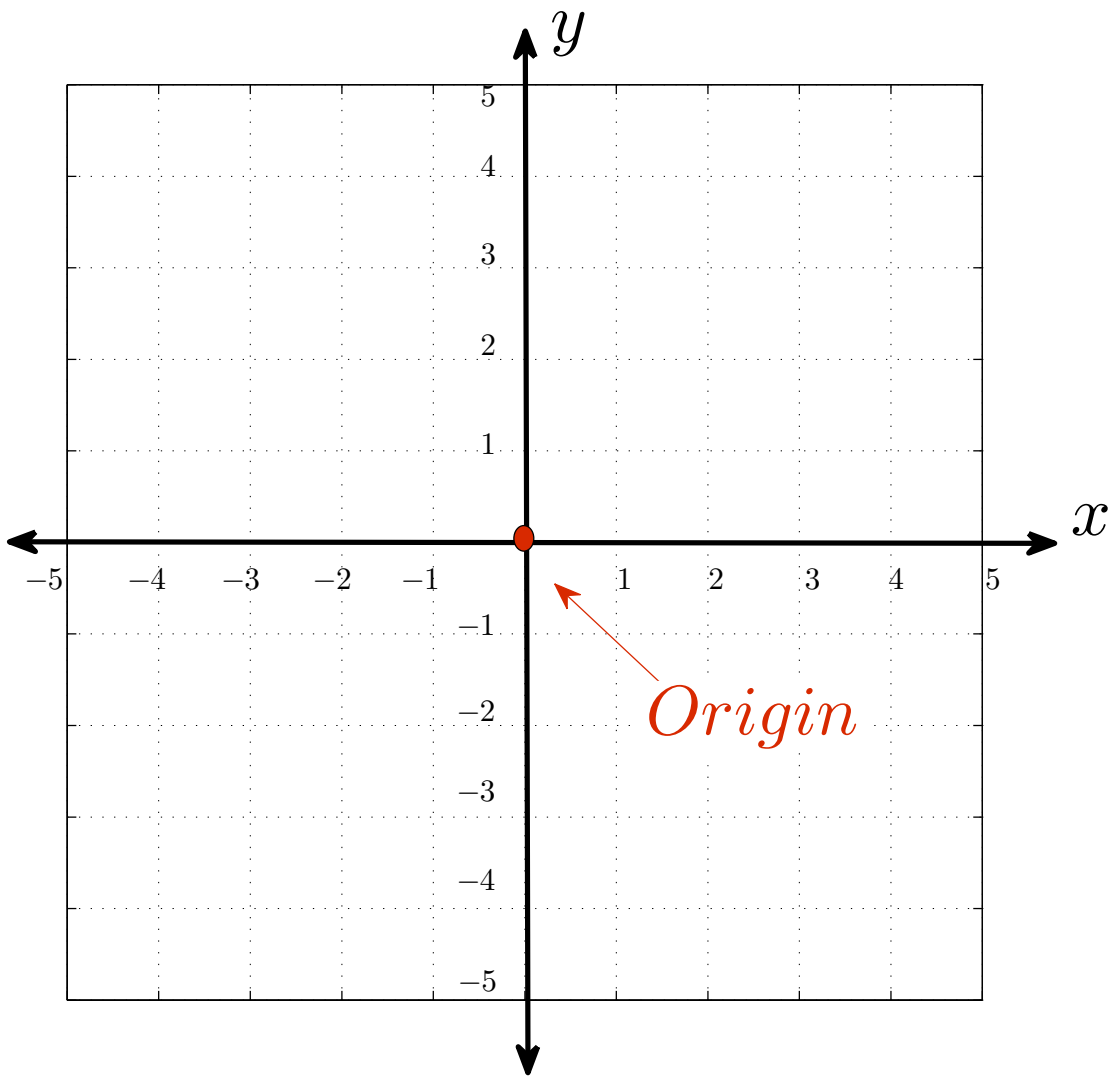

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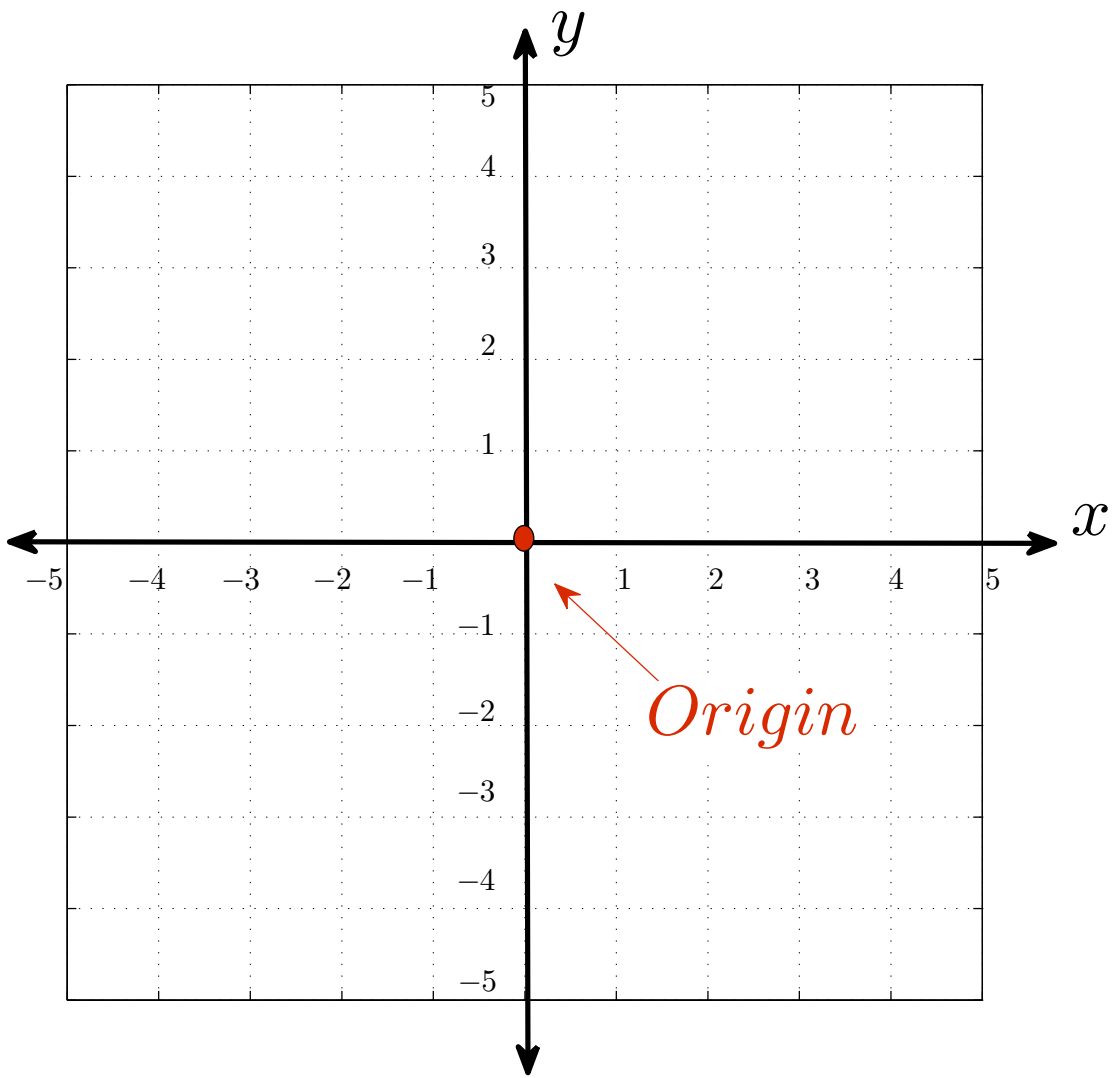

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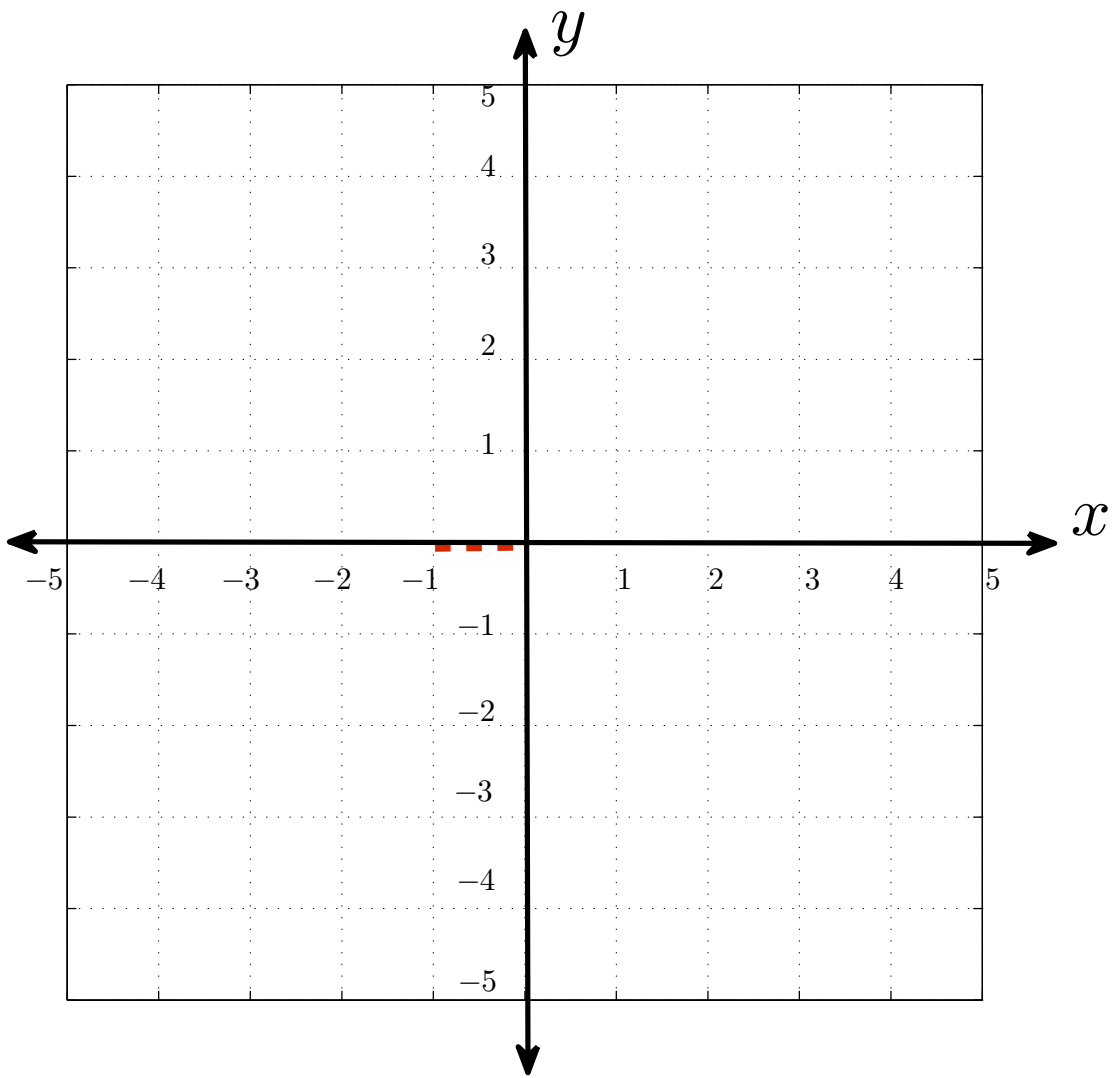

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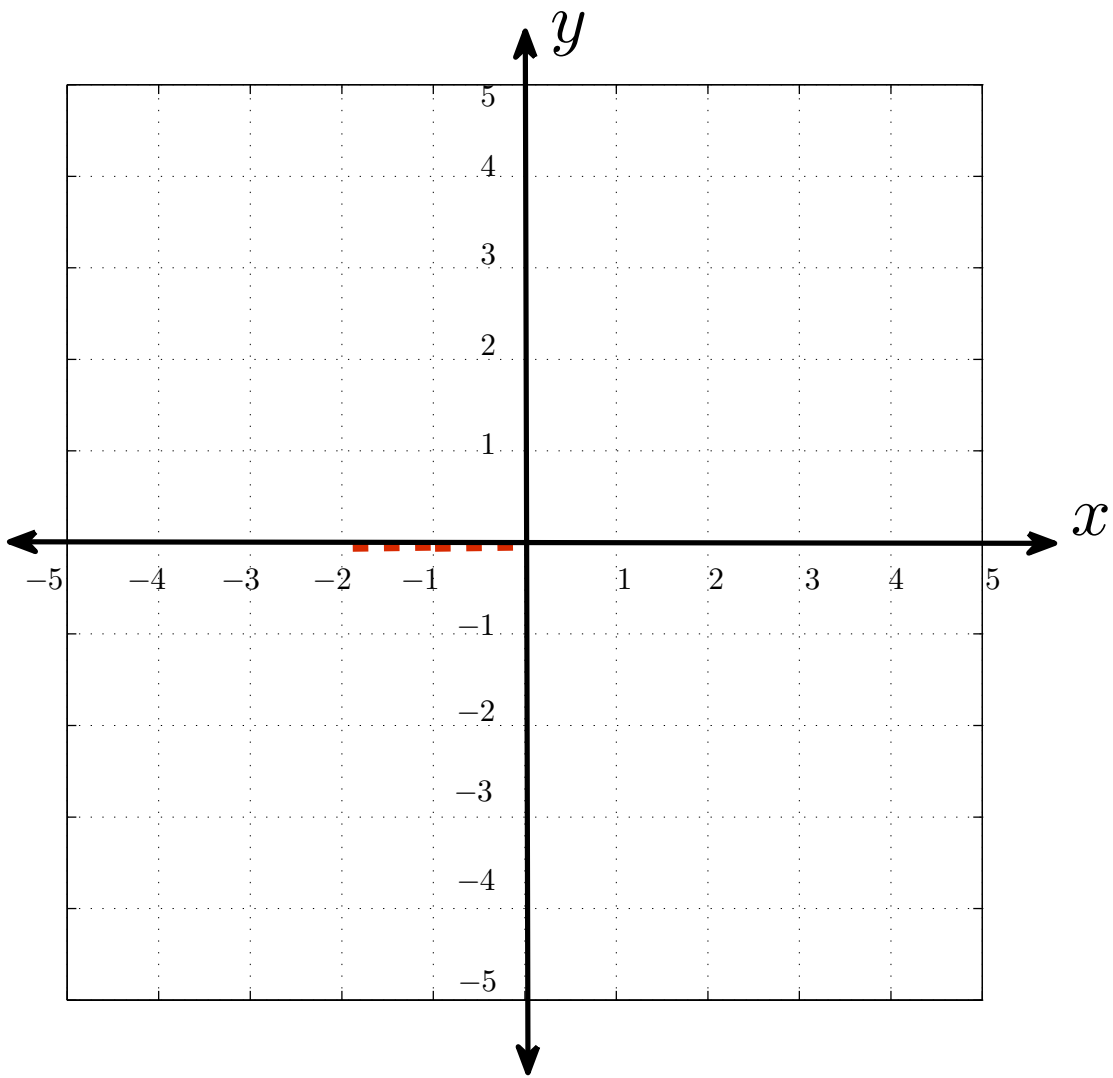

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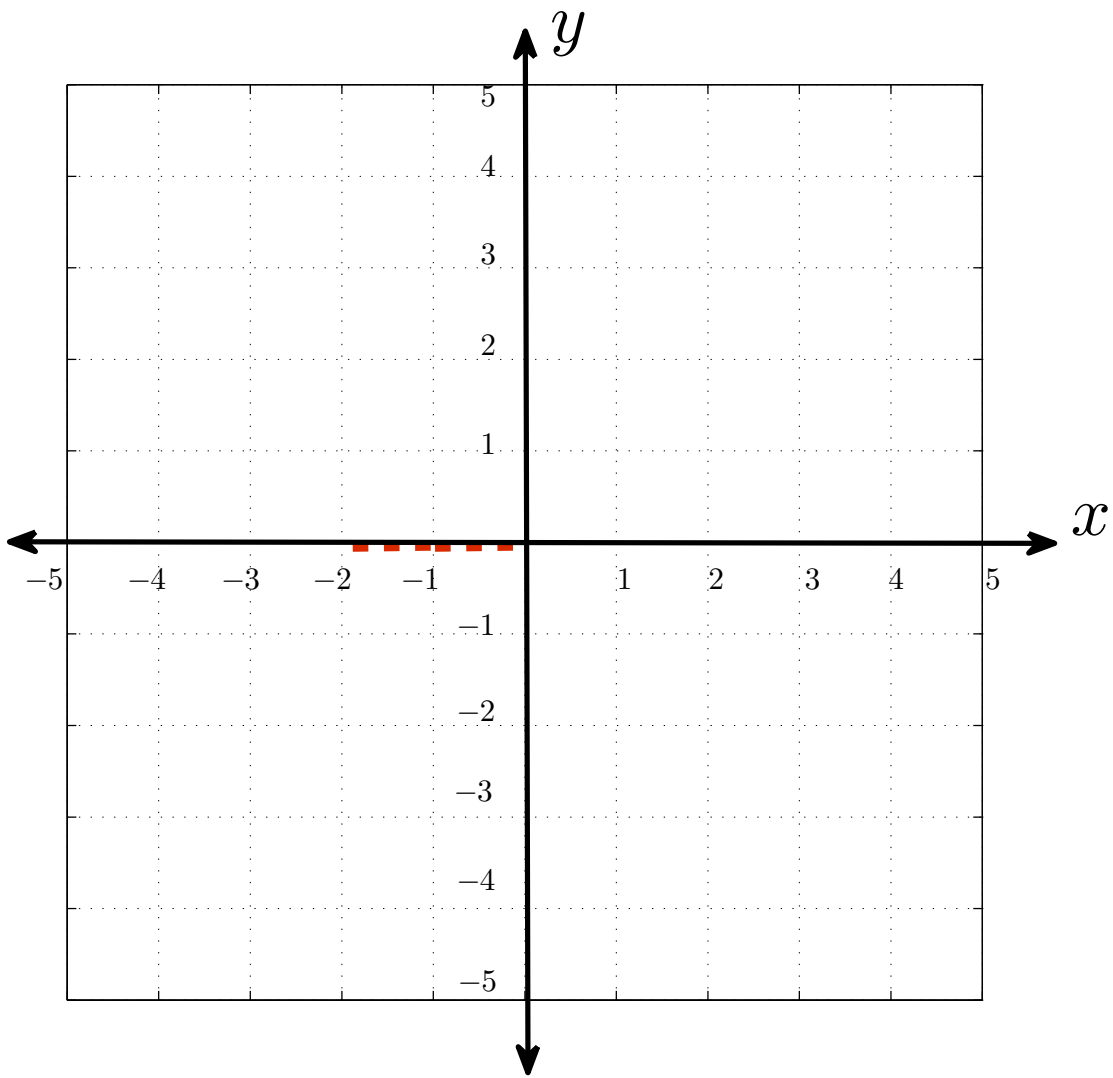

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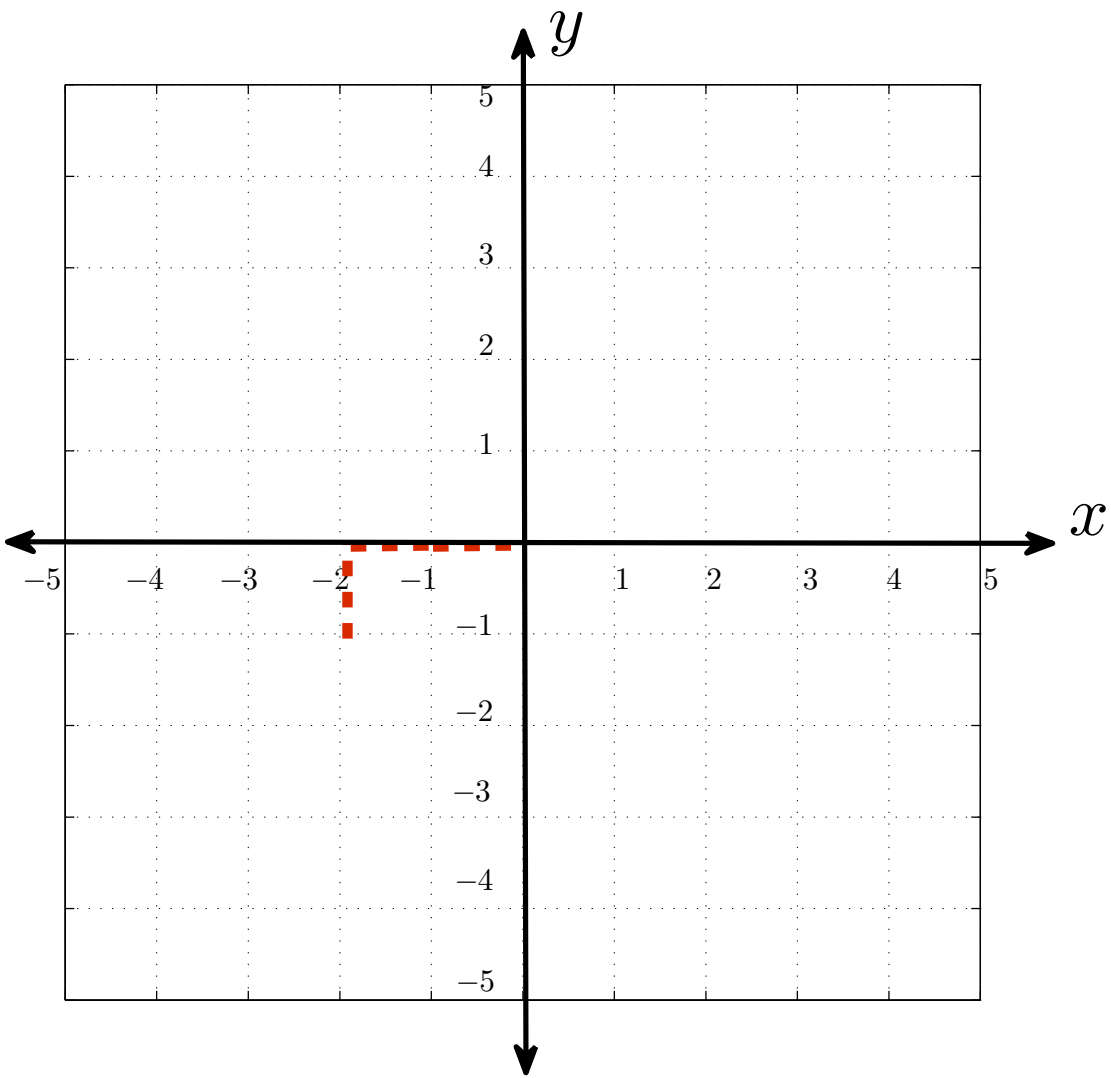

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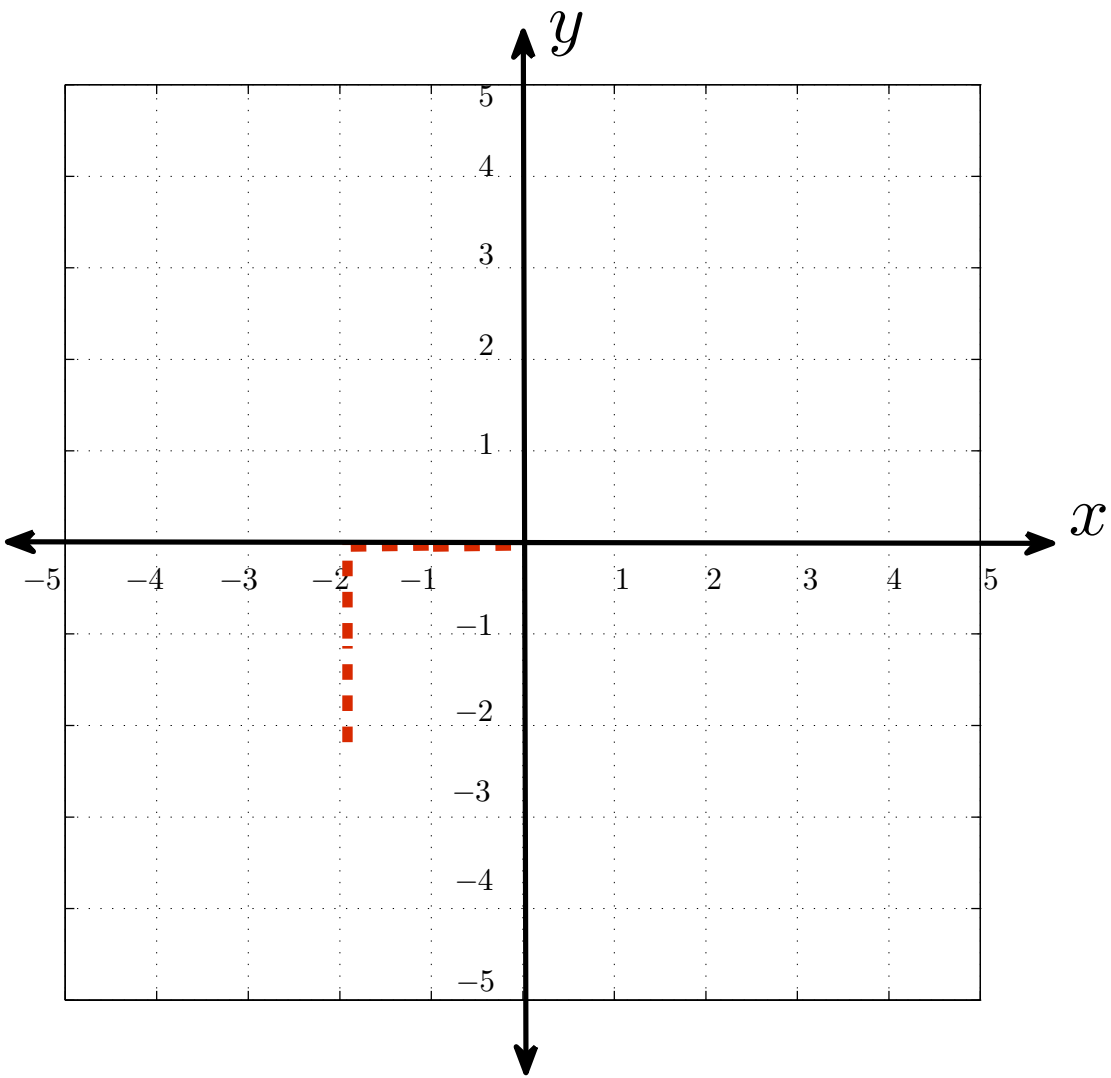

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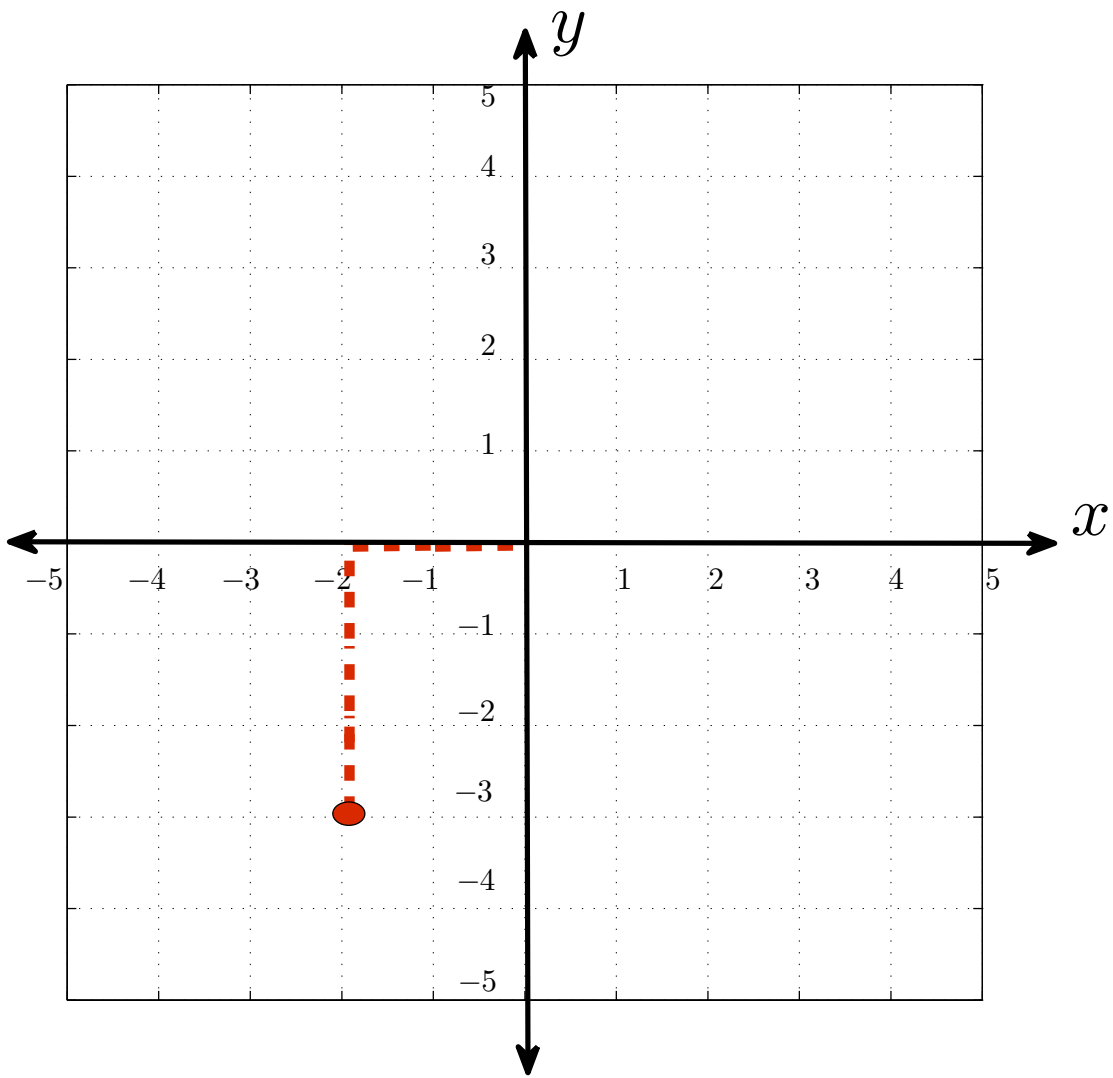

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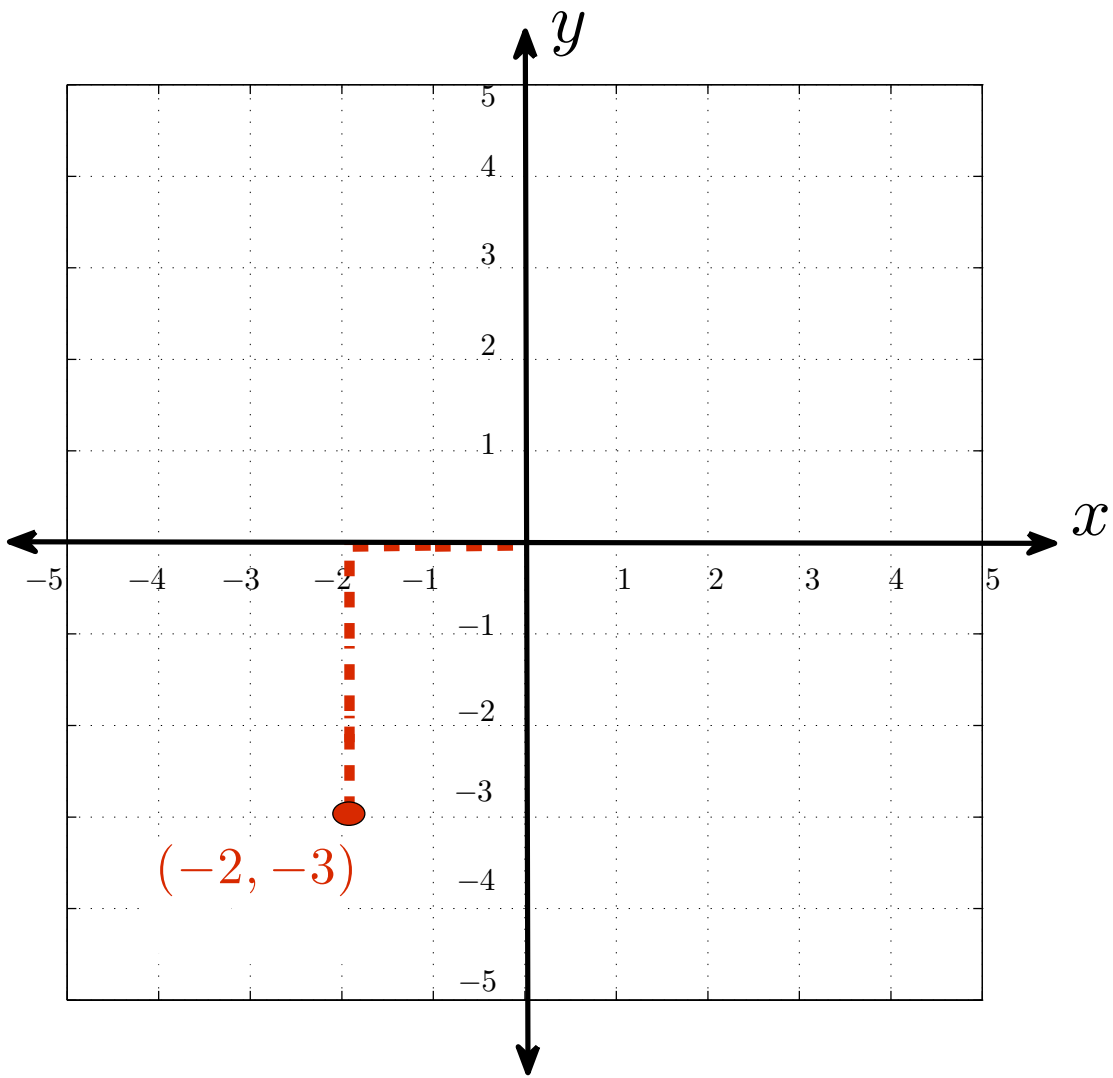

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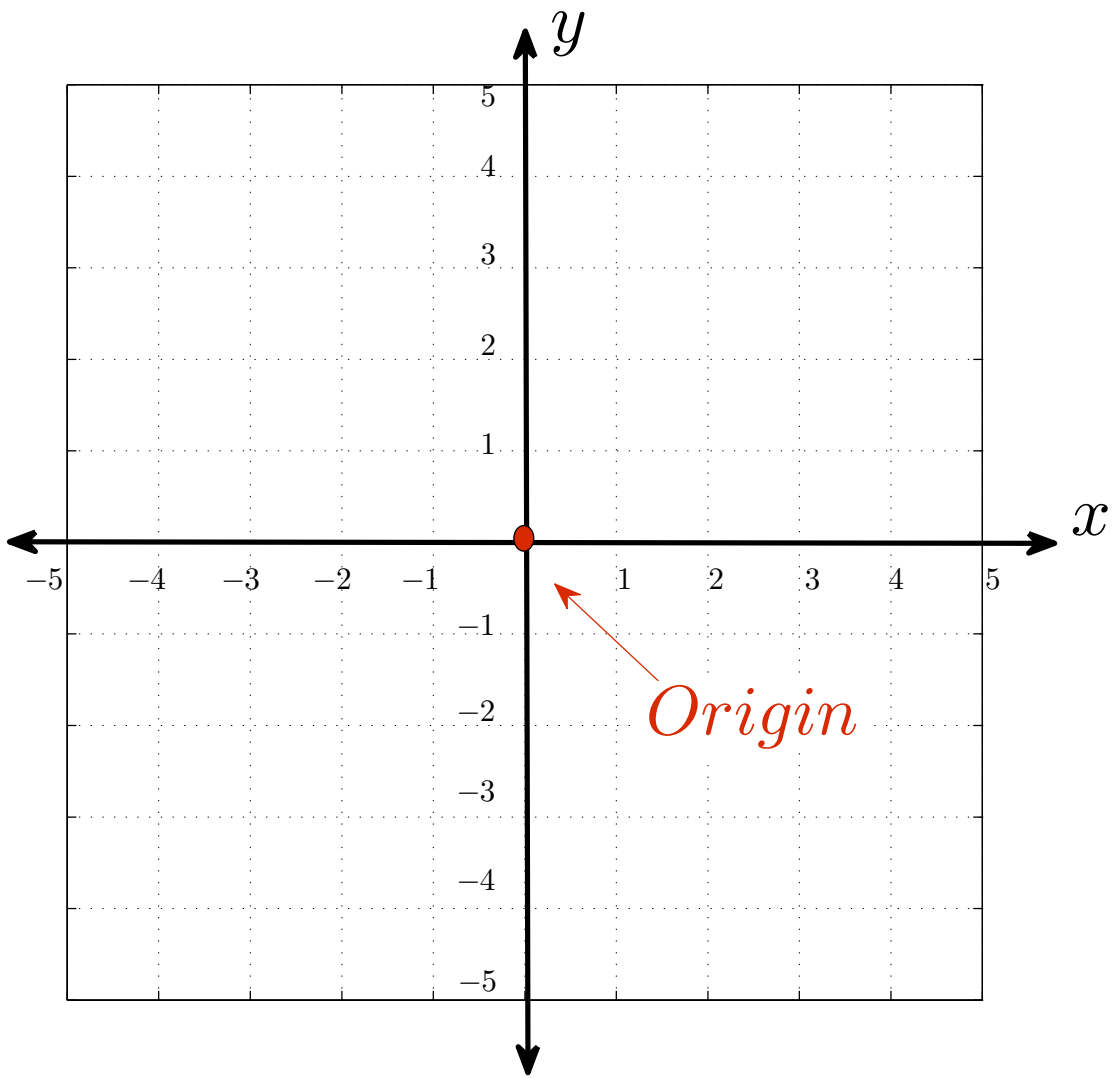

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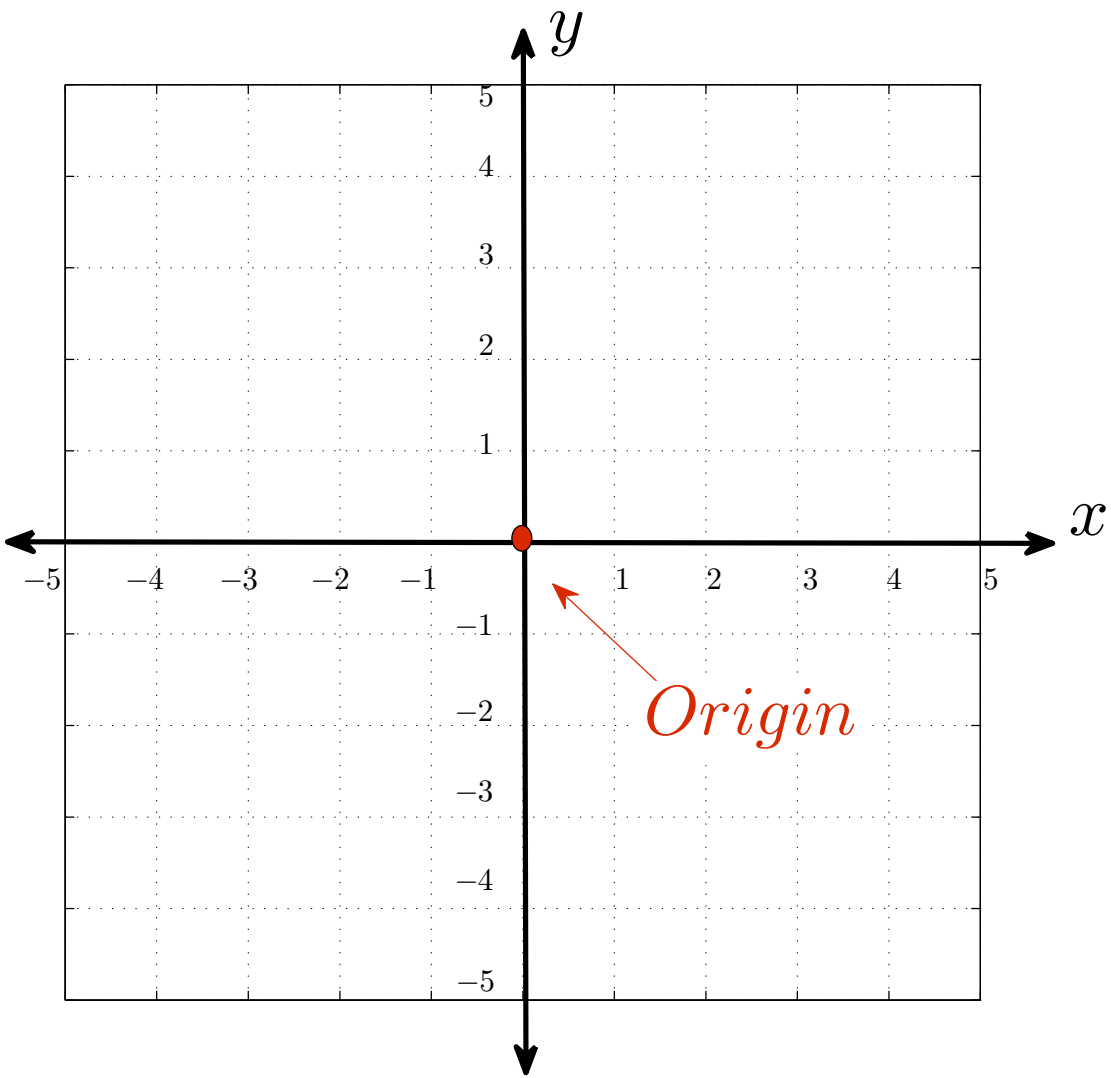

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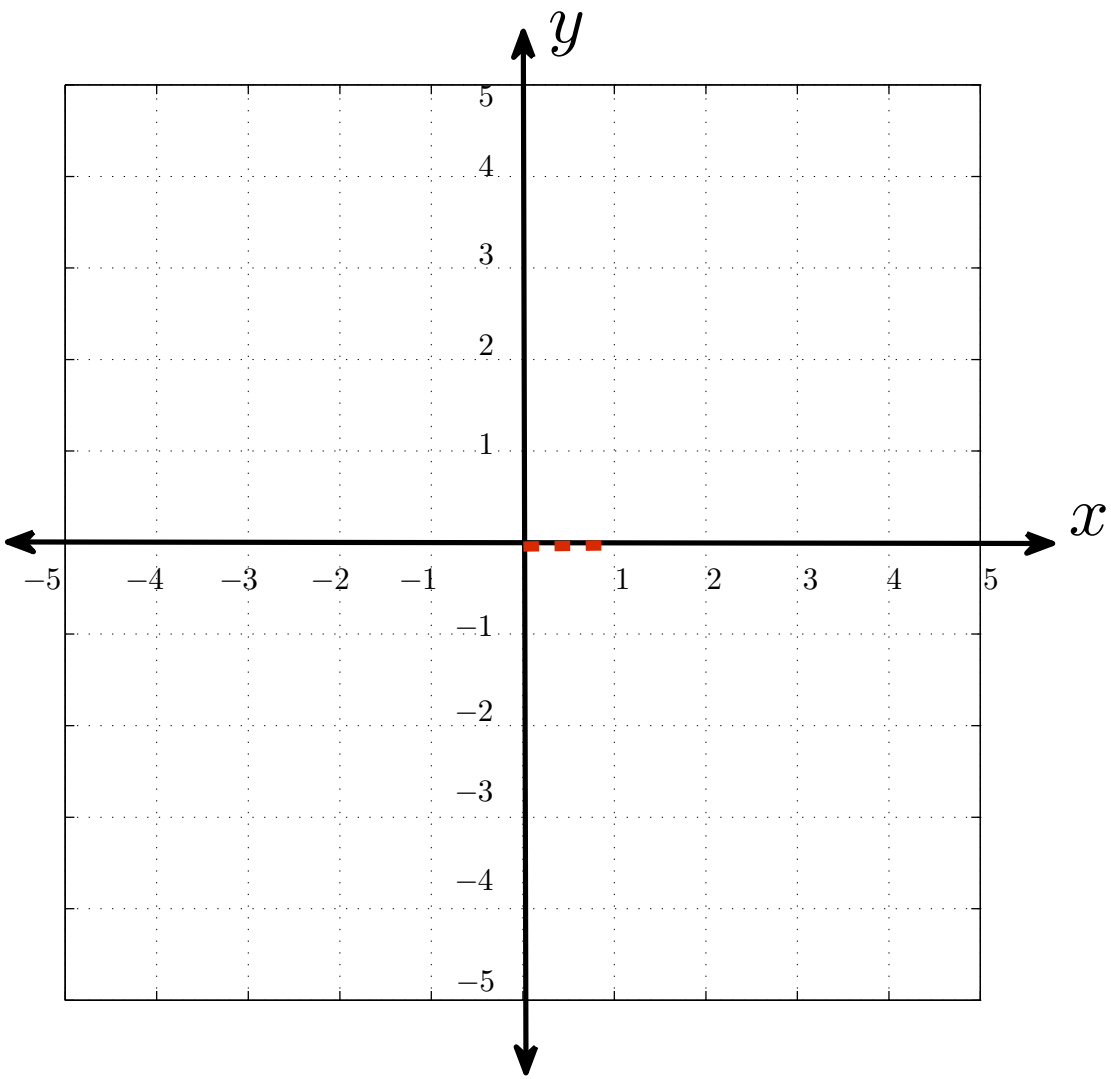

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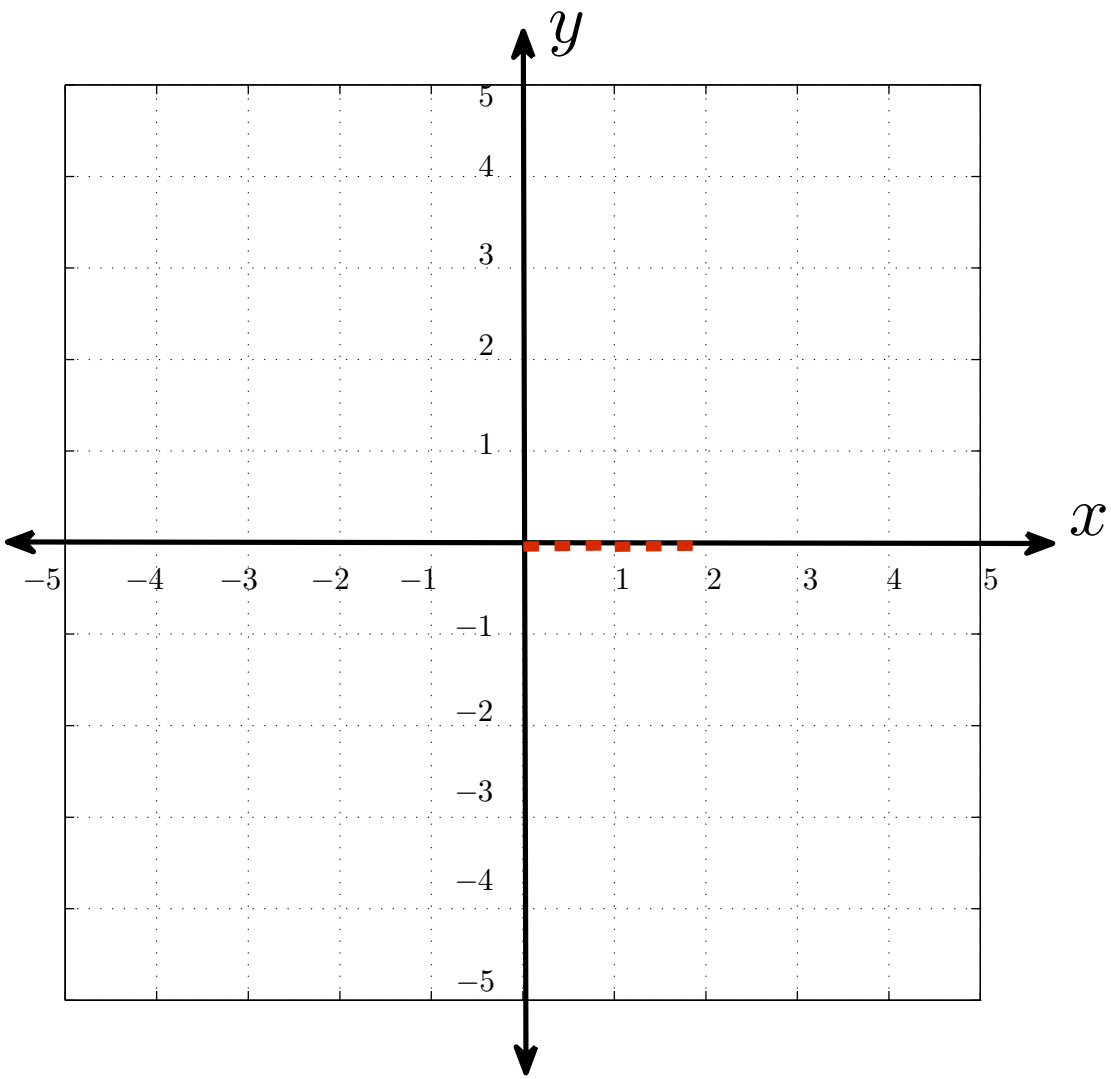

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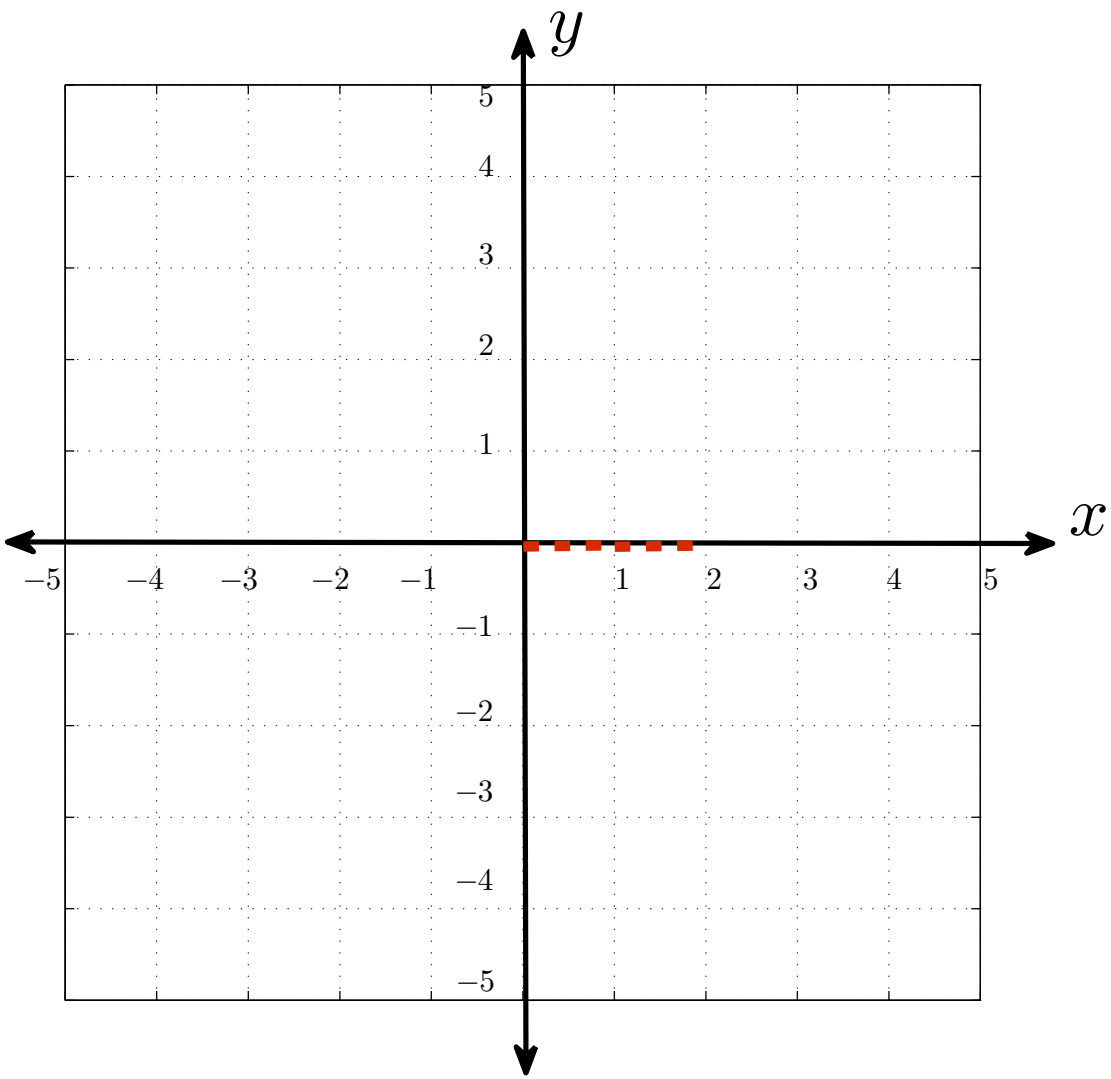

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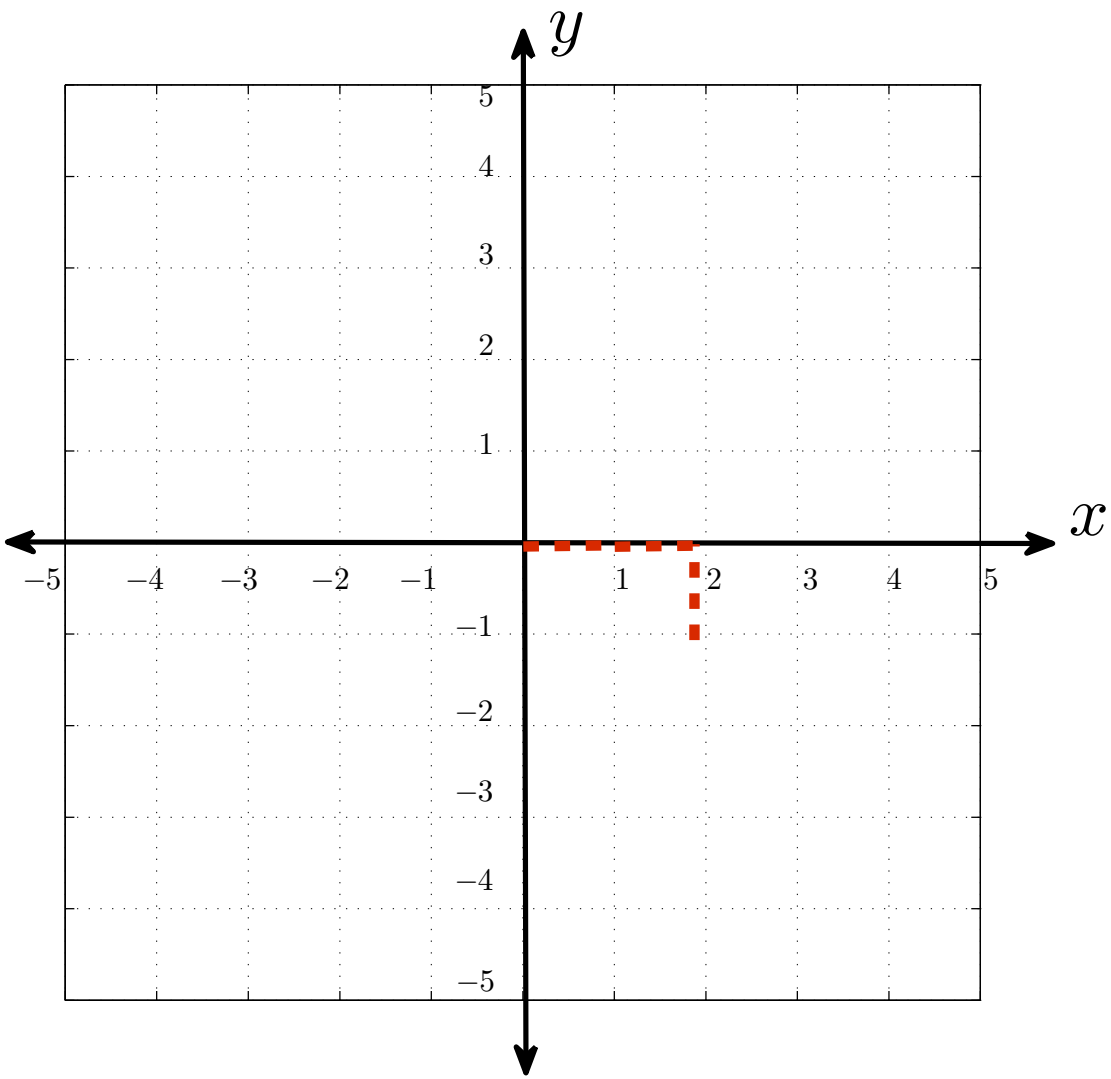

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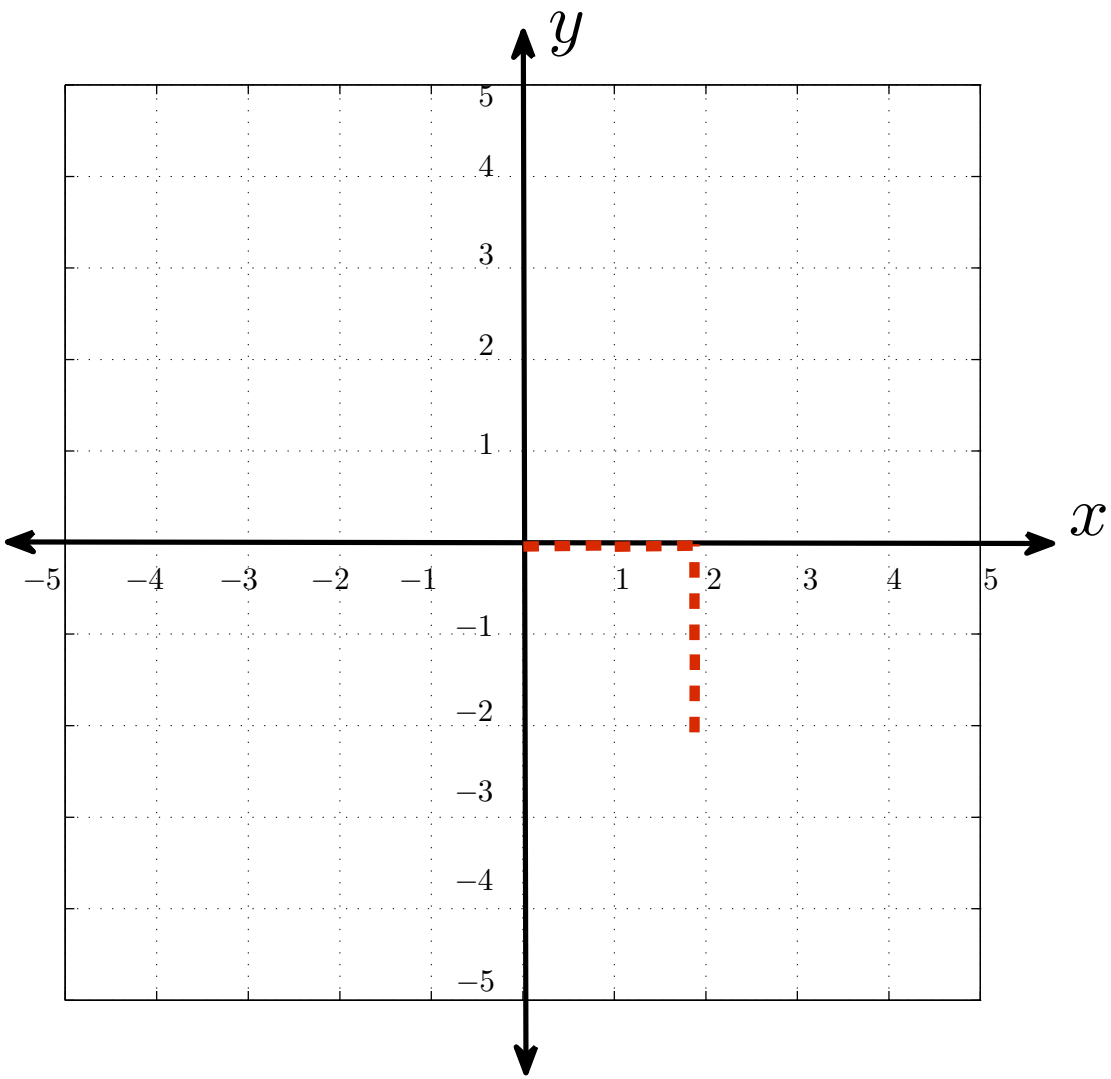

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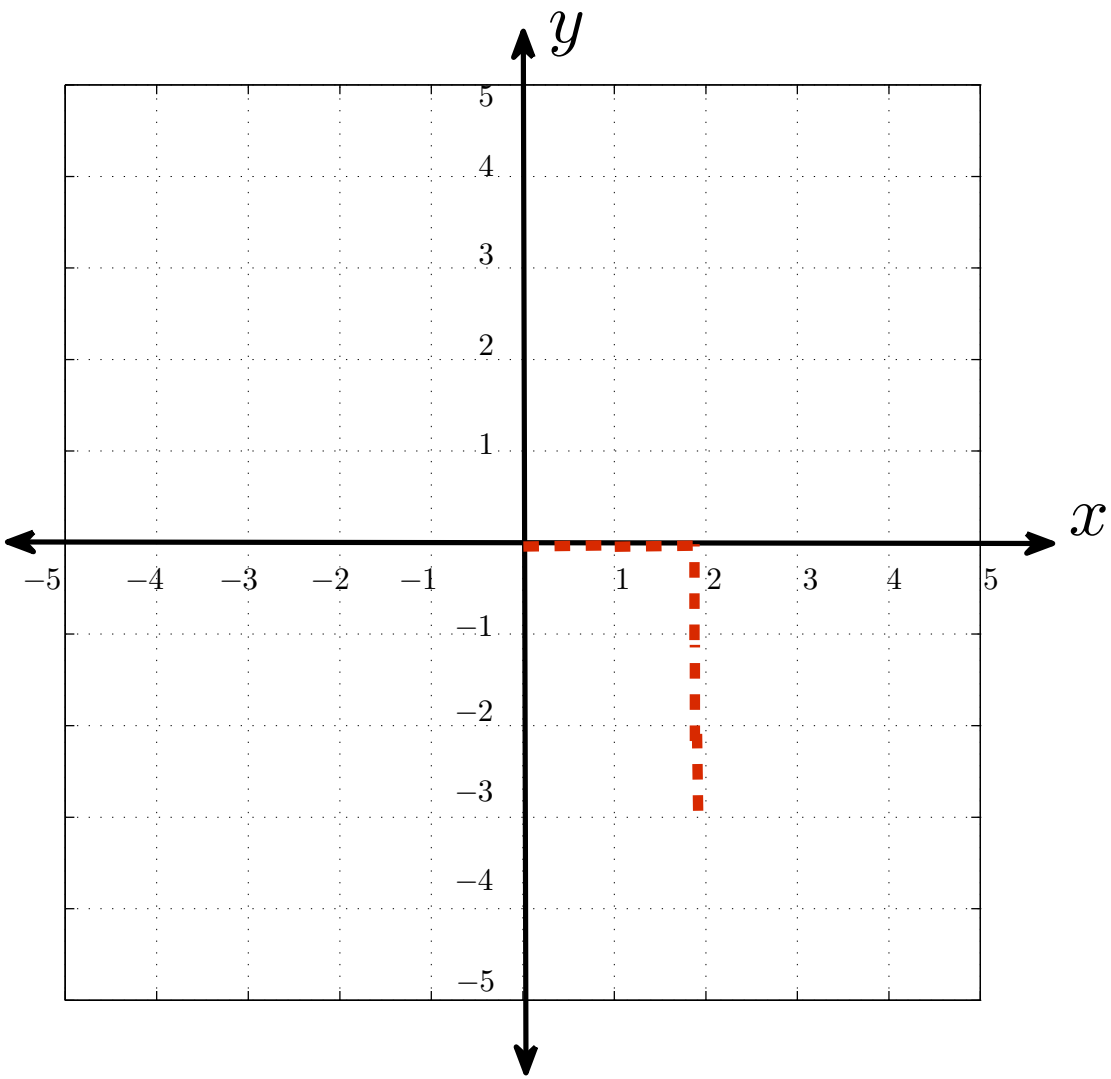

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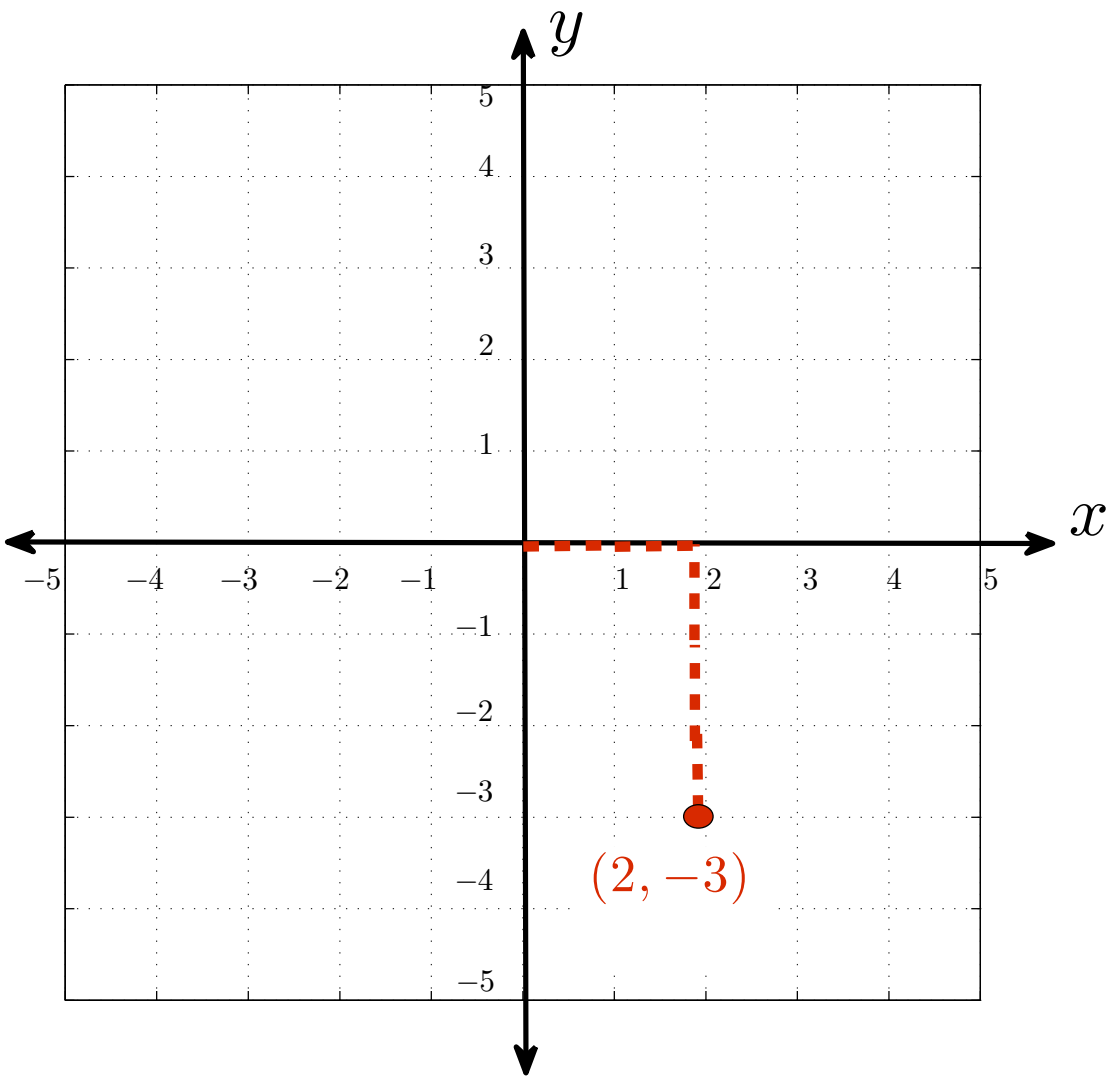
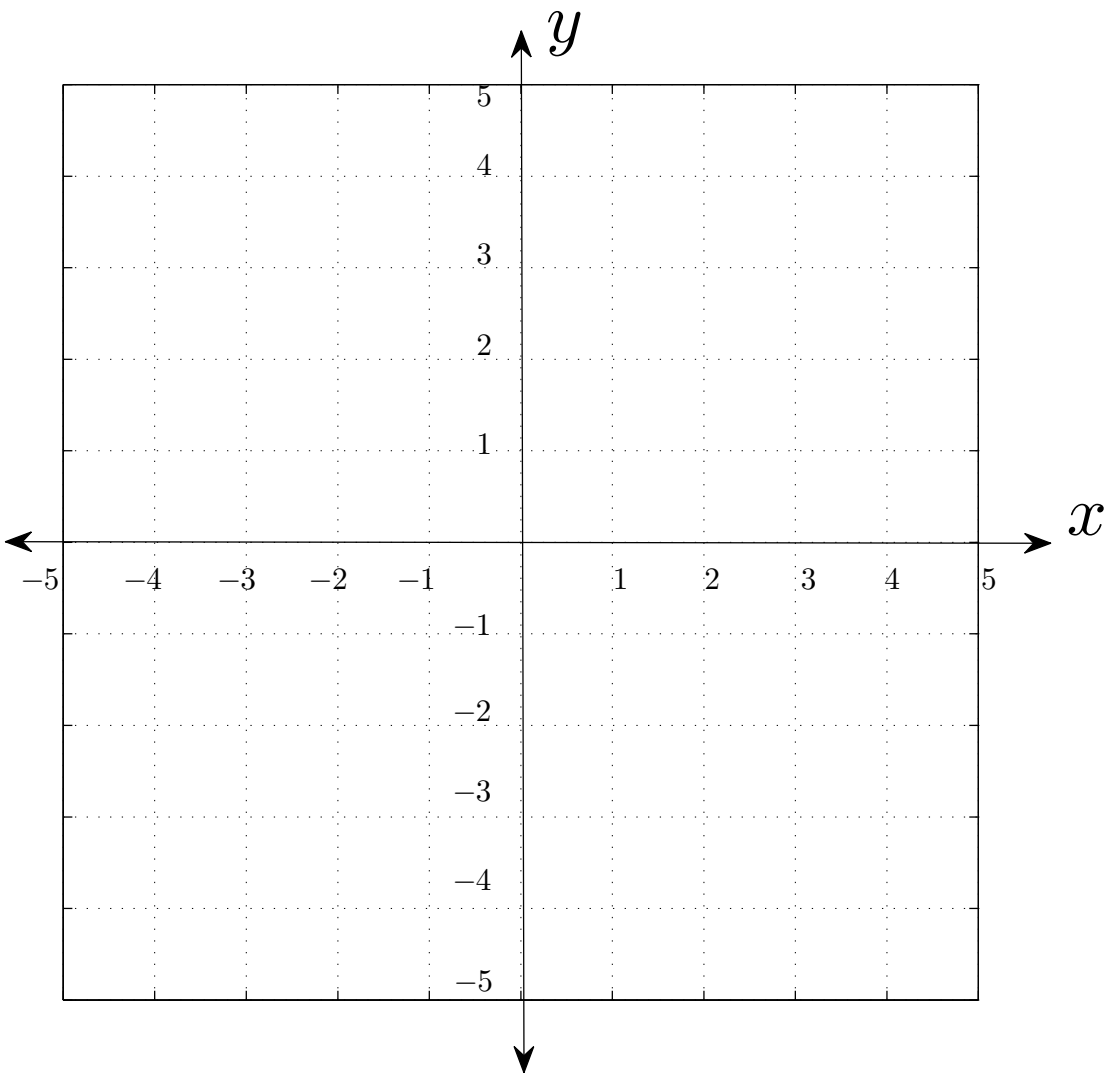
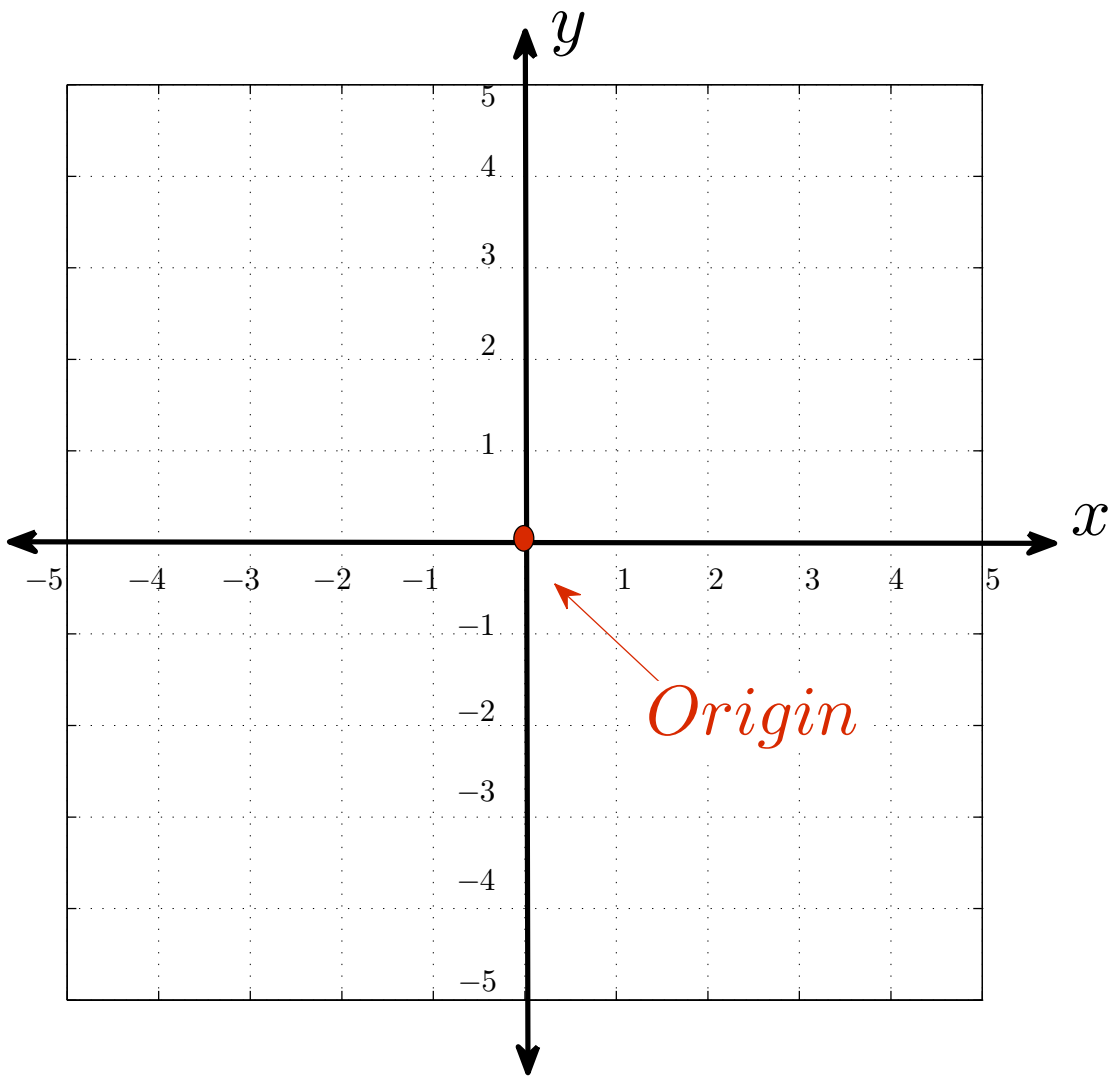


Graphing Ordered Pairs

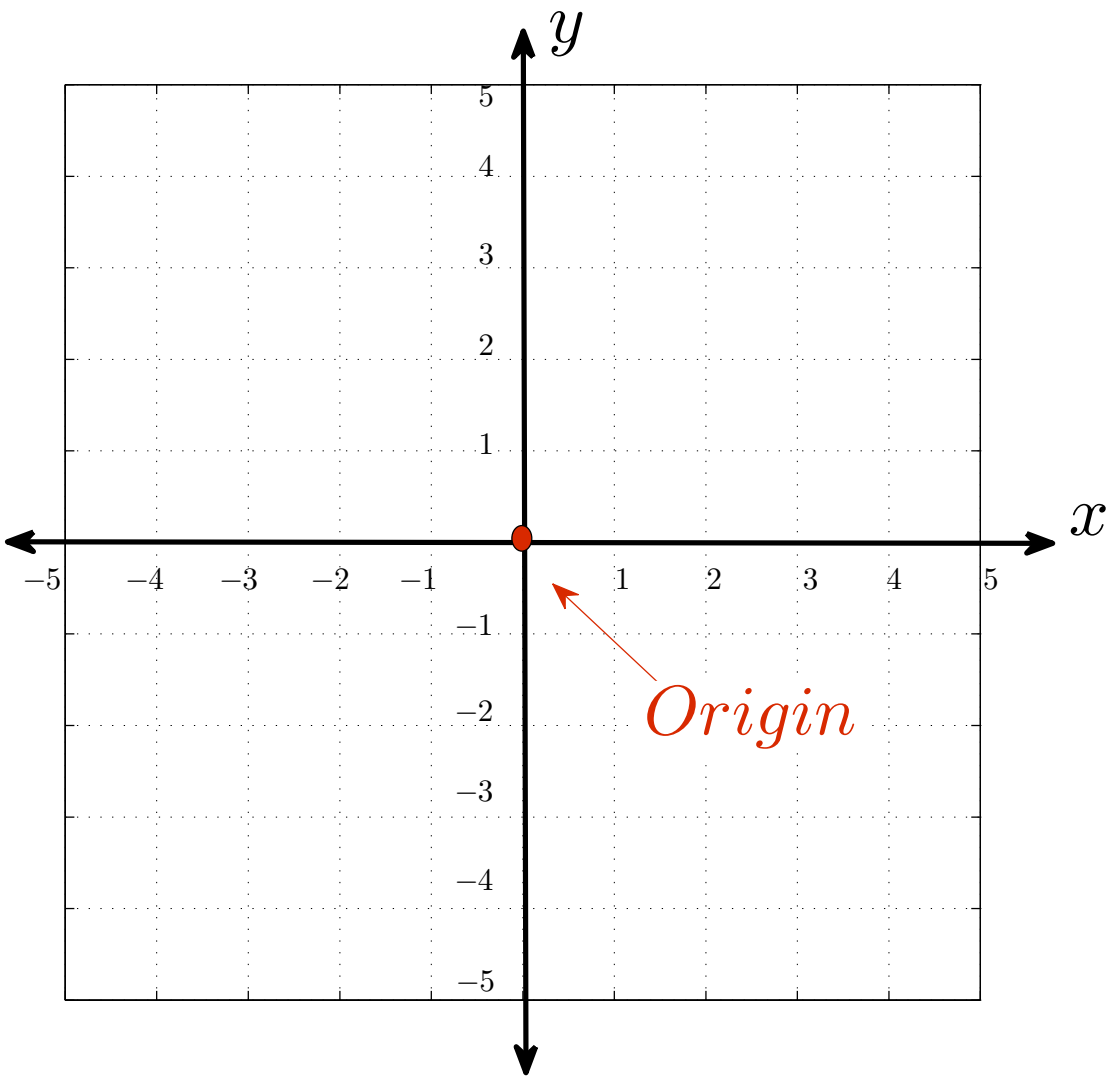
Example 2: Plot (graph) the following ordered pairs: $(3,0),\ (0,2),\ (-3,0),\ (0,-2),$



To plot $(3,0)$, begin at the origin.



To plot $(3,0)$, begin at the origin. Travel along the x-axis 3 units right, in the positive direction.



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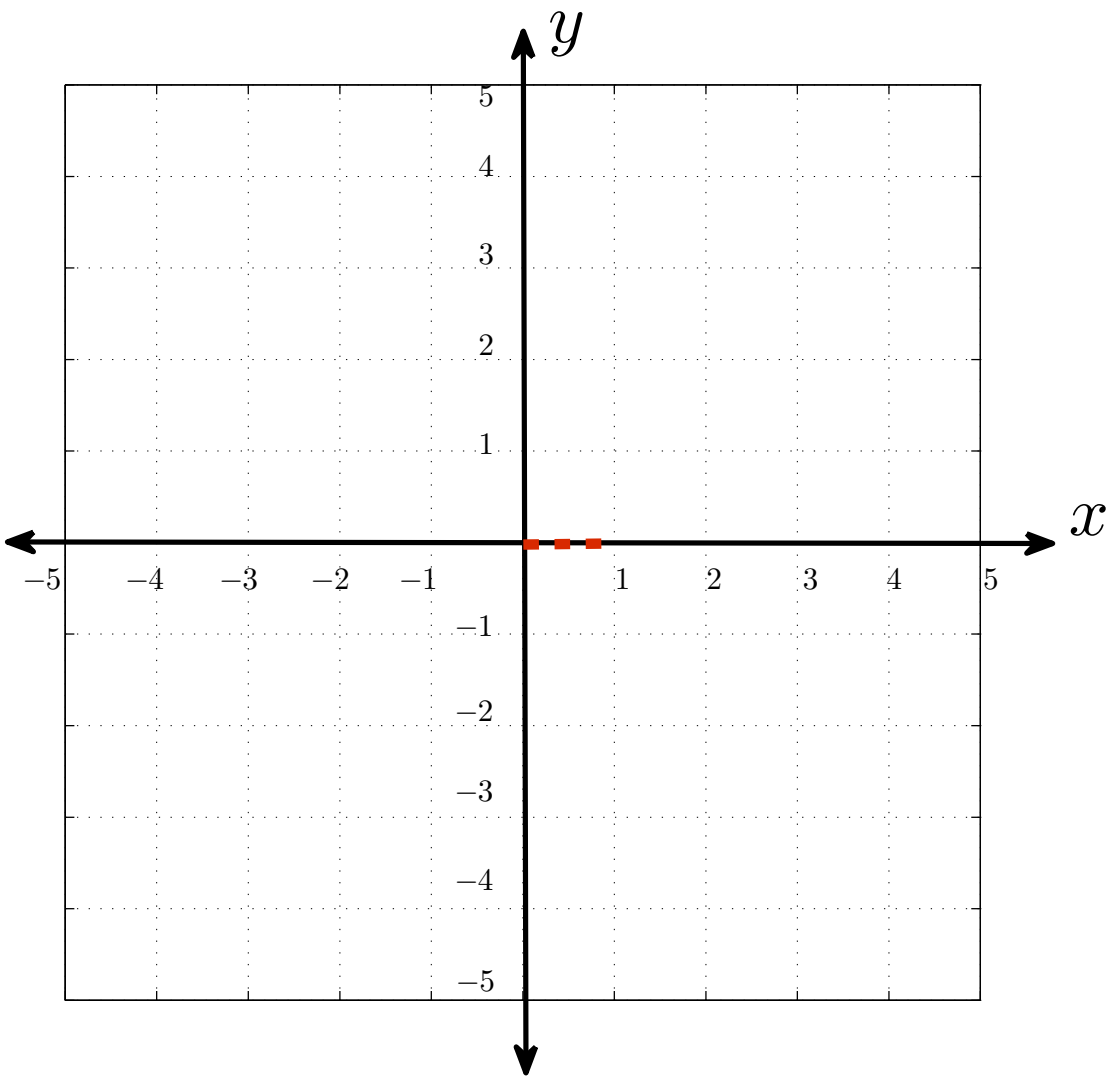

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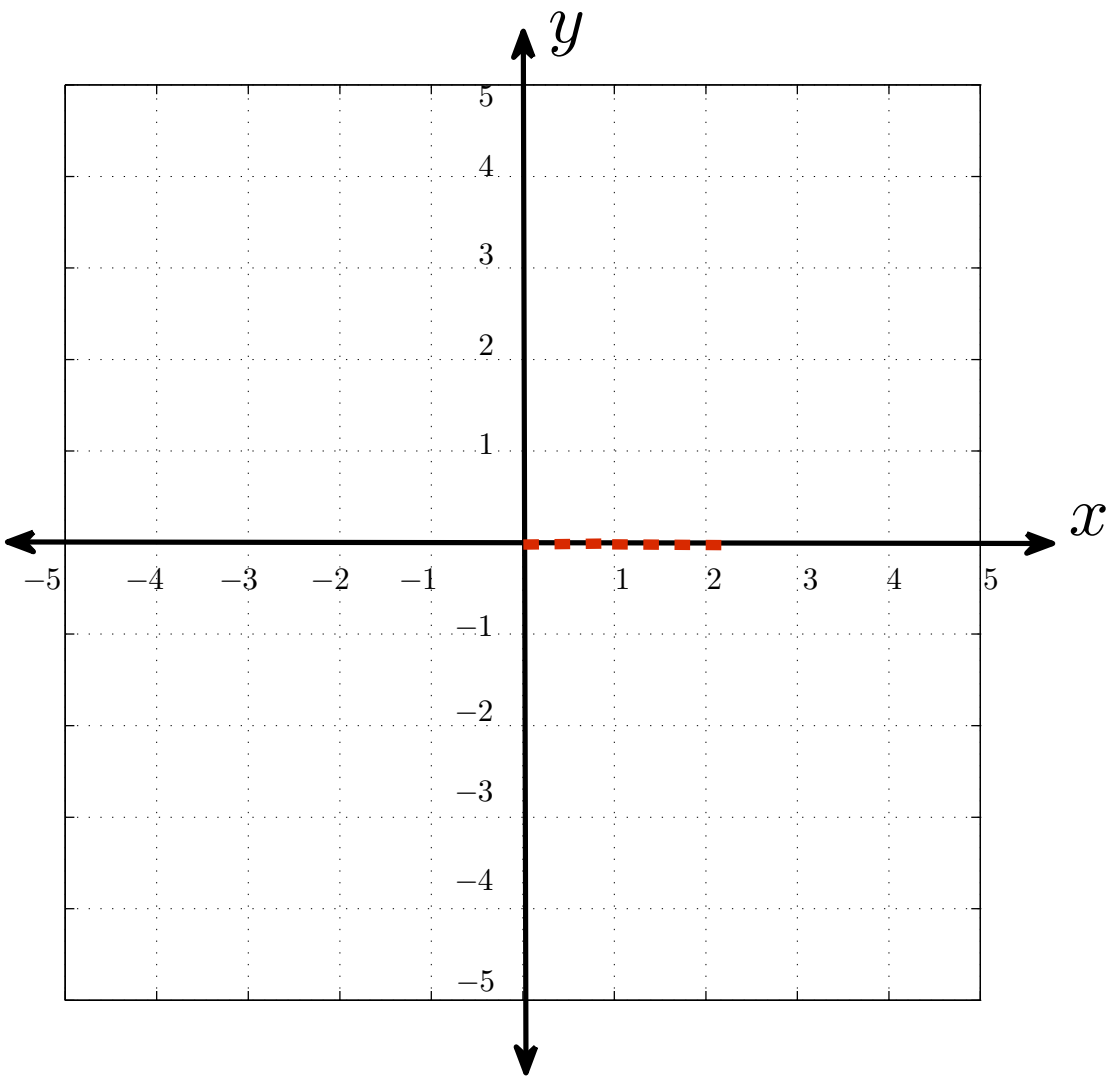

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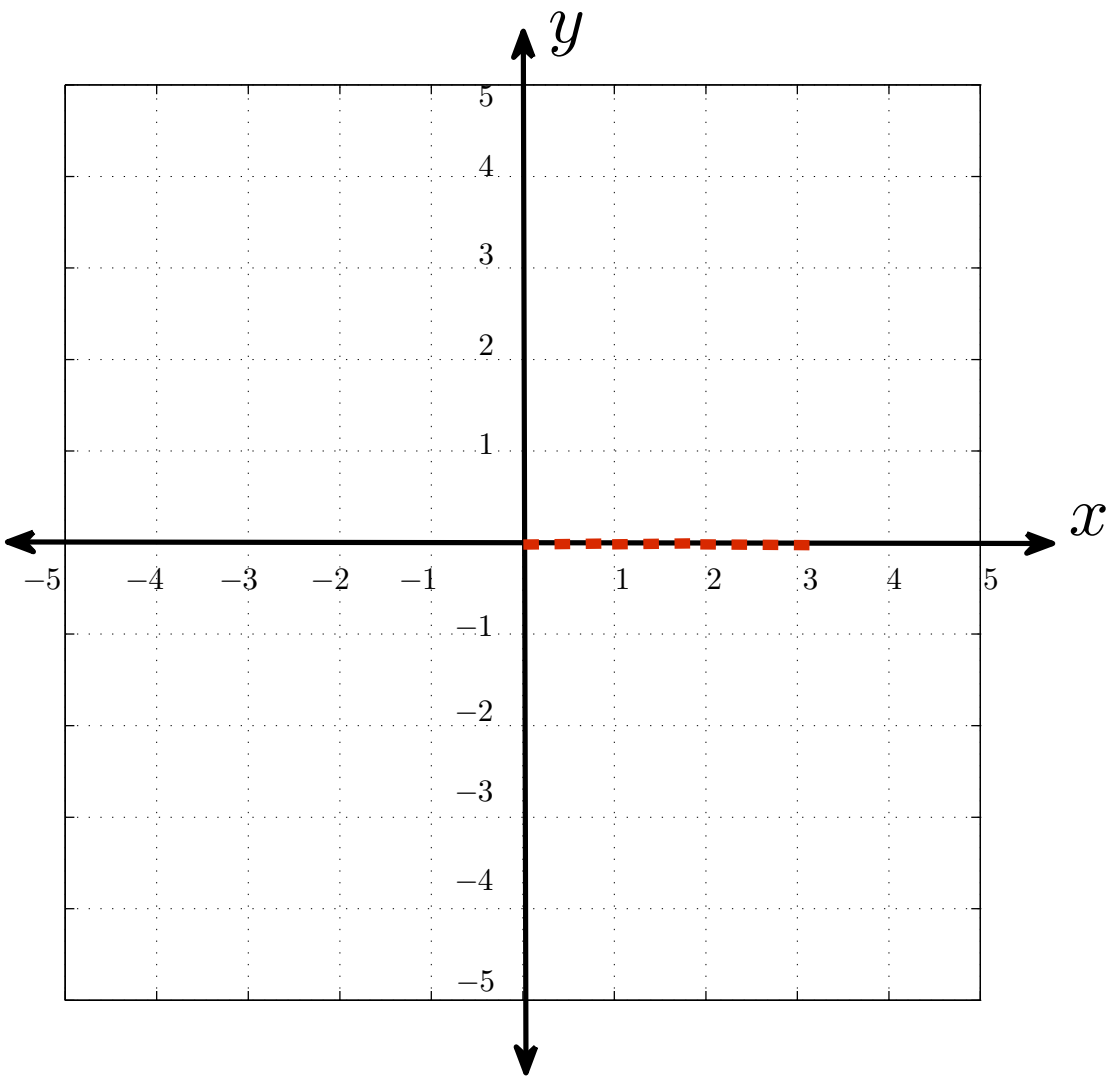

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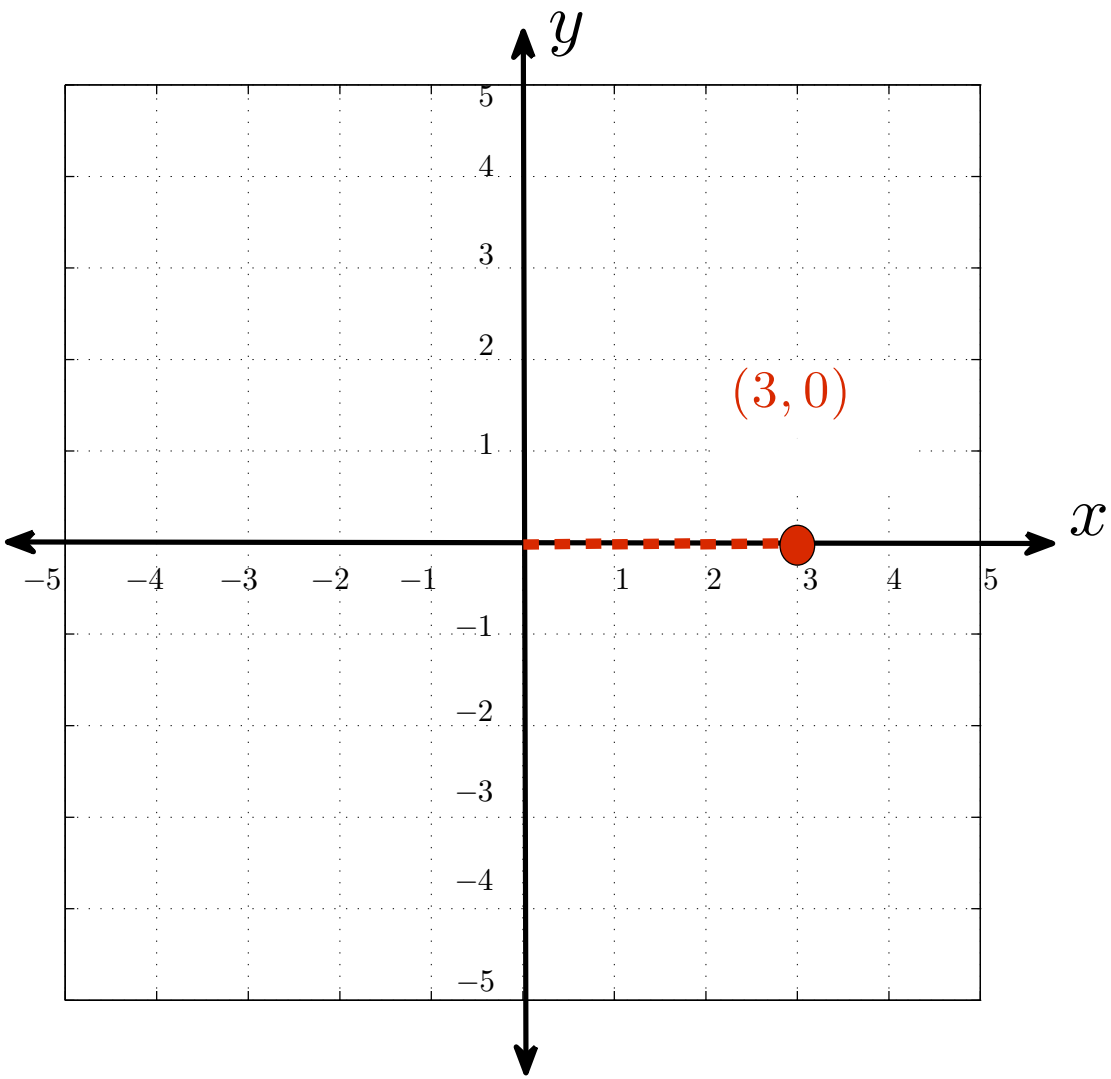

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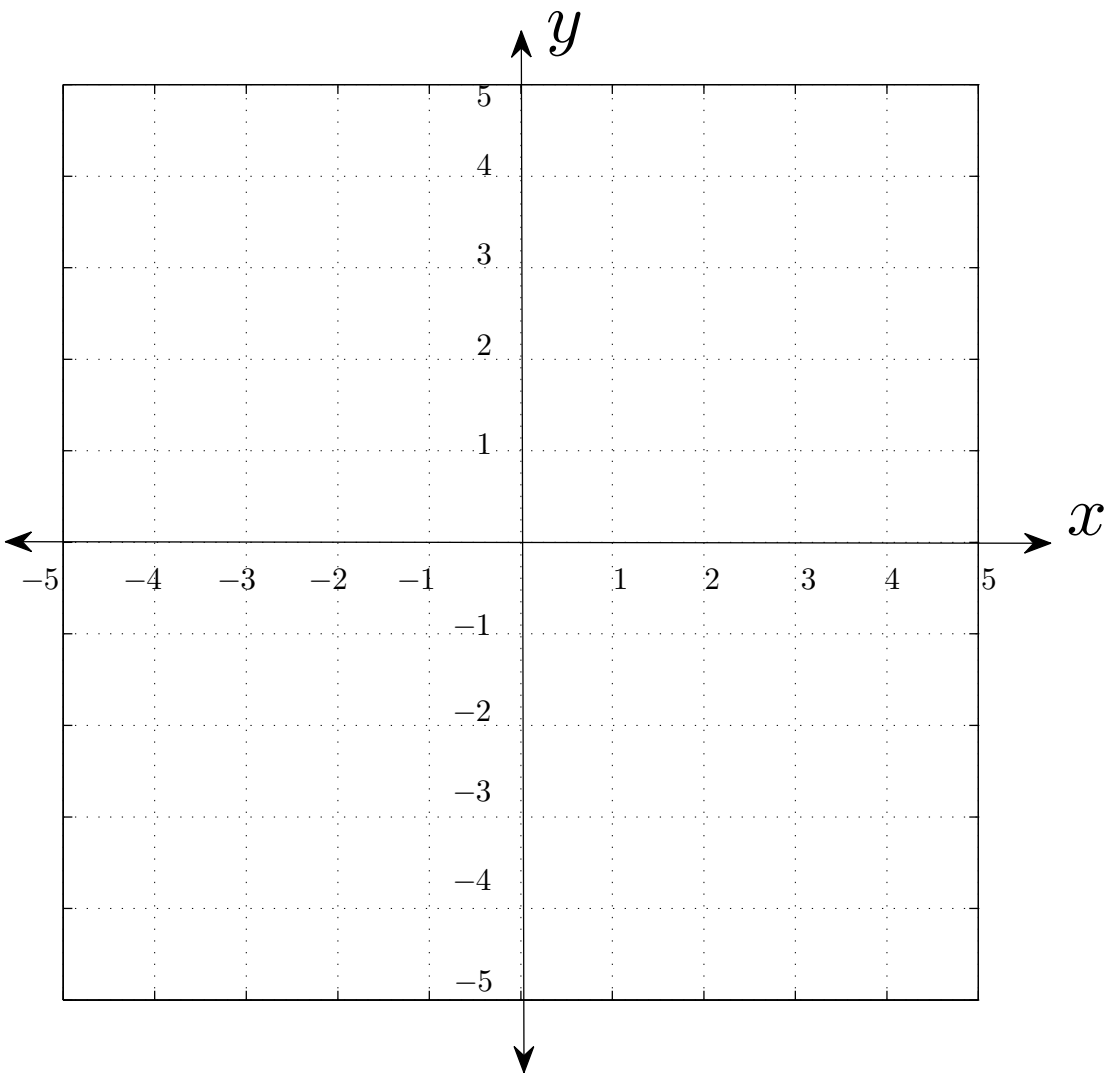
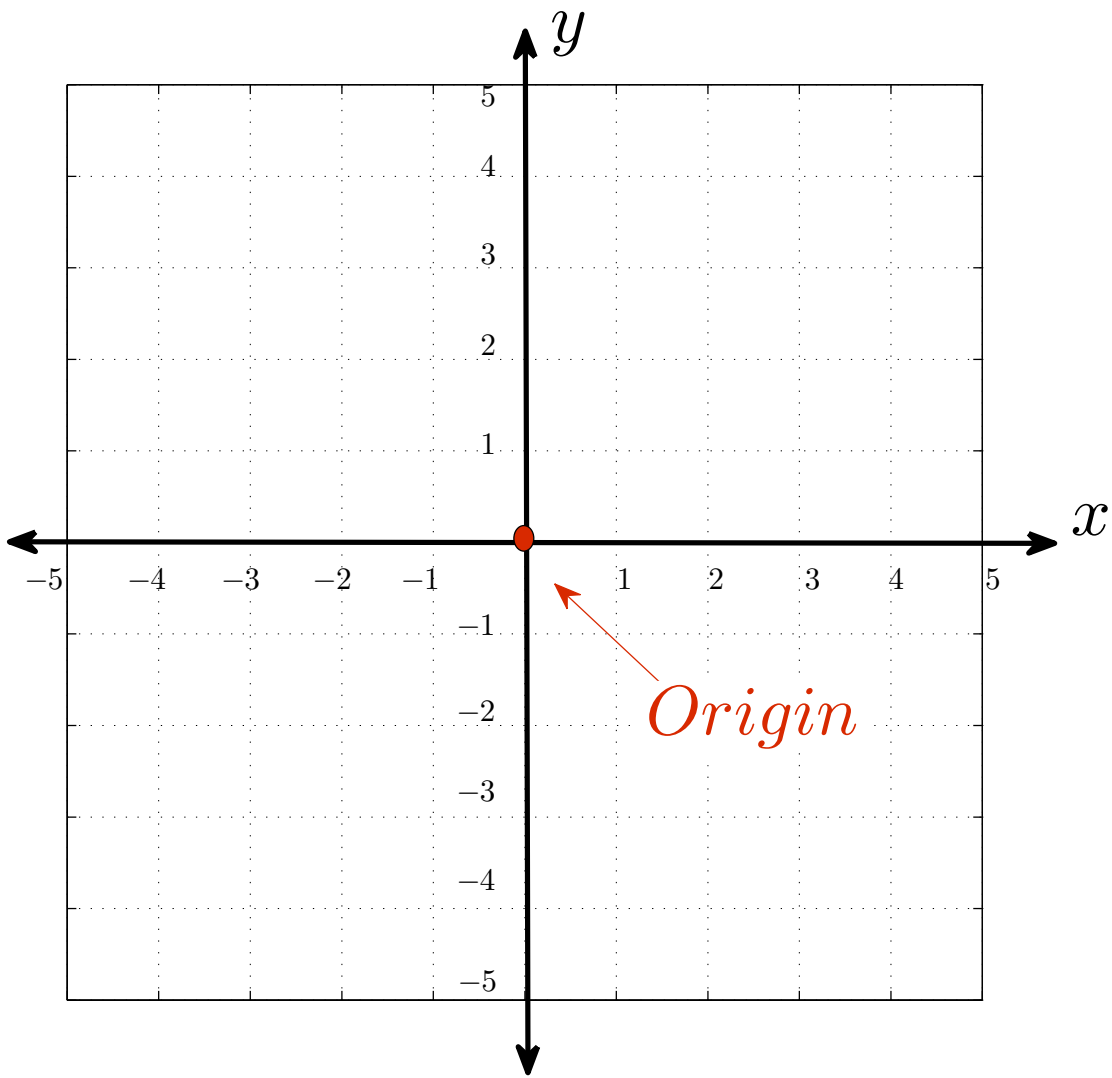
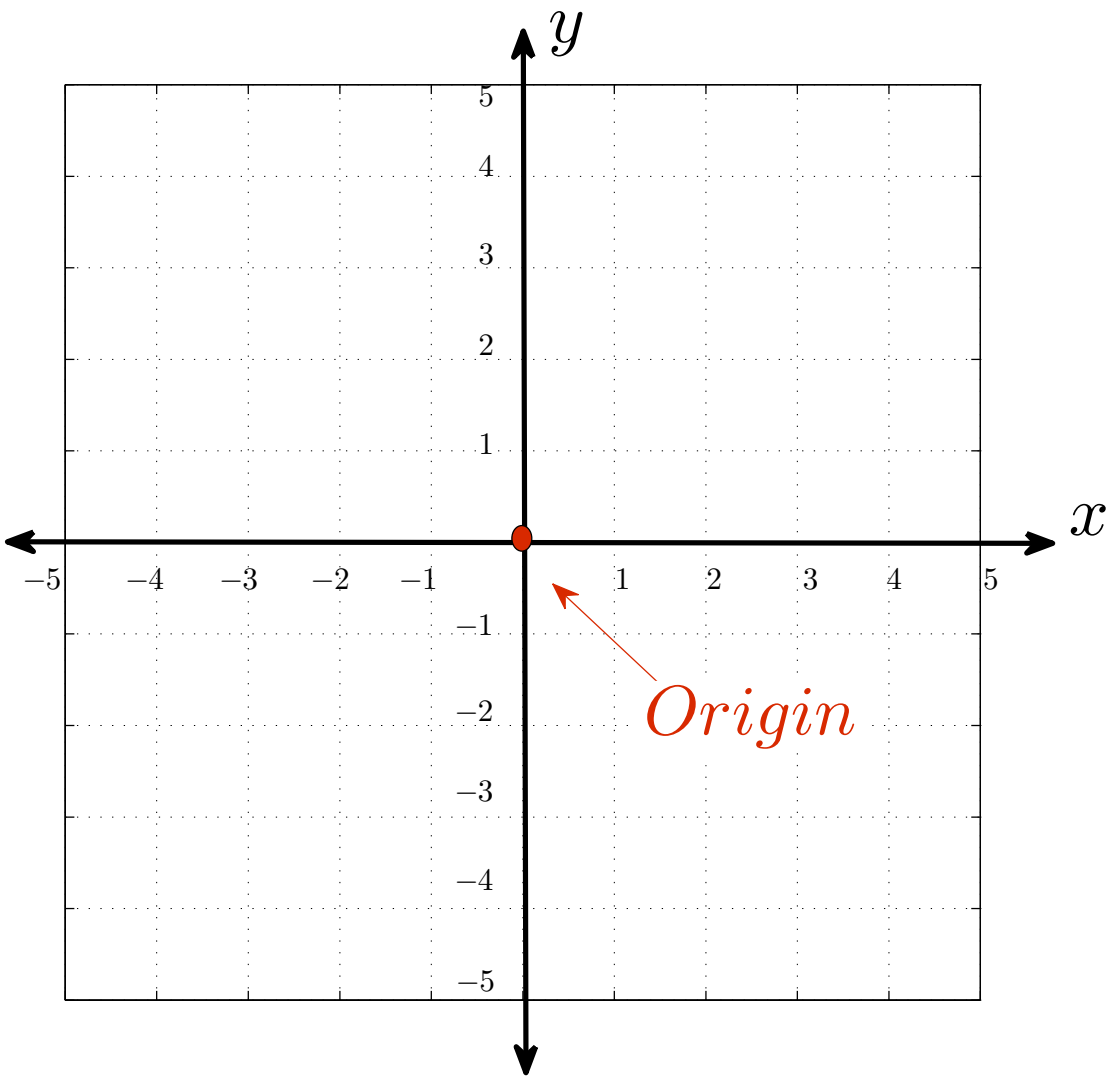


To plot $(0,2)$, begin at the origin. Travel along the x-axis 0 units.



From that point (the origin), move up 2 spaces in the positive y direction.



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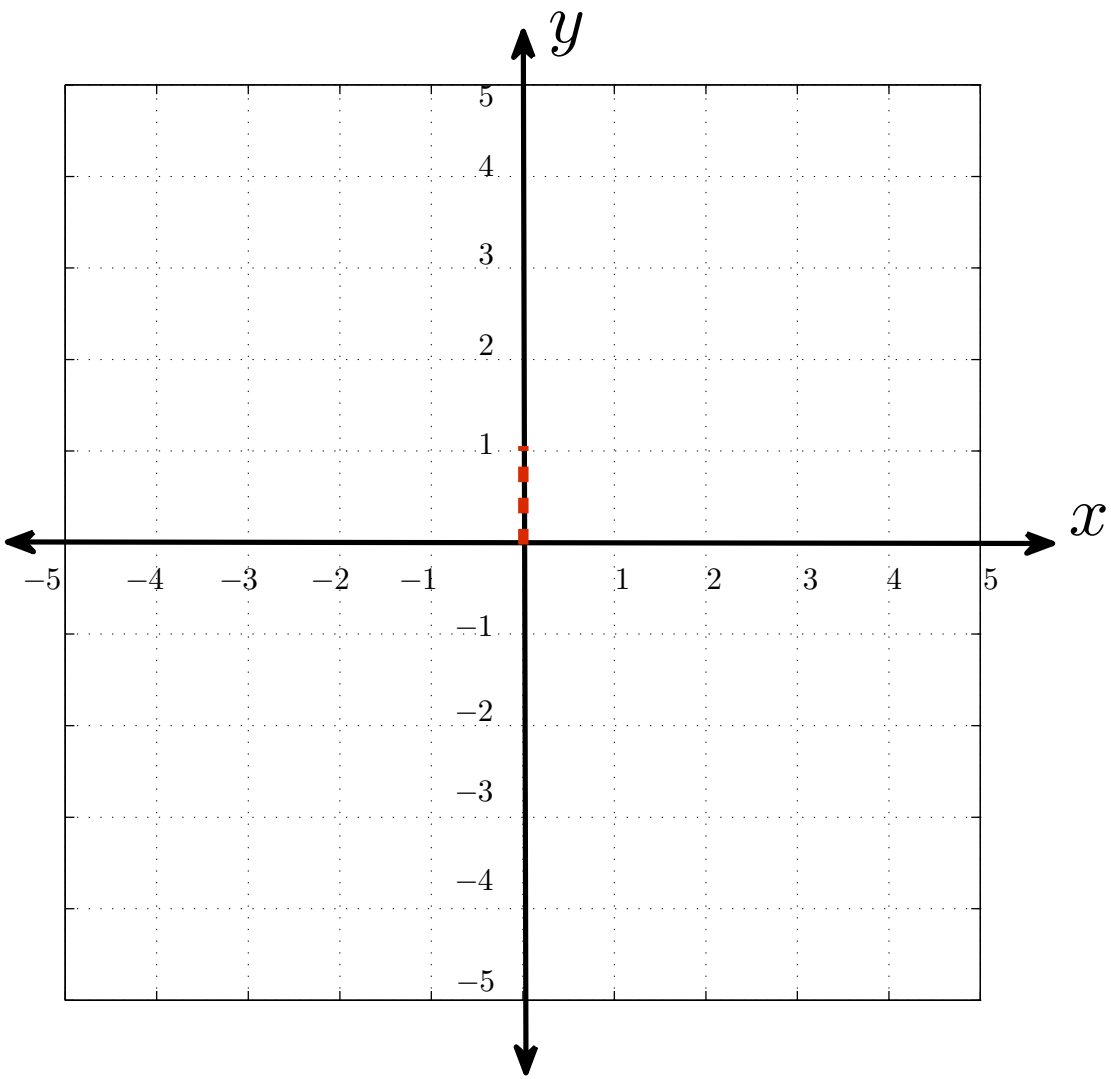

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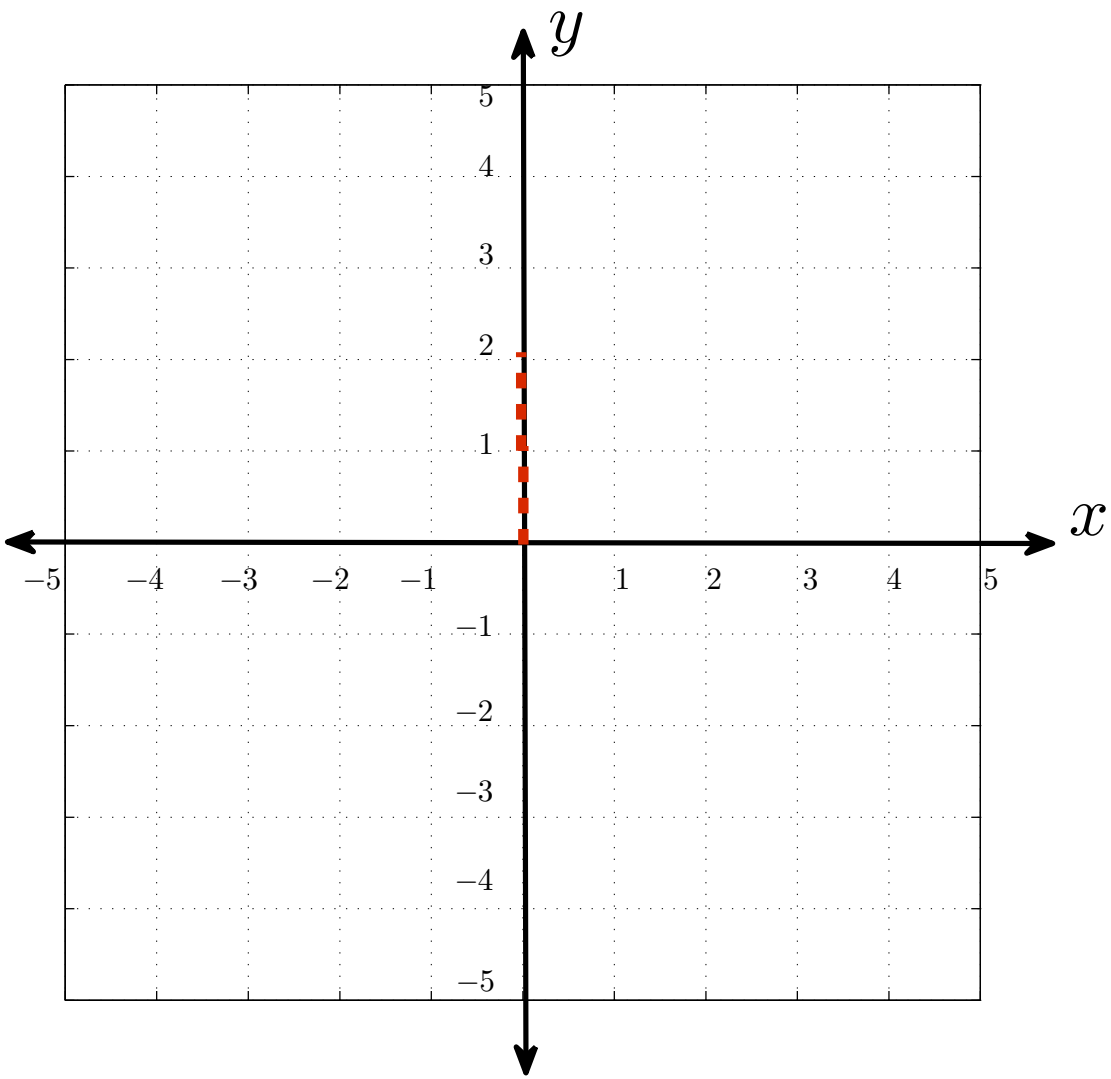

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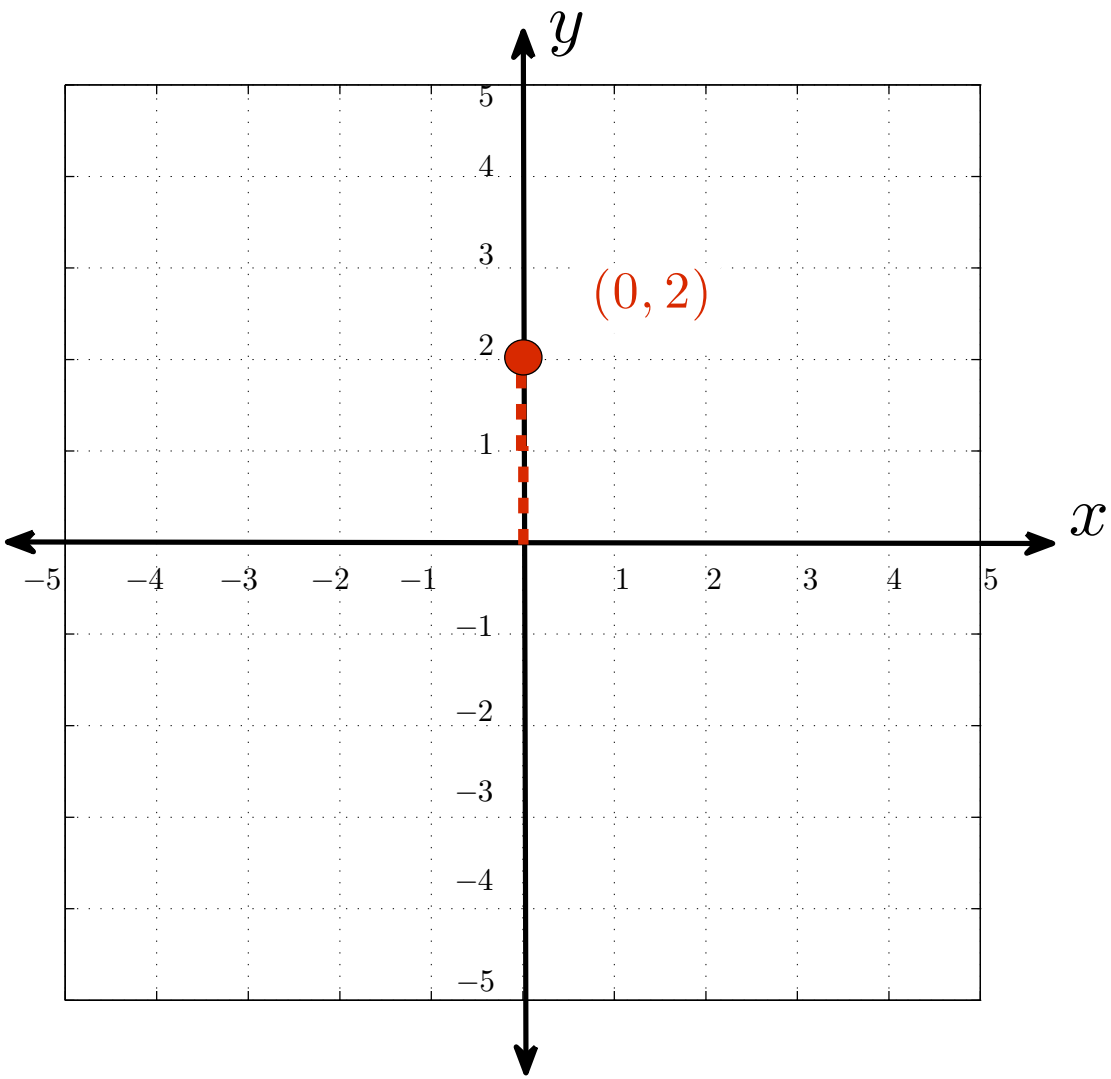

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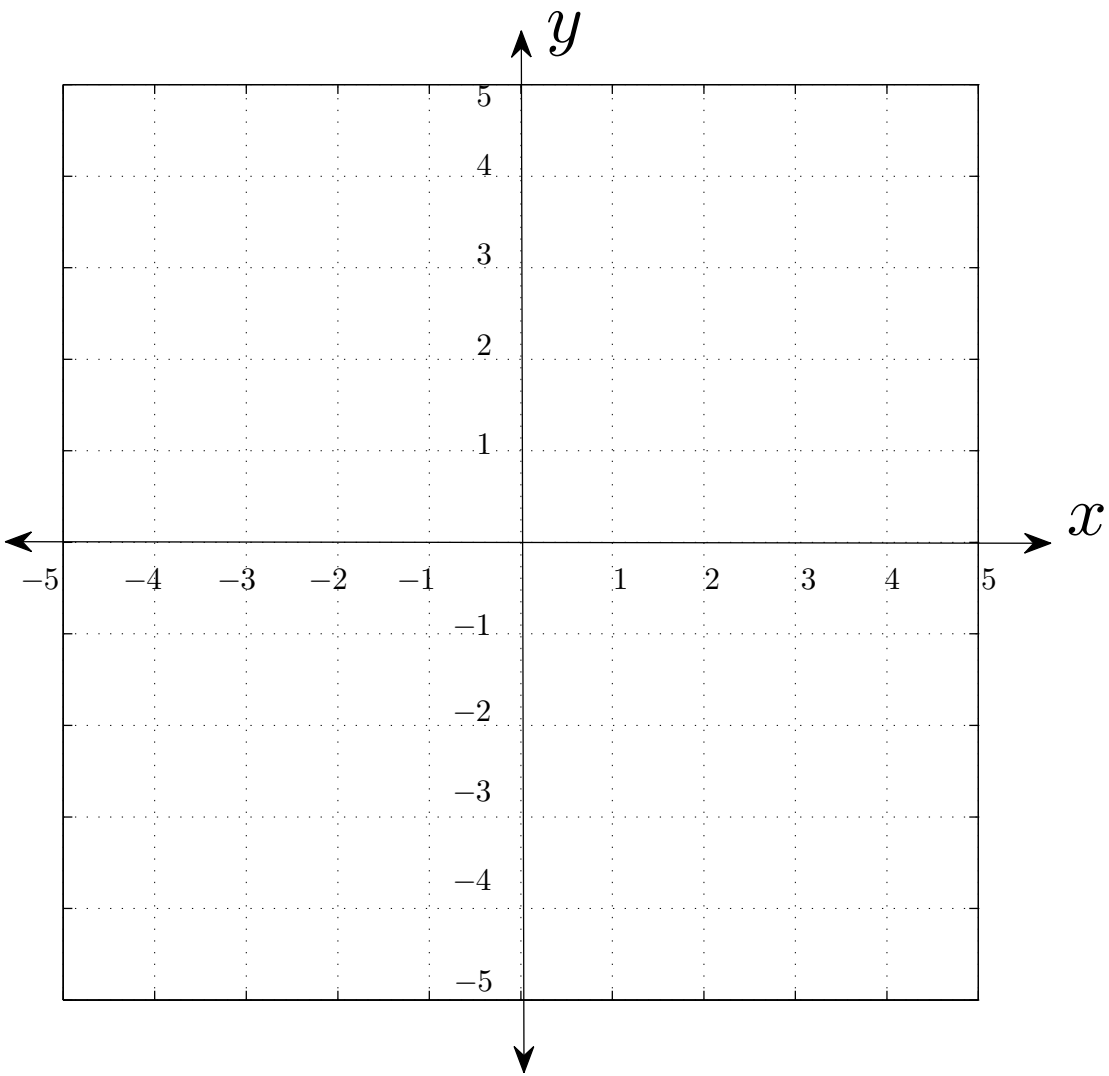
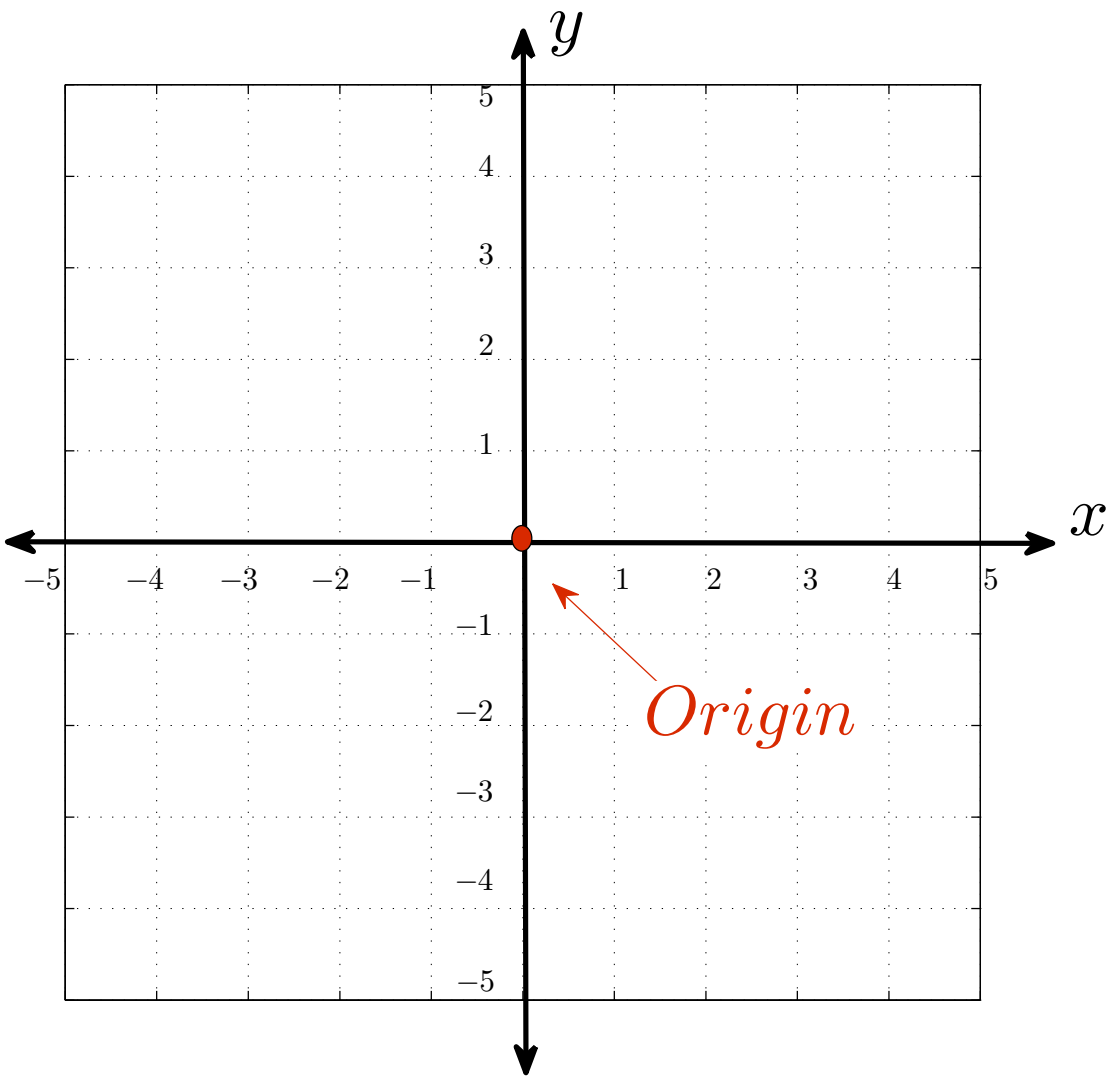
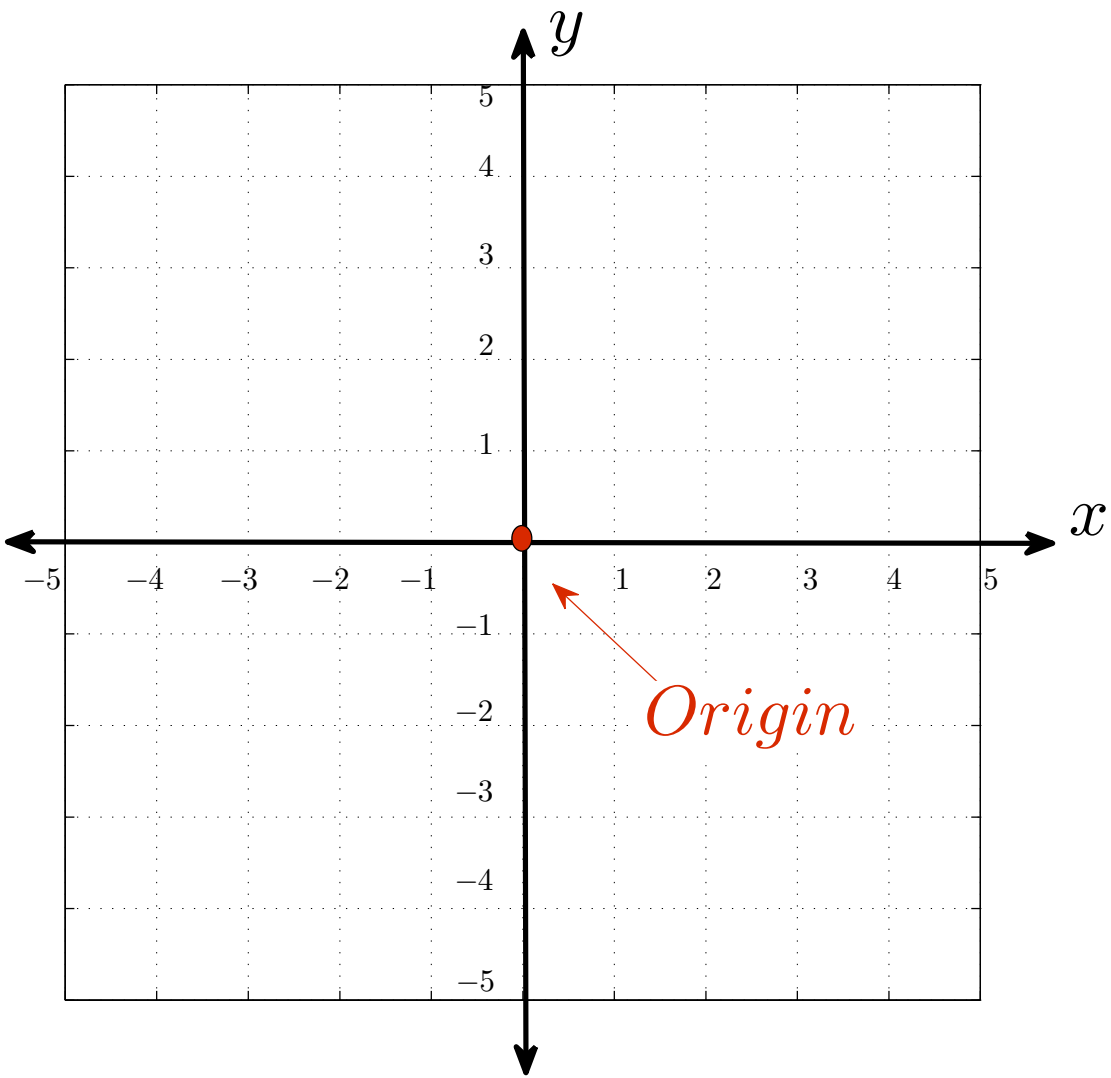


To plot $(-3,0)$, begin at the origin.



To plot $(-3,0)$, begin at the origin. Travel along the x-axis 3 units left (the negative x direction).



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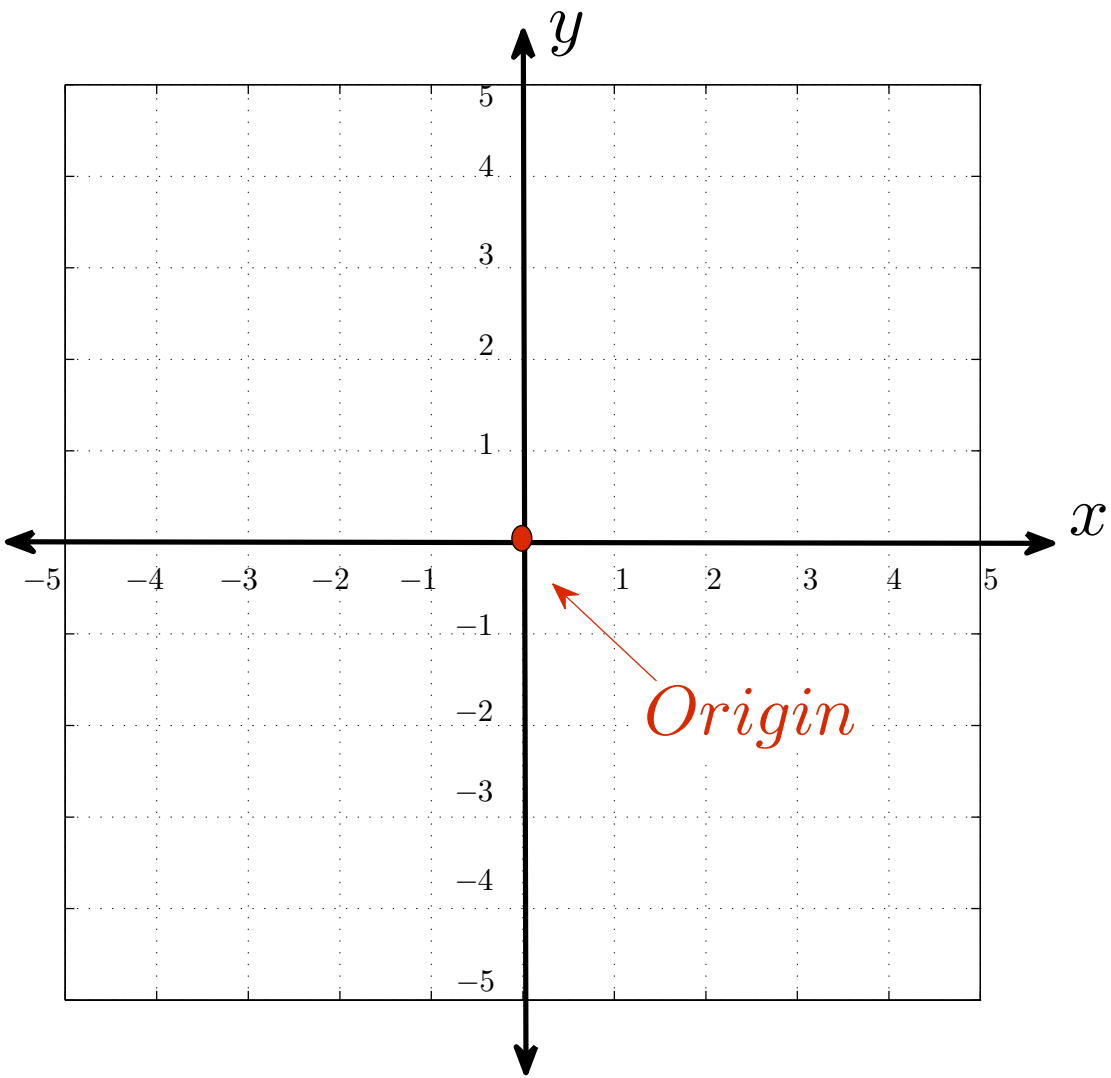

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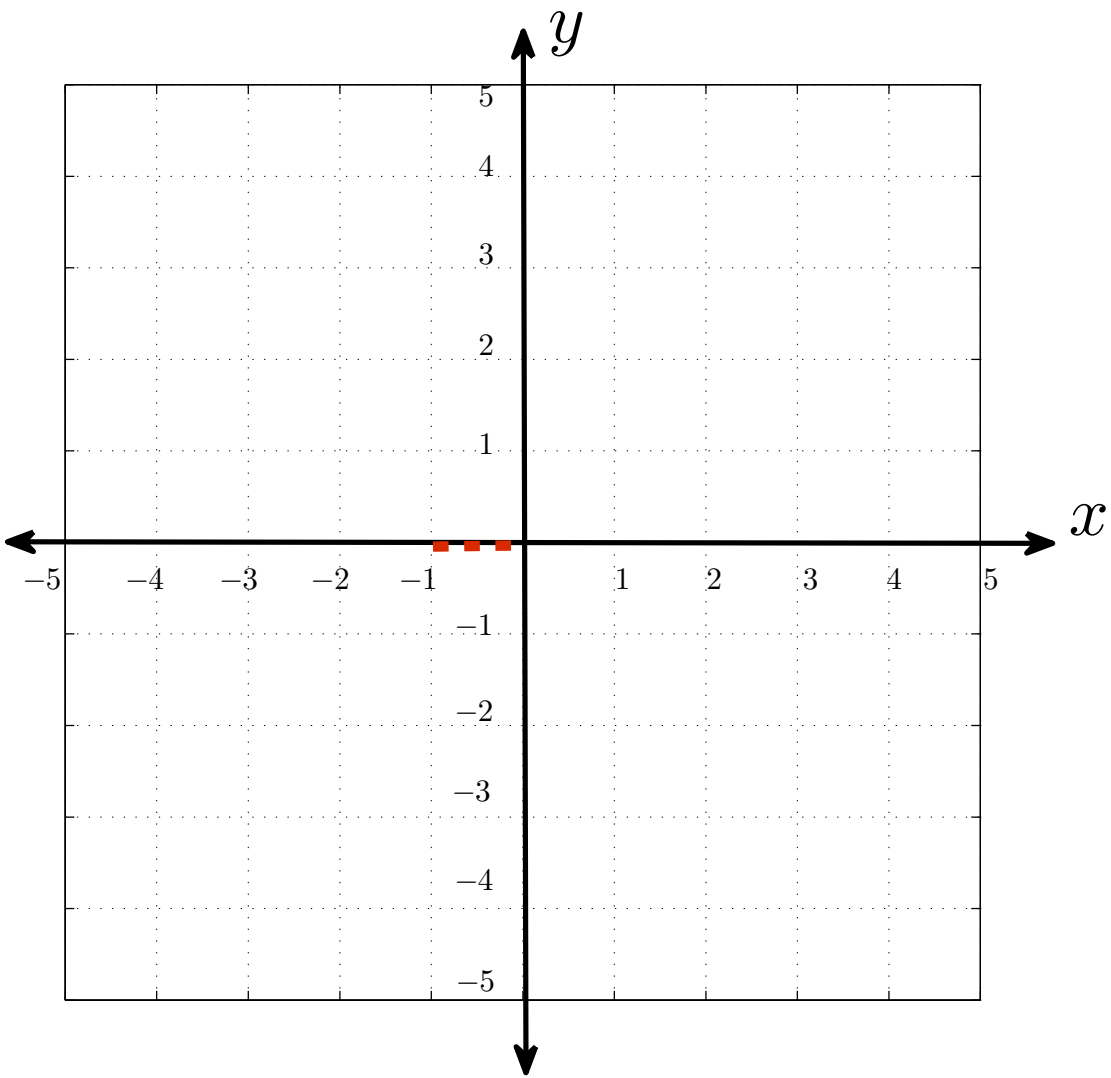

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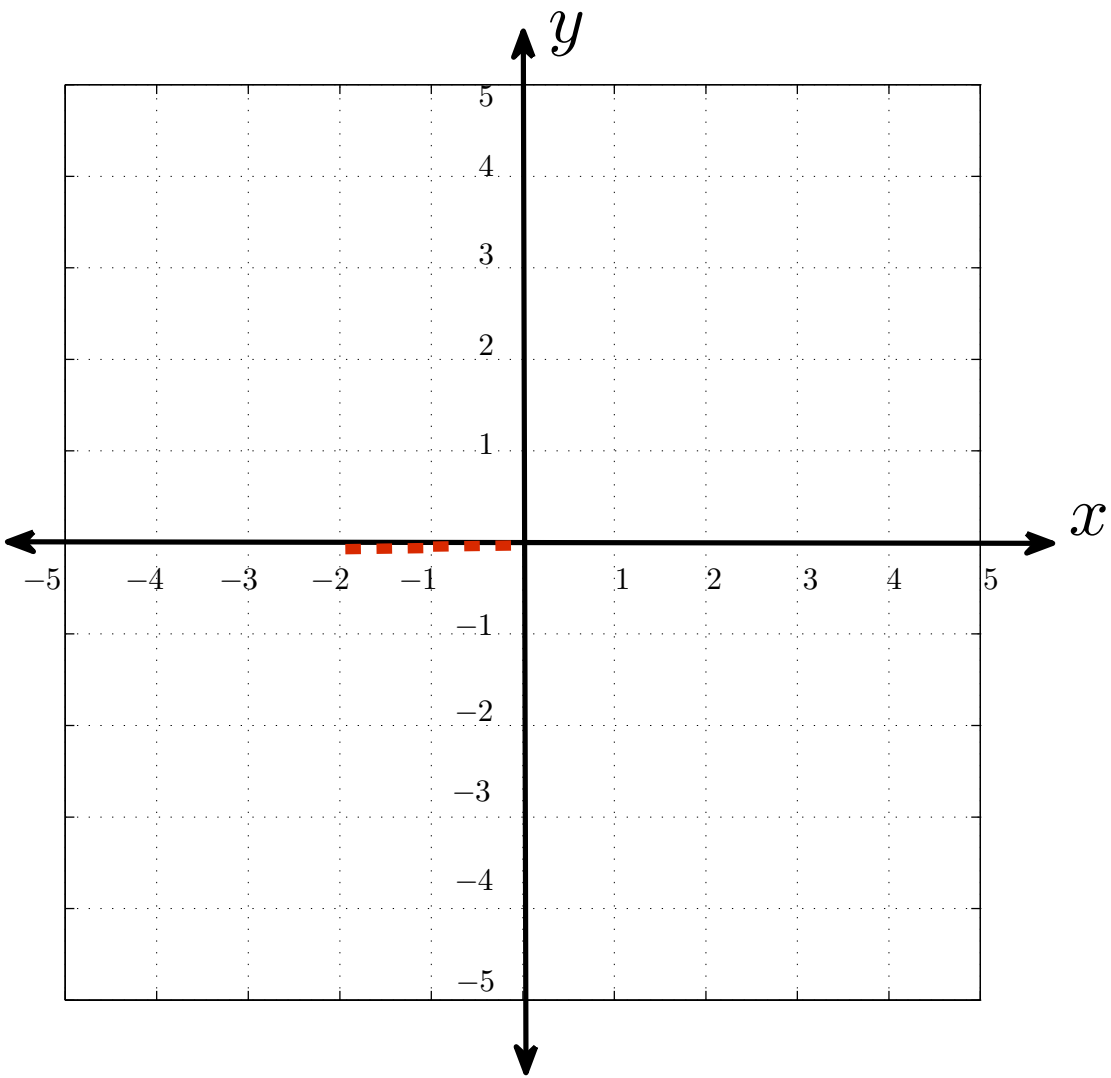

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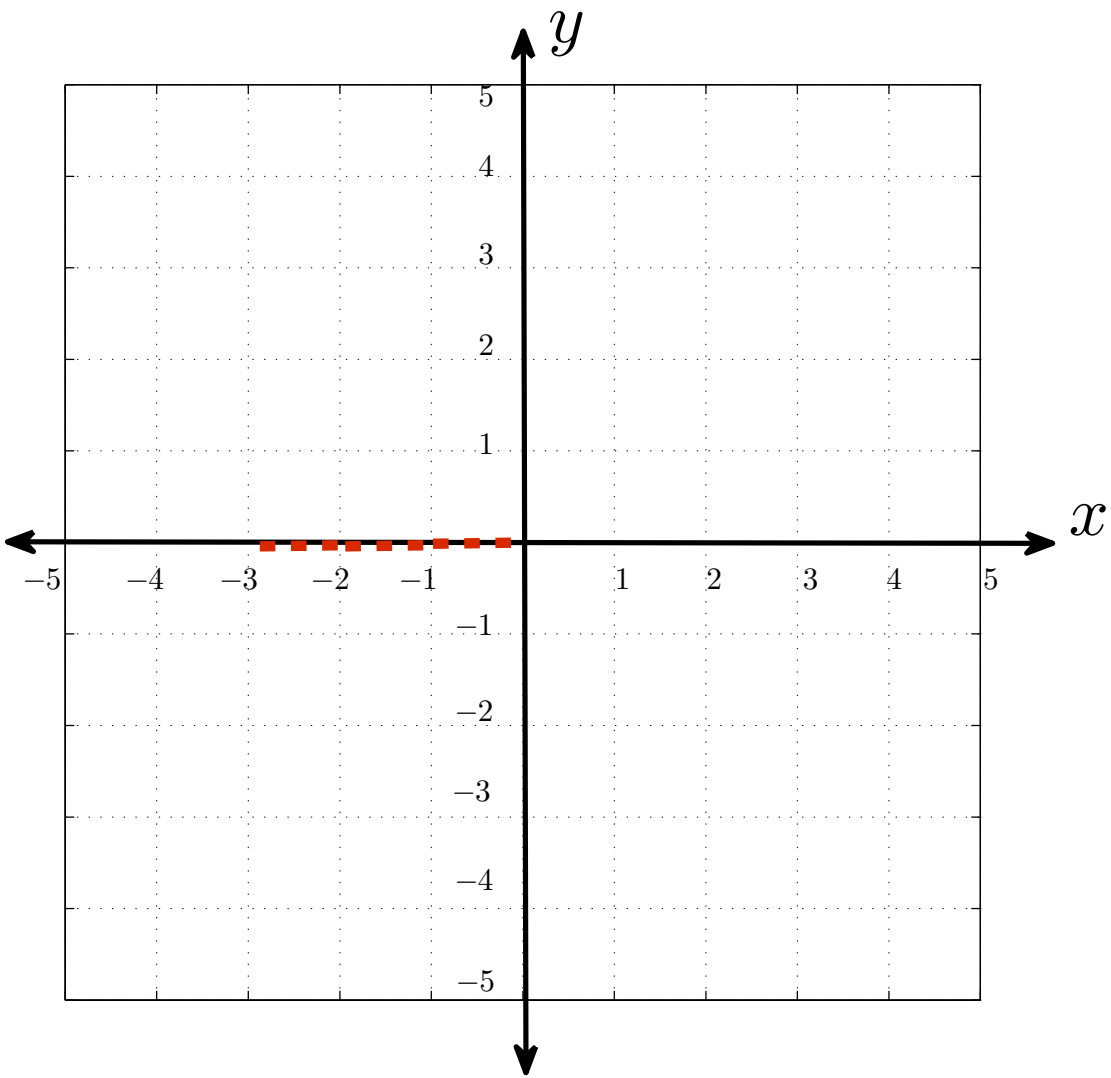
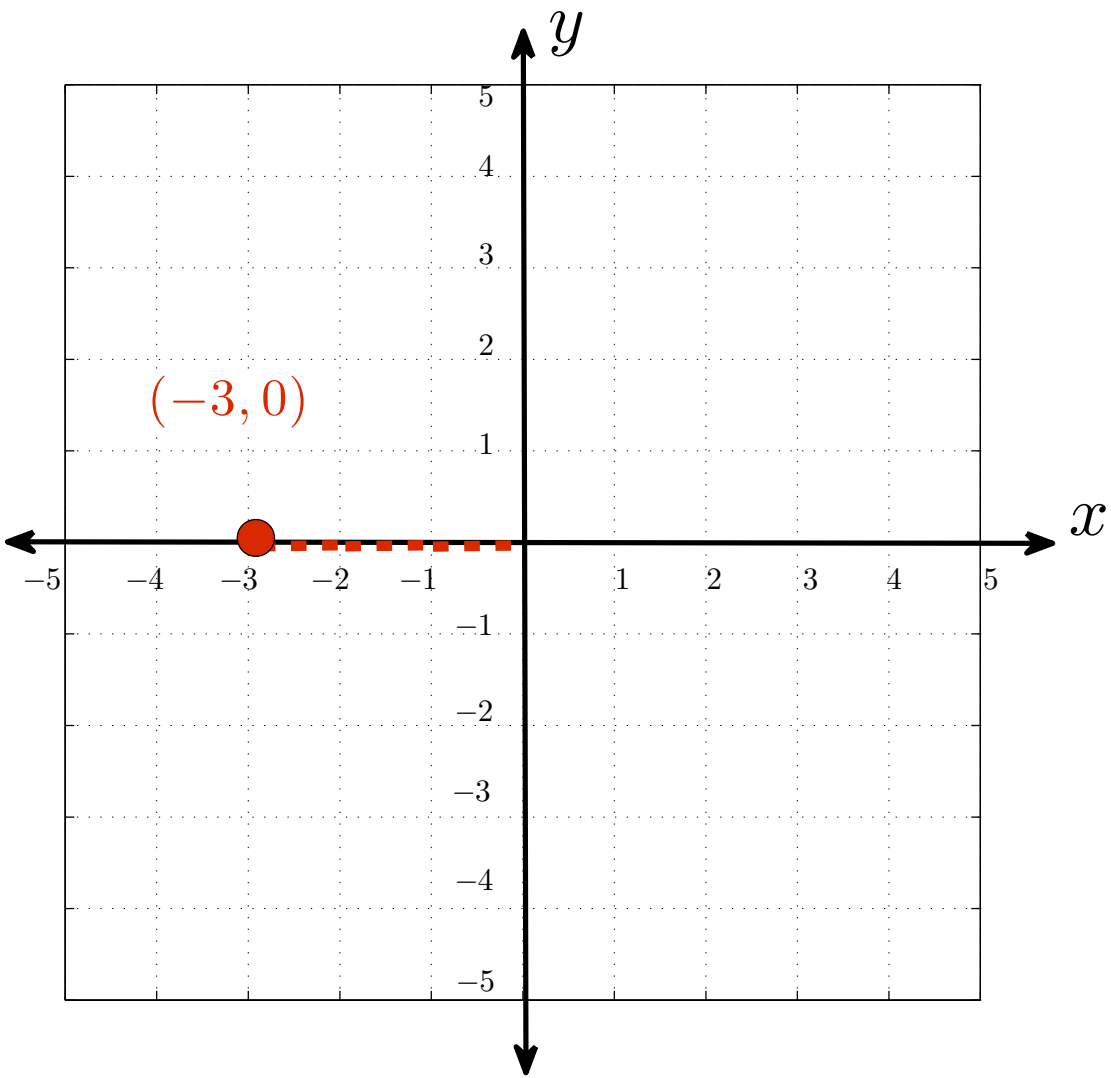


From that point (the origin), move up 0 spaces in the y direction.



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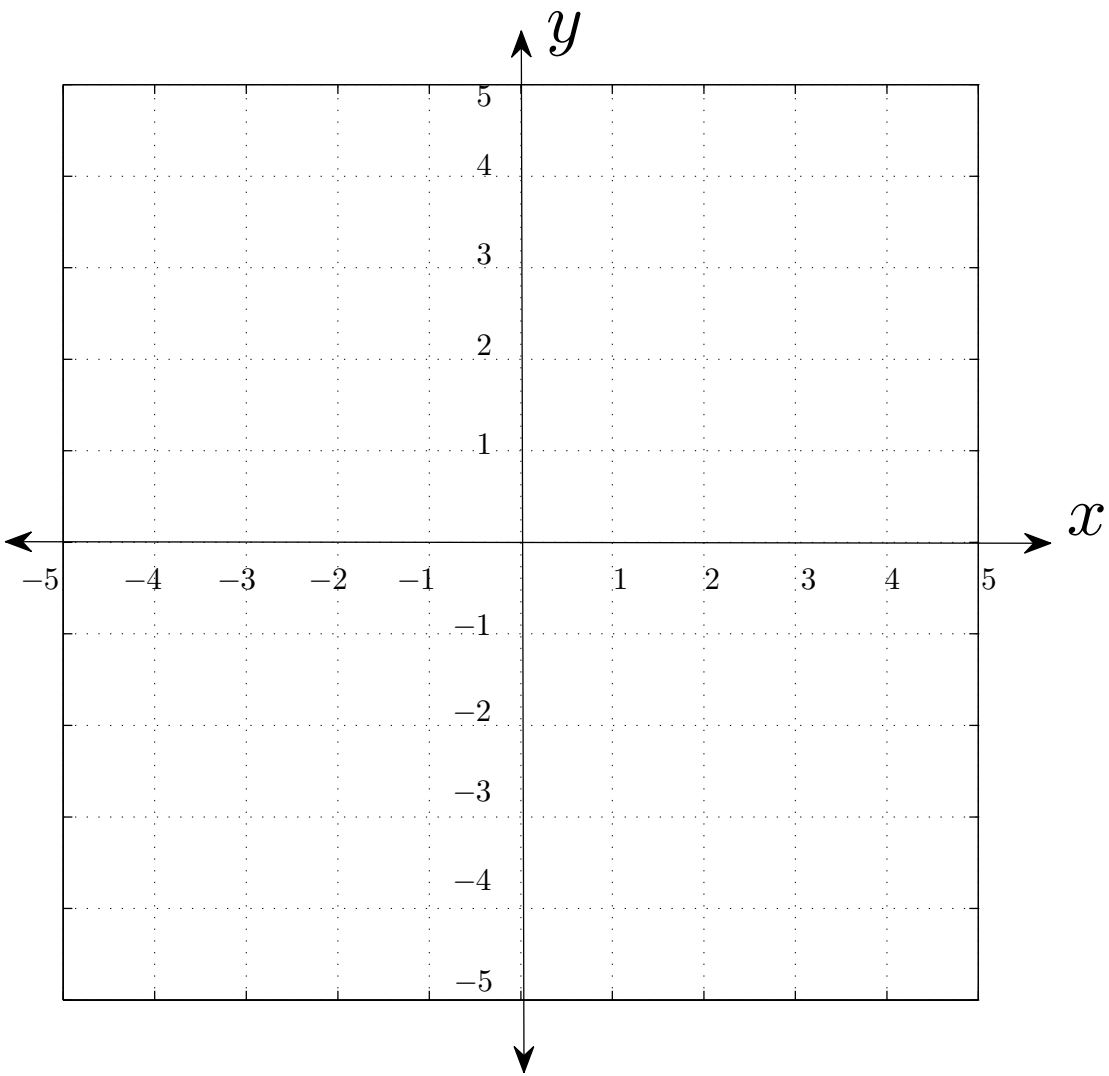
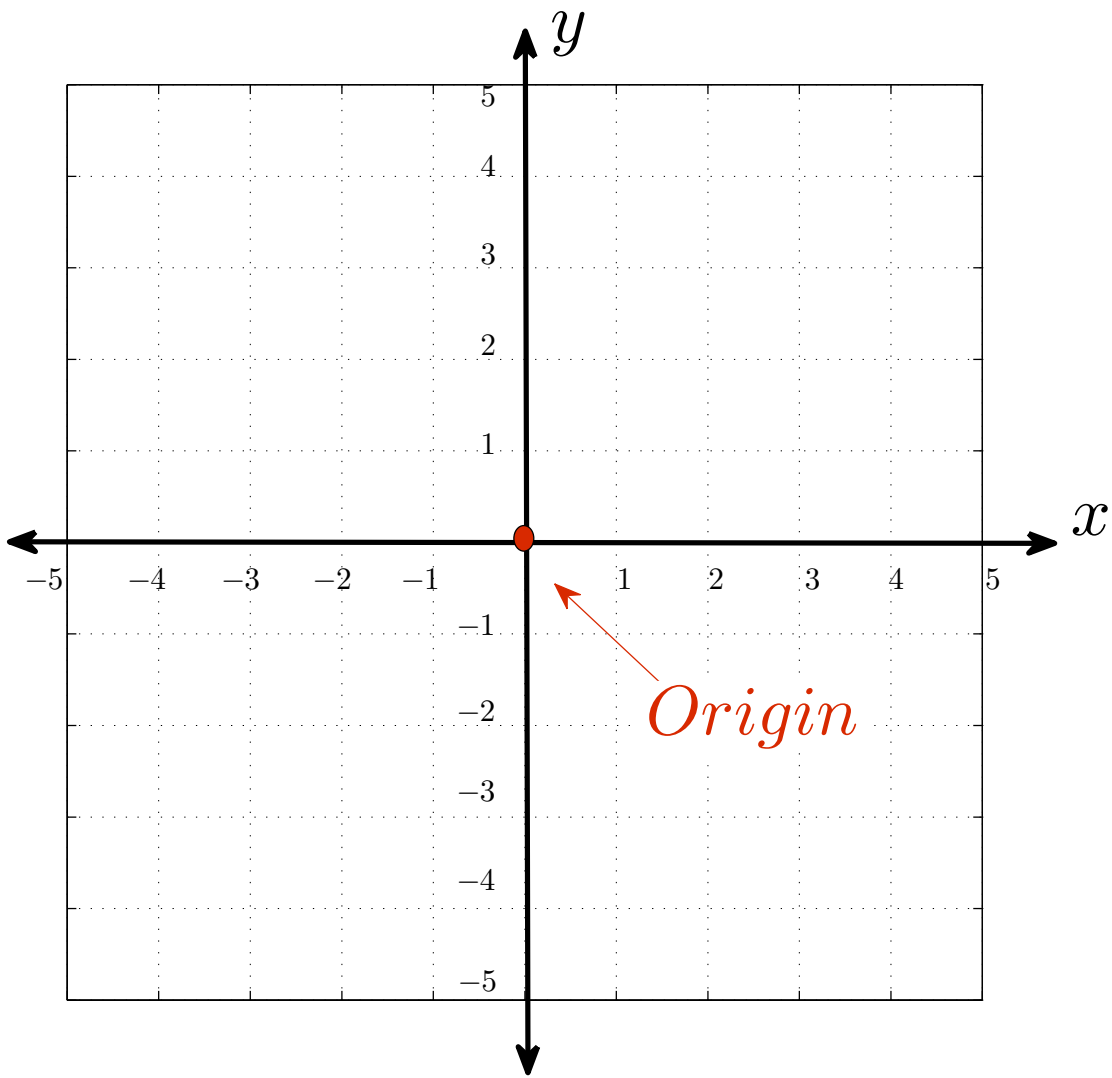
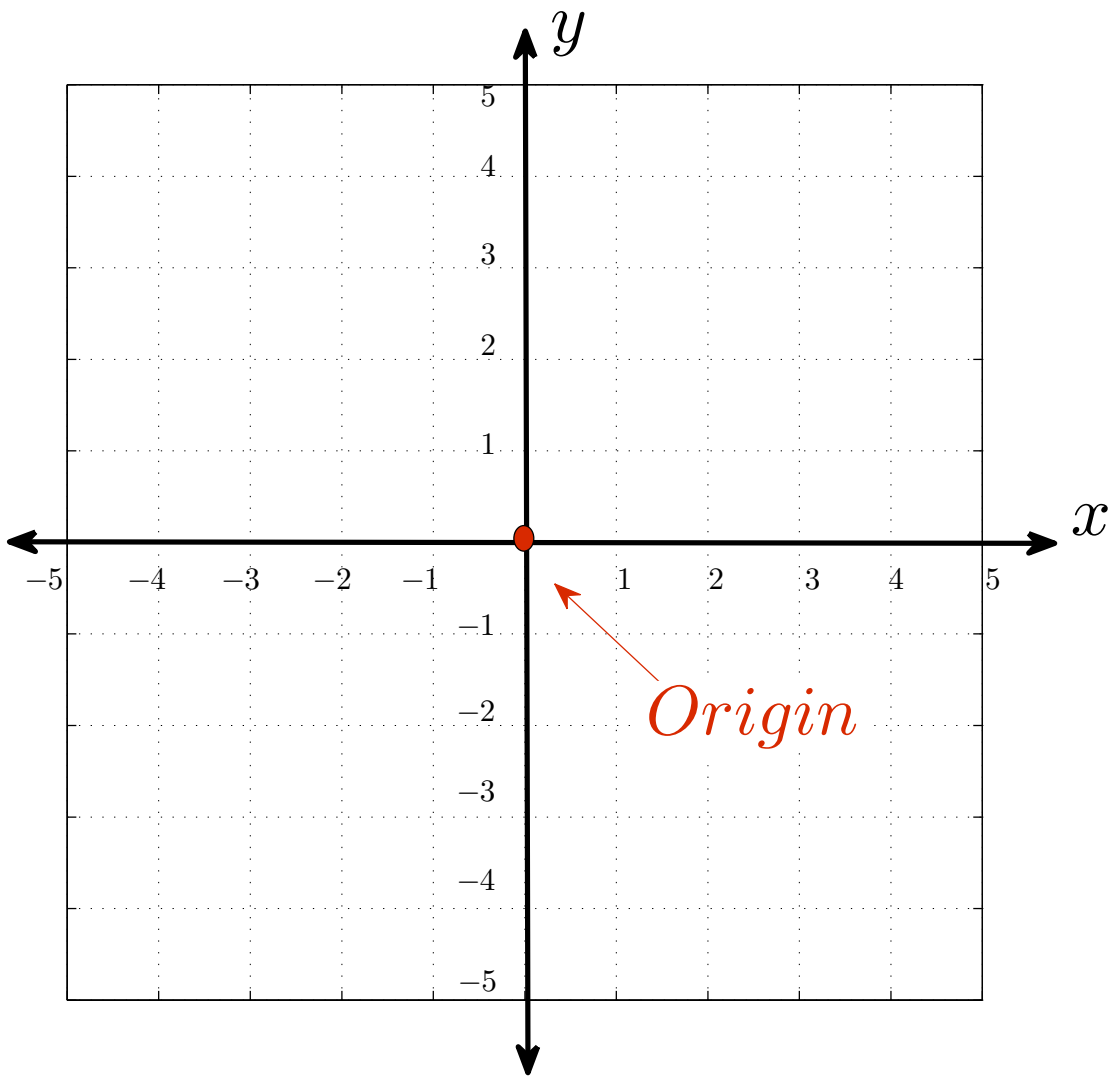


To plot $(0,-2)$, begin at the origin. Travel along the x-axis 0 units.



From that point (the origin), move up 2 spaces in the negative y direction (downwards).



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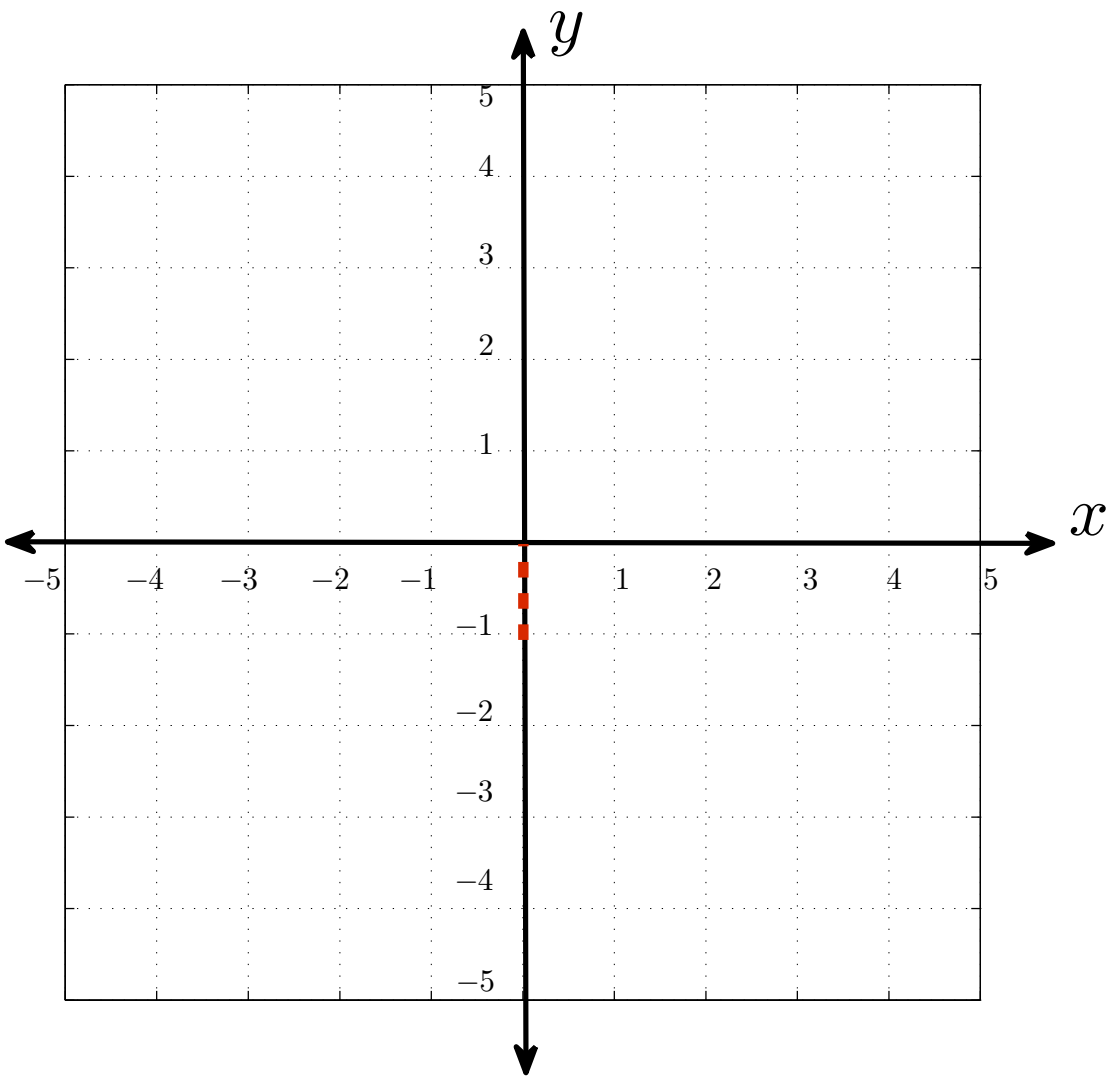

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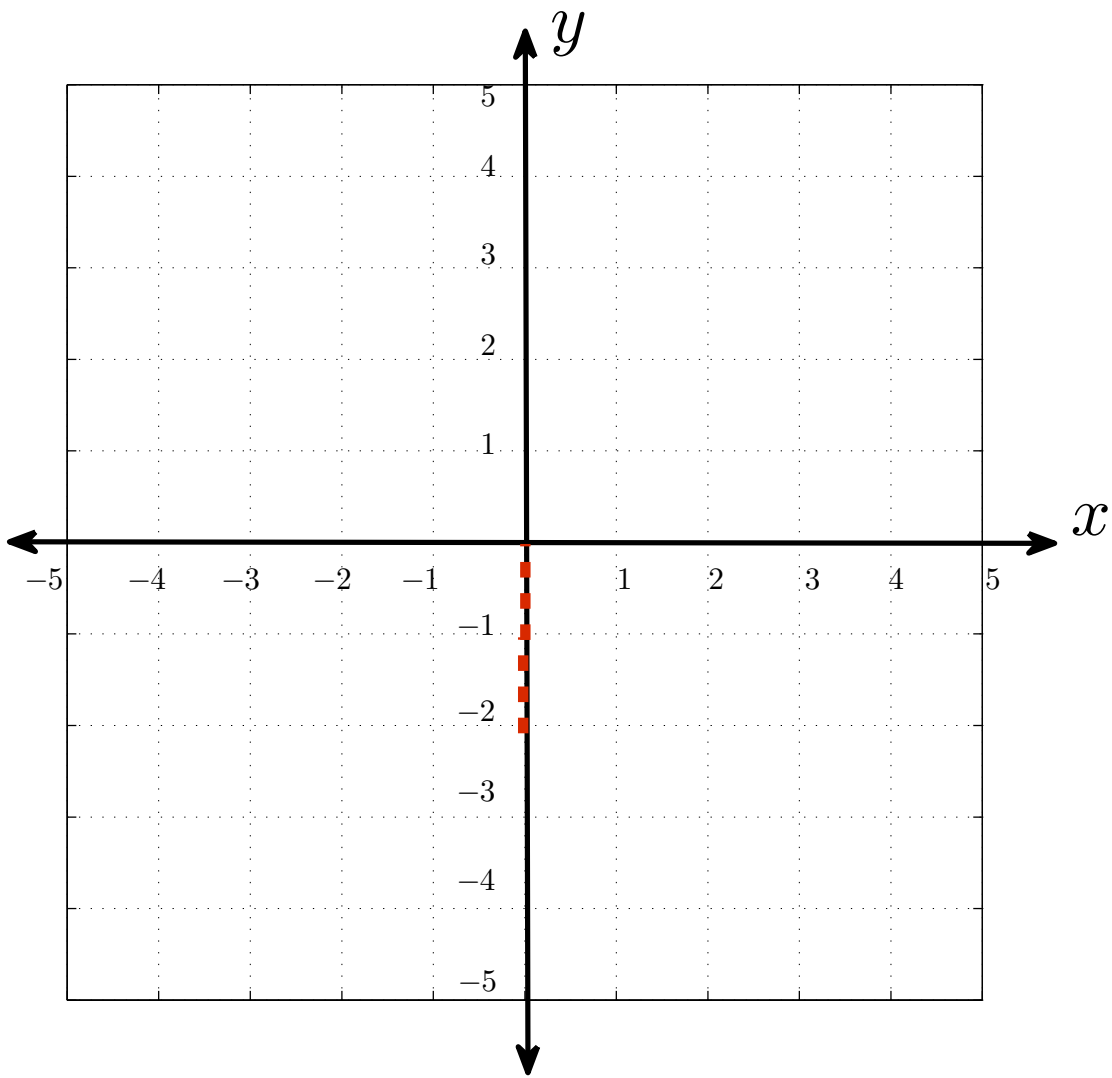

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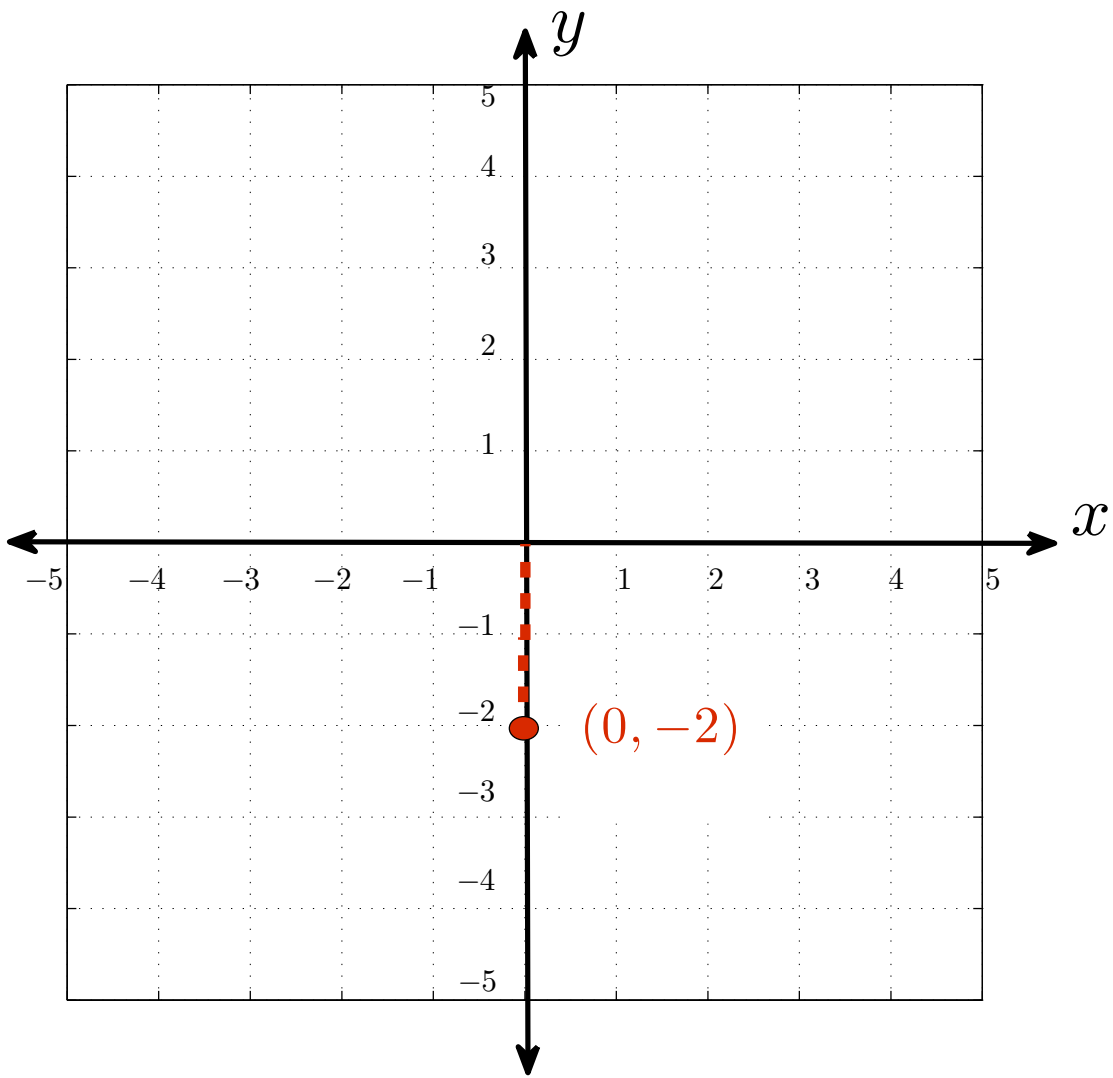
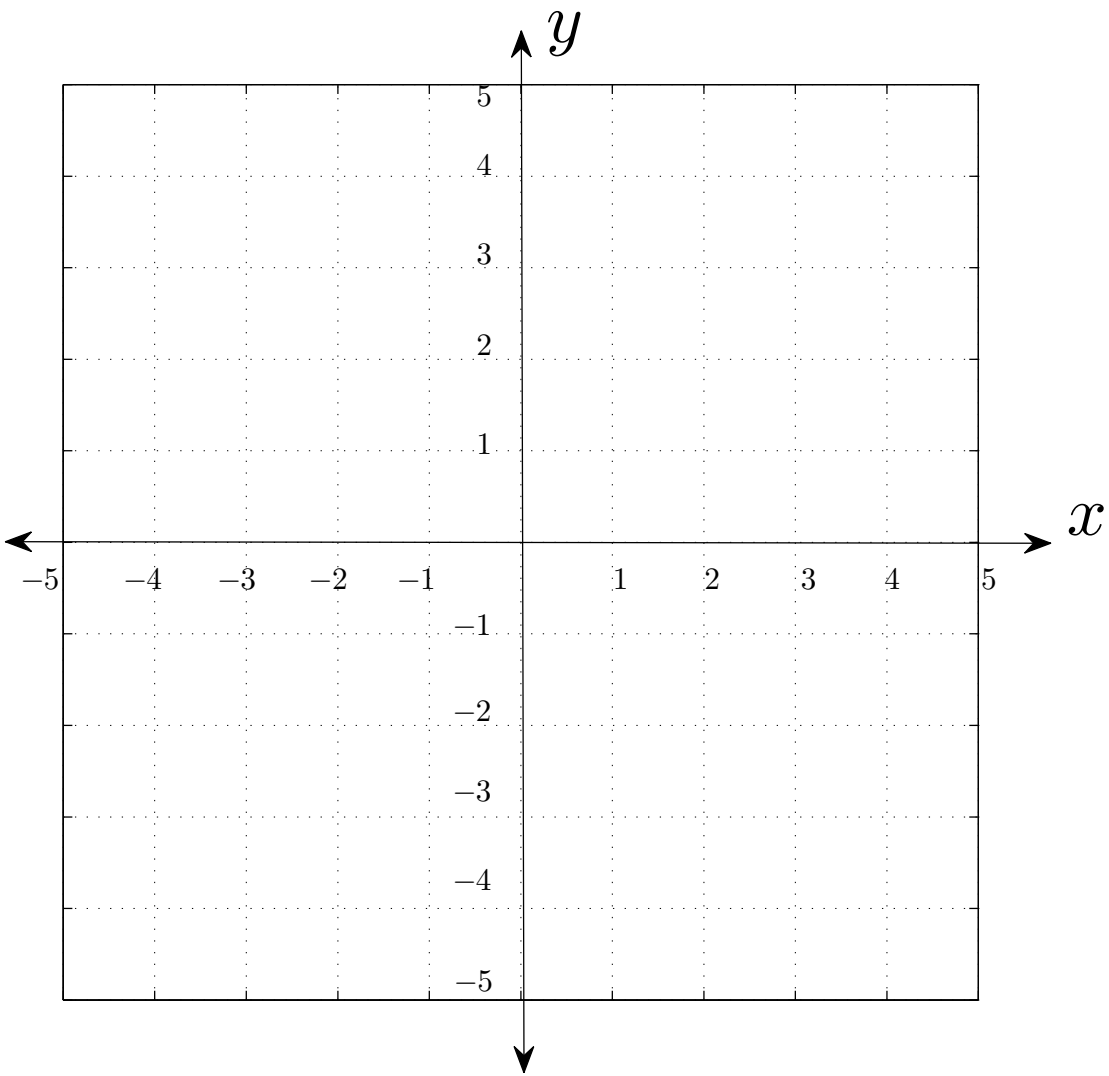


Plot (graph) $(3, 4)$, $(-4, 3)$, $(-1, -4)$ and $(5, -4)$



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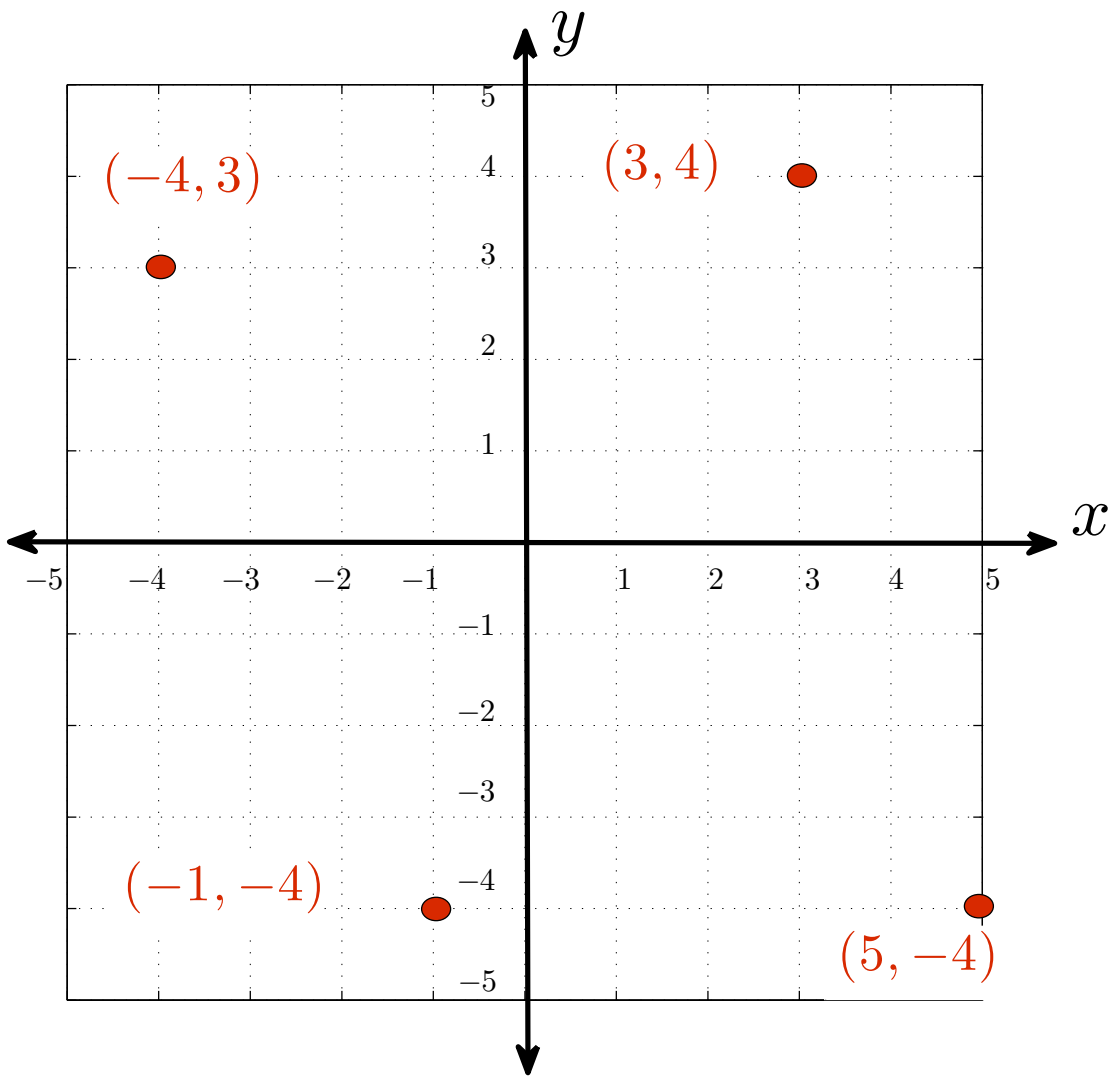
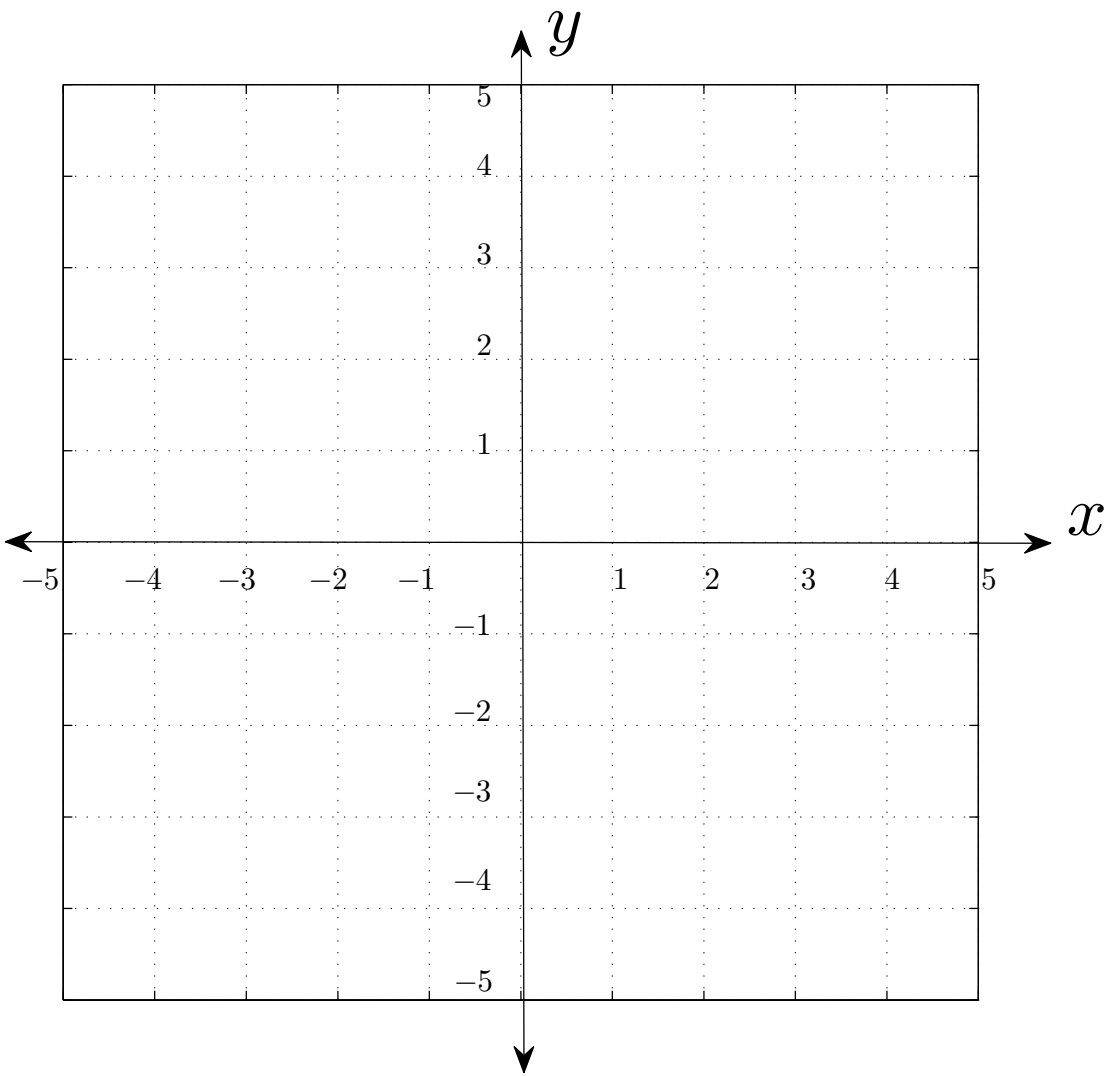
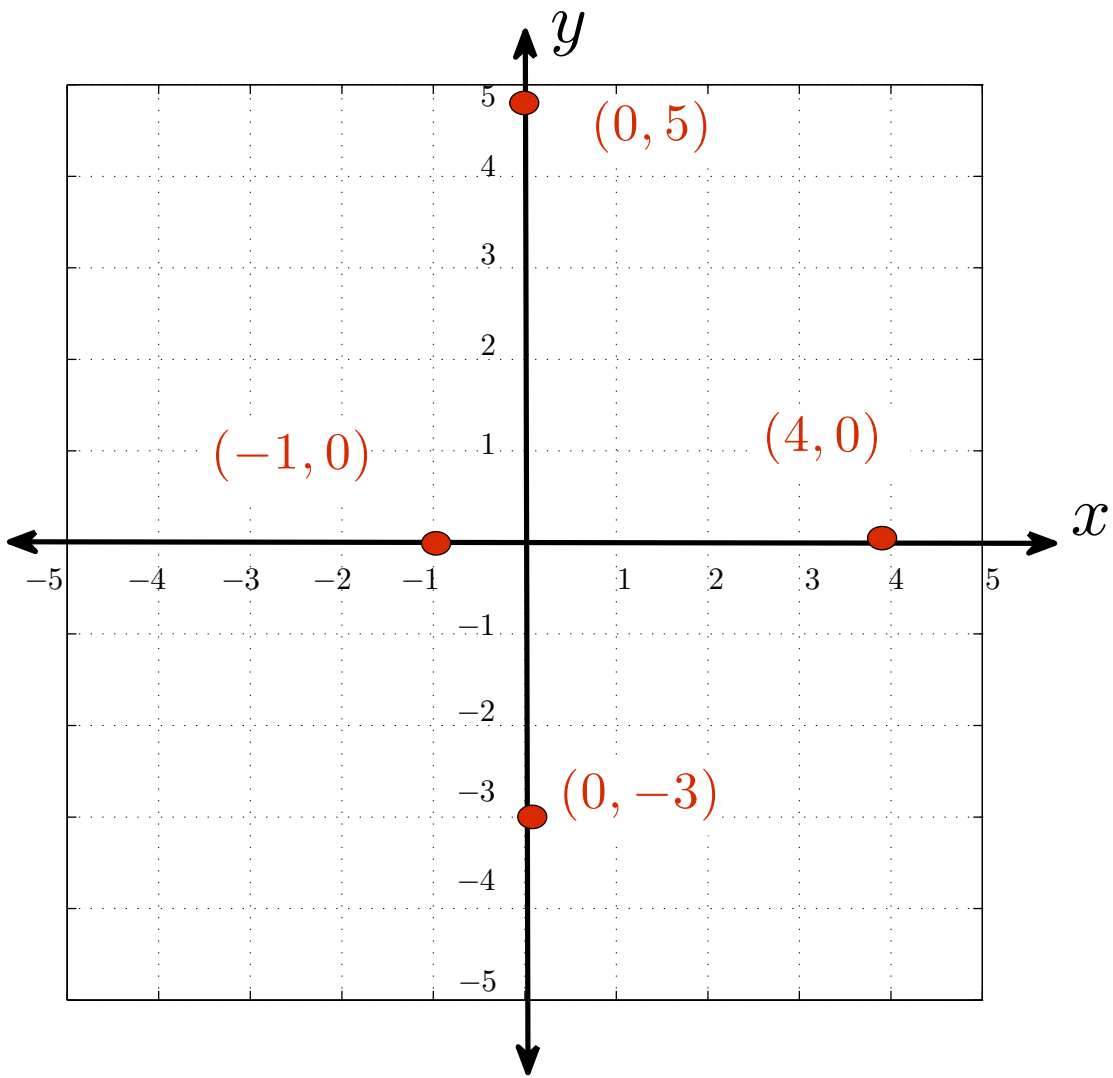


Plot (graph) $(4, 0)$, $(0, -3)$, $(-1, 0)$, and $(0, 5)$



Plot (graph) $(4, 0)$, $(0, -3)$, $(-1, 0)$, and $(0, 5)$



Definition

Suppose A, B and C represent any real numbers. A **linear equation in two variables** is an equation having the *form*

$$A\,x + B\,y = C,$$

For example, $2\,x + 3\,y = 1$ is a linear equation in the two variables x and y.

Solutions of a linear equation in two variables

Any linear equation in two variables always has in infinite number of solutions, and solutions come in the form of ordered pairs.

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Any linear equation in two variables always has in infinite number of solutions, and solutions come in the form of ordered pairs.

Terminology	Definition	Illustration
Solution of an equation in x and y	An ordered pair (a, b) that yields a true statement if $x = a$ and $y = b$	(1,4) is a solution of $y = 5x - 1$, since substituting x=1 and $y = 4$ renders the LHS = 4 and the RHS $= 5(1) - 1 = 4$

LHS is an abbreviation for "left-hand side" (of the equation)

RHS is an abbreviation for "right-hand side" (of the equation)

Equations and Graphs

Definition

For each ordered-pair solution, (a, b), of an equation in x and y there is a point (a, b) in a rectangular coordinate plane. The set of all such points is called a **graph of the equation**.

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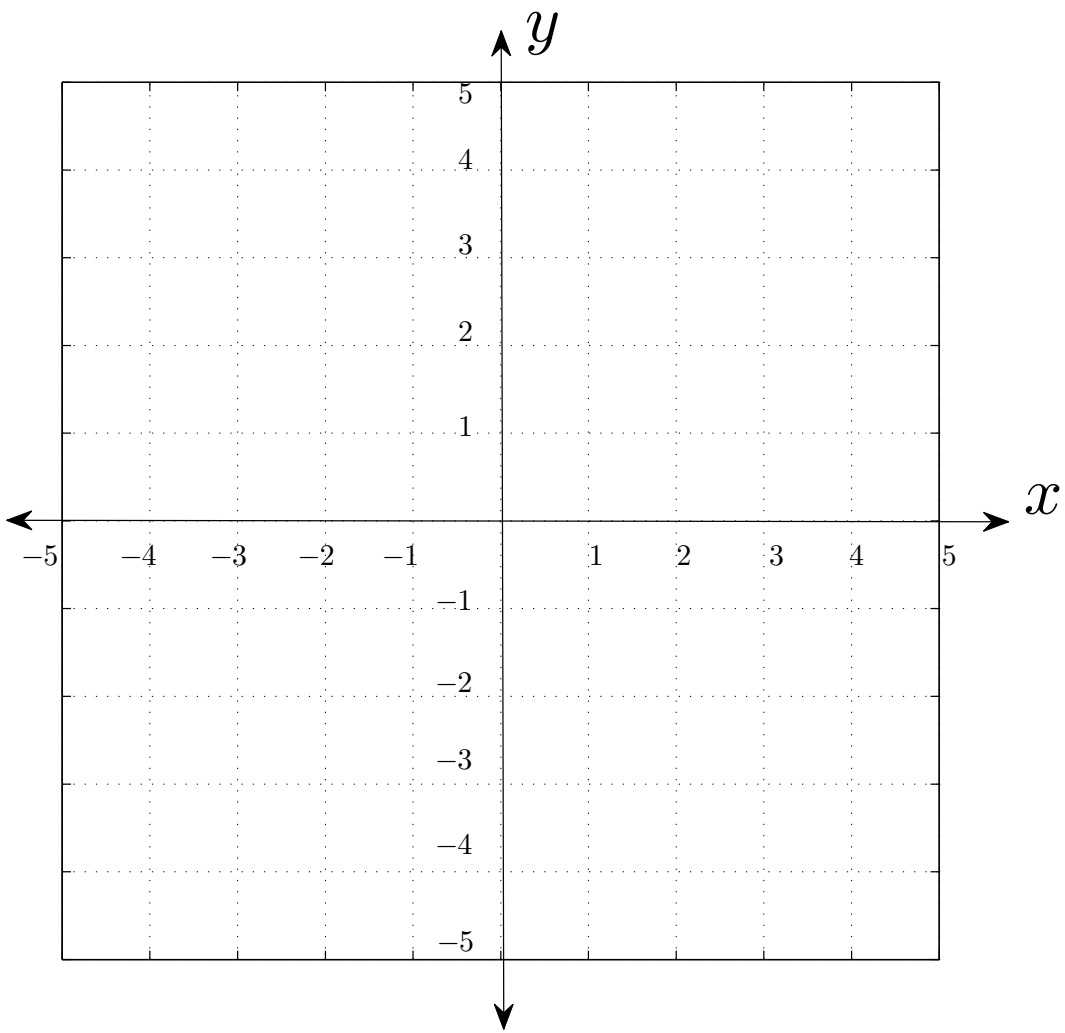
We can graph a linear equation by finding 3 ordered-pair solutions of the equation, plot the corresponding points on the rectangular grid, then draw a line between the three points.

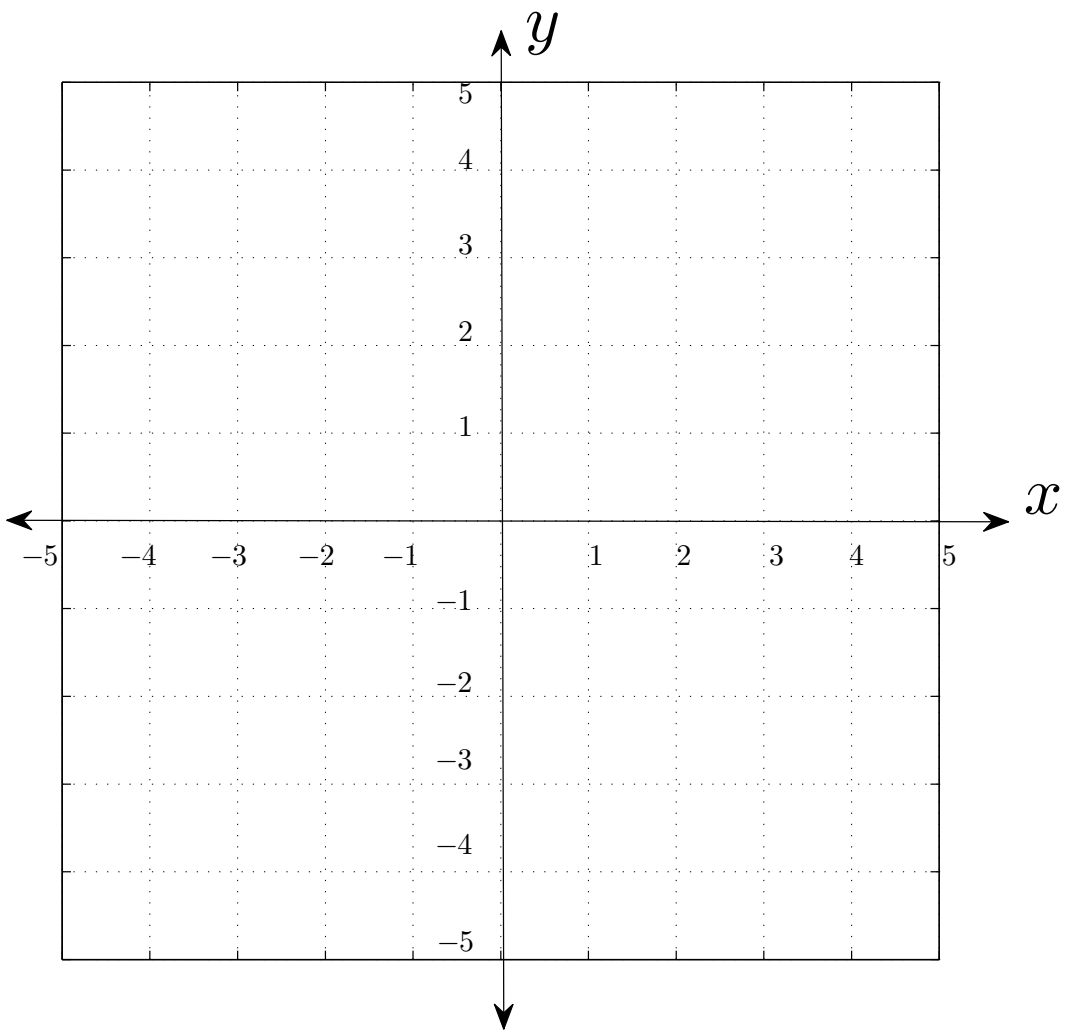
We use the third point for "insurance." If all three points line up in a straight we have not made a mistake!

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$



Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

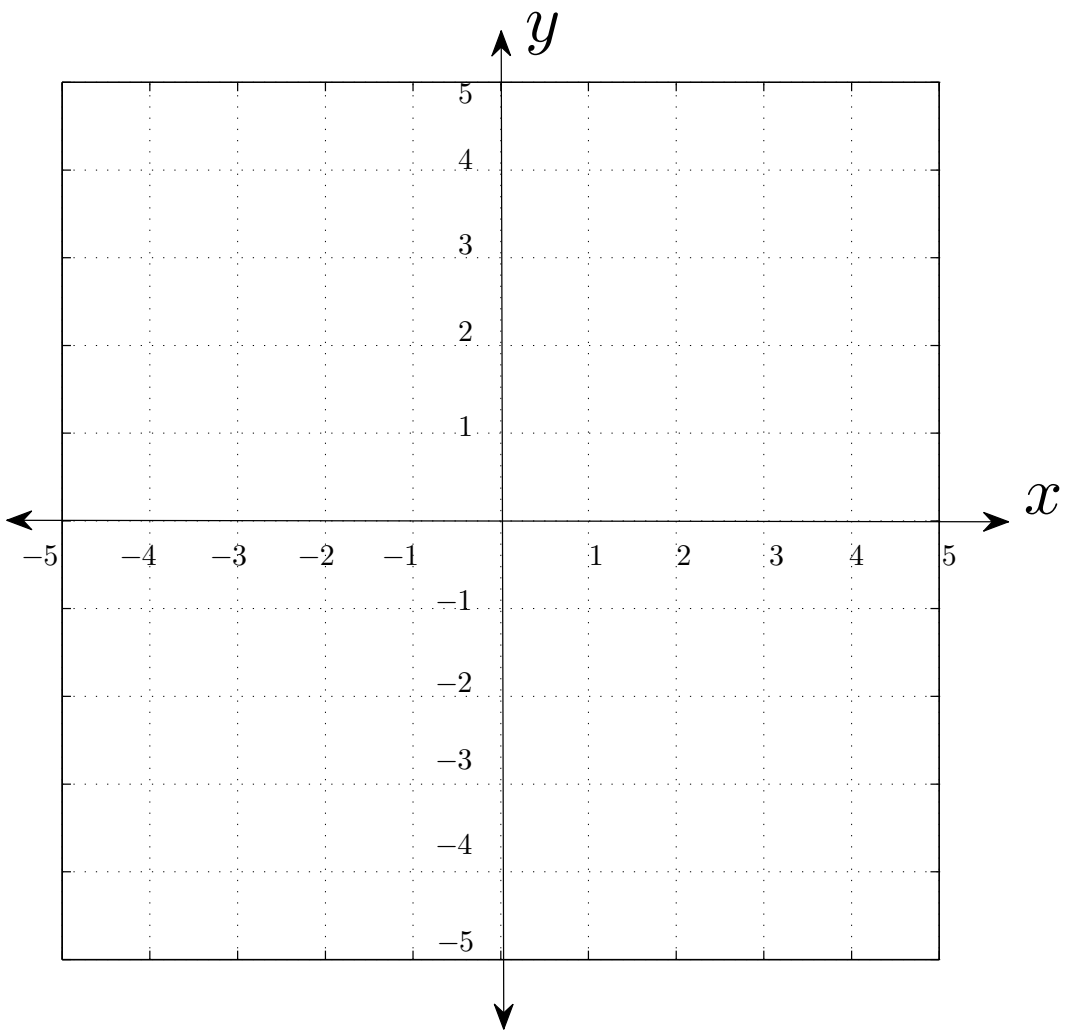
We begin by a making a table that summarizes x and y values. Since every value of x we substitute into the equation will be multiplied by $-\frac{1}{2}$, we use numbers for x that are divisible by 2.



x	y	(x, y)
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

That way, when we multiply by $-\frac{1}{2}$, the result will be an integer.



x	y	(x, y)
-2		
0		
2		

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We let $x = -2$ in the equation to find the y-value of the ordered pair which is associated with x-coordinate -2.

x	y	(x, y)
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We let $x = -2$ in the equation to find the y-value of the ordered pair which is associated with x-coordinate -2.

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (-2) - 3$$

$$= 1 - 3$$

$$= -2$$

x	y	(x, y)
-2		
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Upon simplification, we get the ordered pair solution (-2,-2)

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (-2) - 3$$

$$= 1 - 3$$

$$= -2$$

x	y	(x, y)
-2	-2	(-2,-2)
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Next, we let $x = 0$ in the equation to find the y-value of the ordered pair which is associated with x-coordinate 0.

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (0) - 3$$

$$= 0 - 3$$

$$= -3$$

x	y	(x, y)
-2	-2	(-2,-2)
0		
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

This gives us the ordered pair solution (0,-3)

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (0) - 3$$

$$= 0 - 3$$

$$= -3$$

x	y	(x, y)
-2	-2	(-2,-2)
0	-3	(0,-3)
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Afterwards, we let $x = 2$ in the equation.

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (2) - 3$$

$$= -1 - 3$$

$$= -4$$

x	y	(x, y)
-2	-2	(-2,-2)
0	-3	(0,-3)
2		

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

Upon simplification, we get the ordered pair solution (2,-4)

$$y = -\frac{1}{2} \cdot (x) - 3$$

$$= -\frac{1}{2} \cdot (2) - 3$$

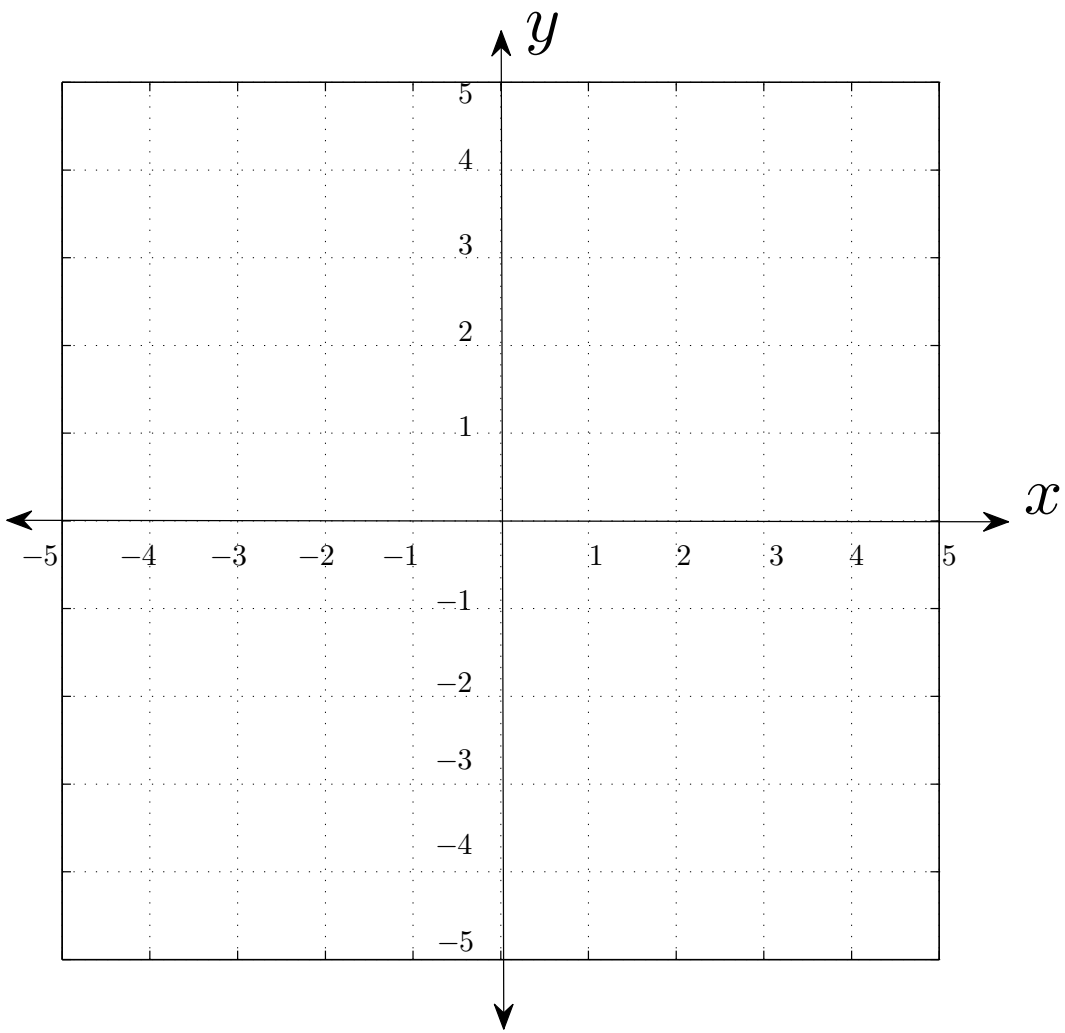
$$= -1 - 3$$

$$= -4$$

x	y	(x, y)
-2	-2	(-2,-2)
0	-3	(0,-3)
2	-4	(2,-4)

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We now locate the three ordered pair solutions (points) on the rectangular coordinate grid, then draw a line through the solutions.



x	y	(x, y)
-2	-2	(-2, -2)
0	-3	(0,-3)
2	-4	(2, -4)

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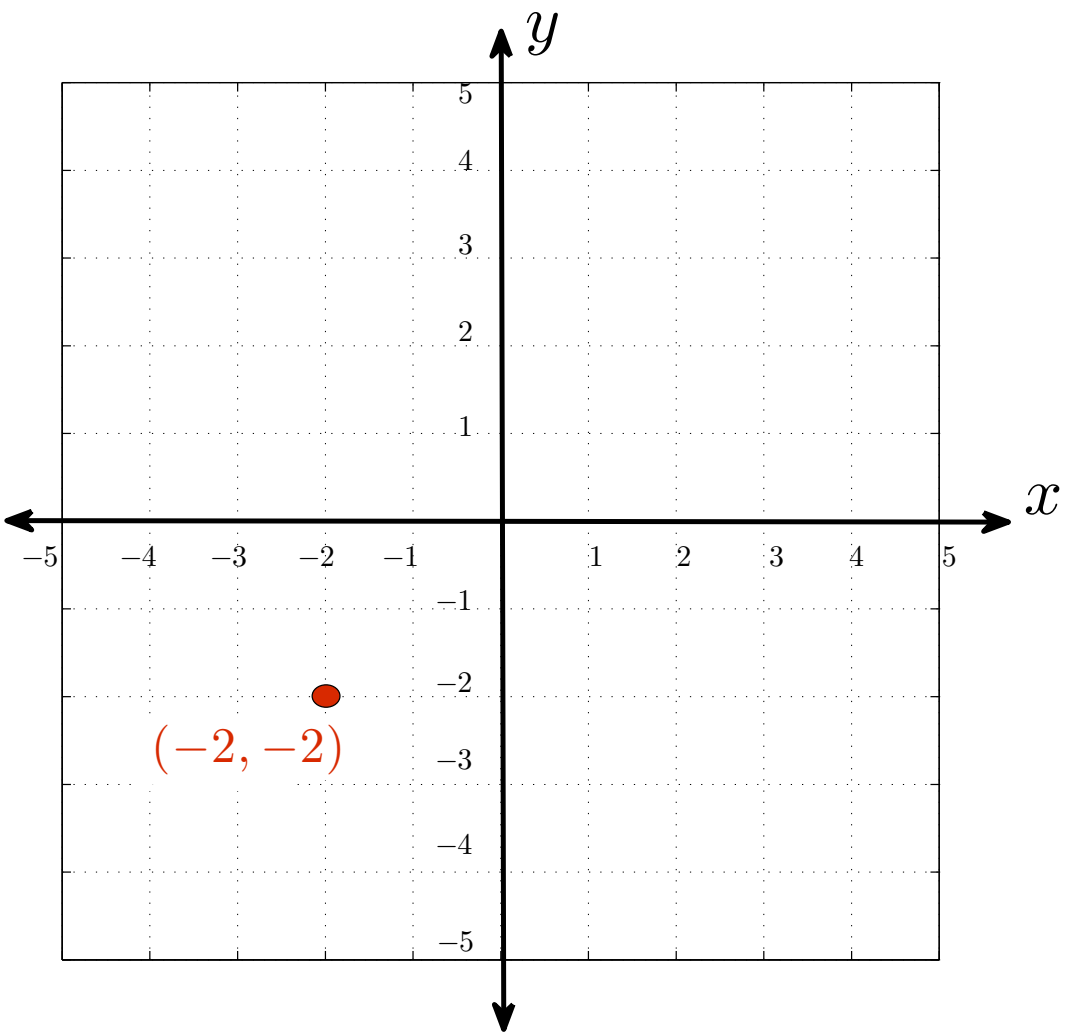
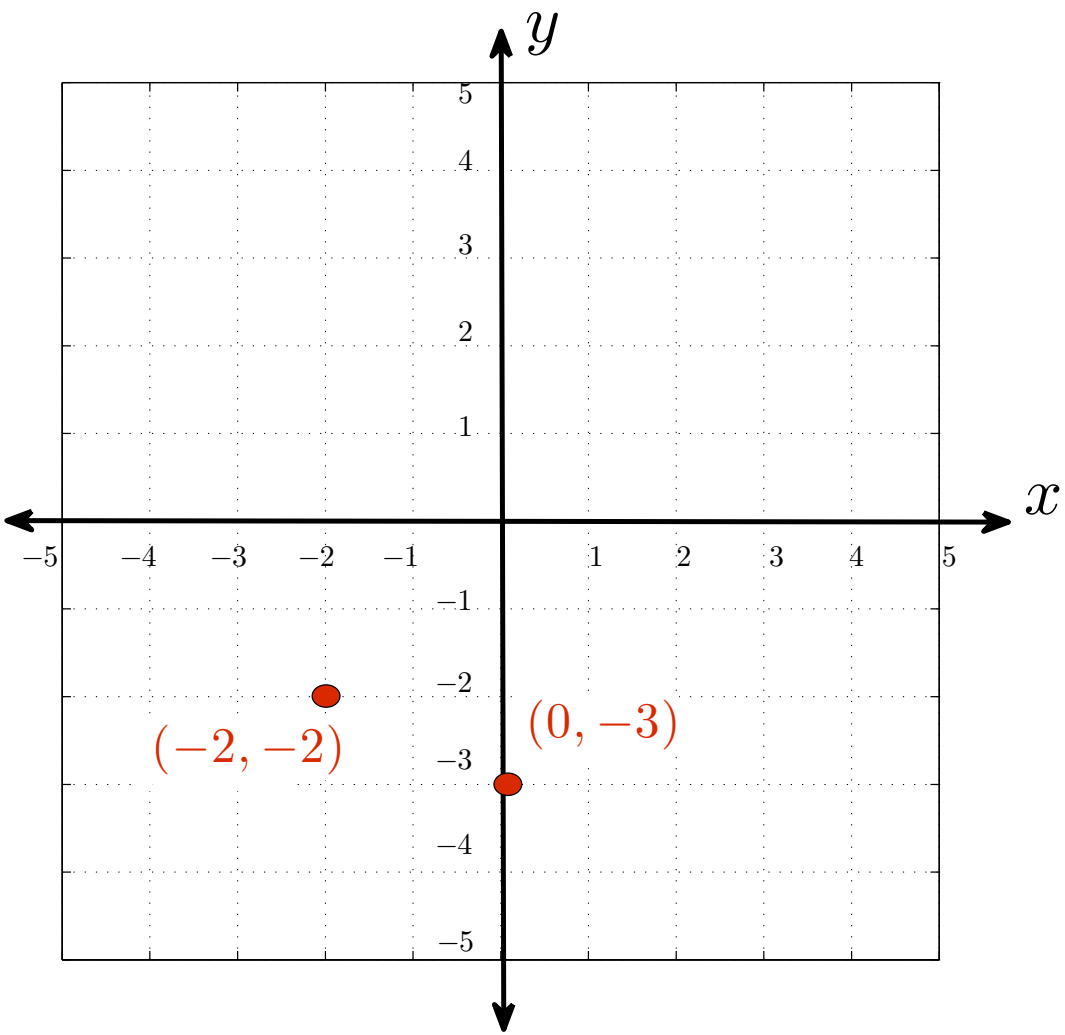


x	y	(x, y)
-2	-2	(-2, -2)
0	-3	(0,-3)
2	-4	(2, -4)

Example 3: Graph the linear equation $y = -\frac{1}{2}x - 3$

We now locate the three ordered pair solutions (points) on the rectangular coordinate grid, then draw a line through the solutions.



x	y	(x, y)
-2	-2	(-2, -2)
0	-3	(0,-3)
2	-4	(2, -4)

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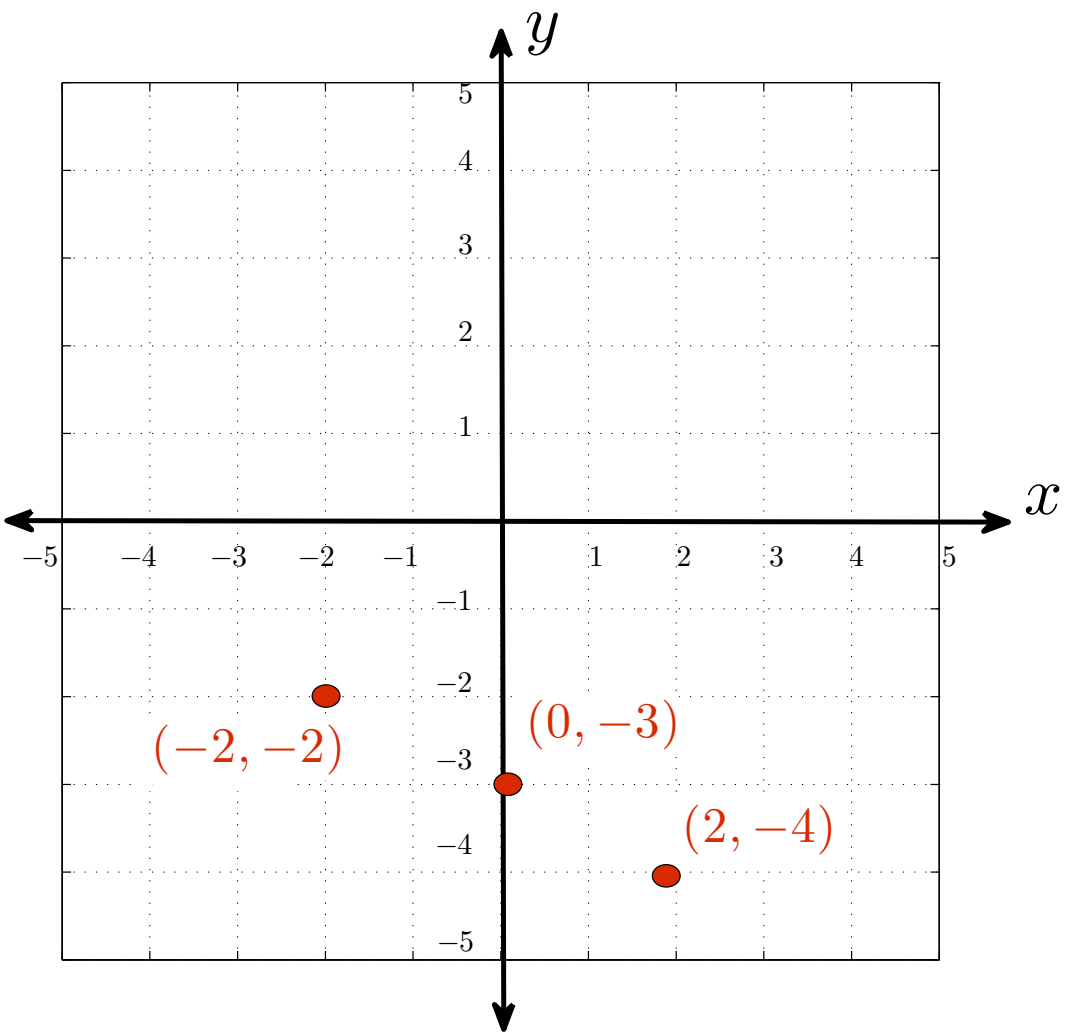

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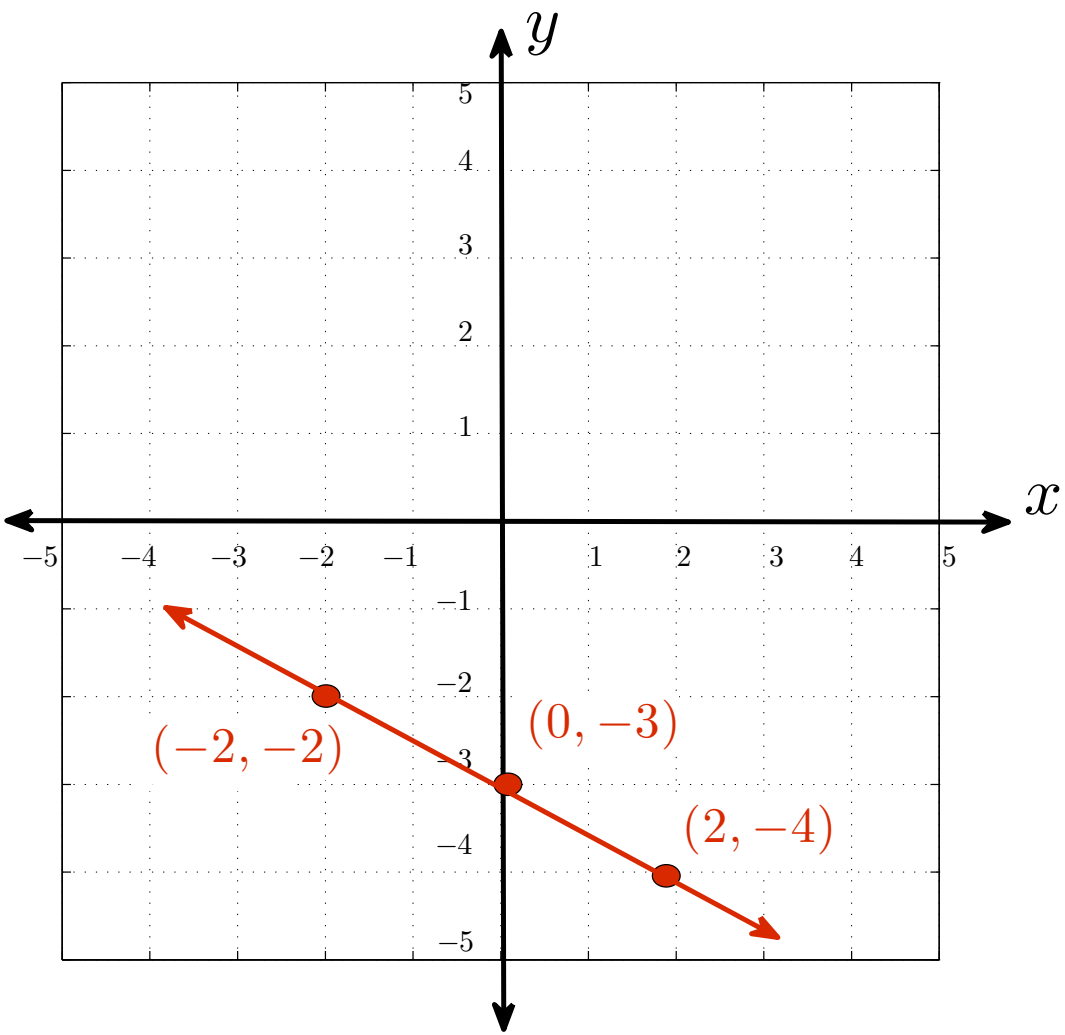

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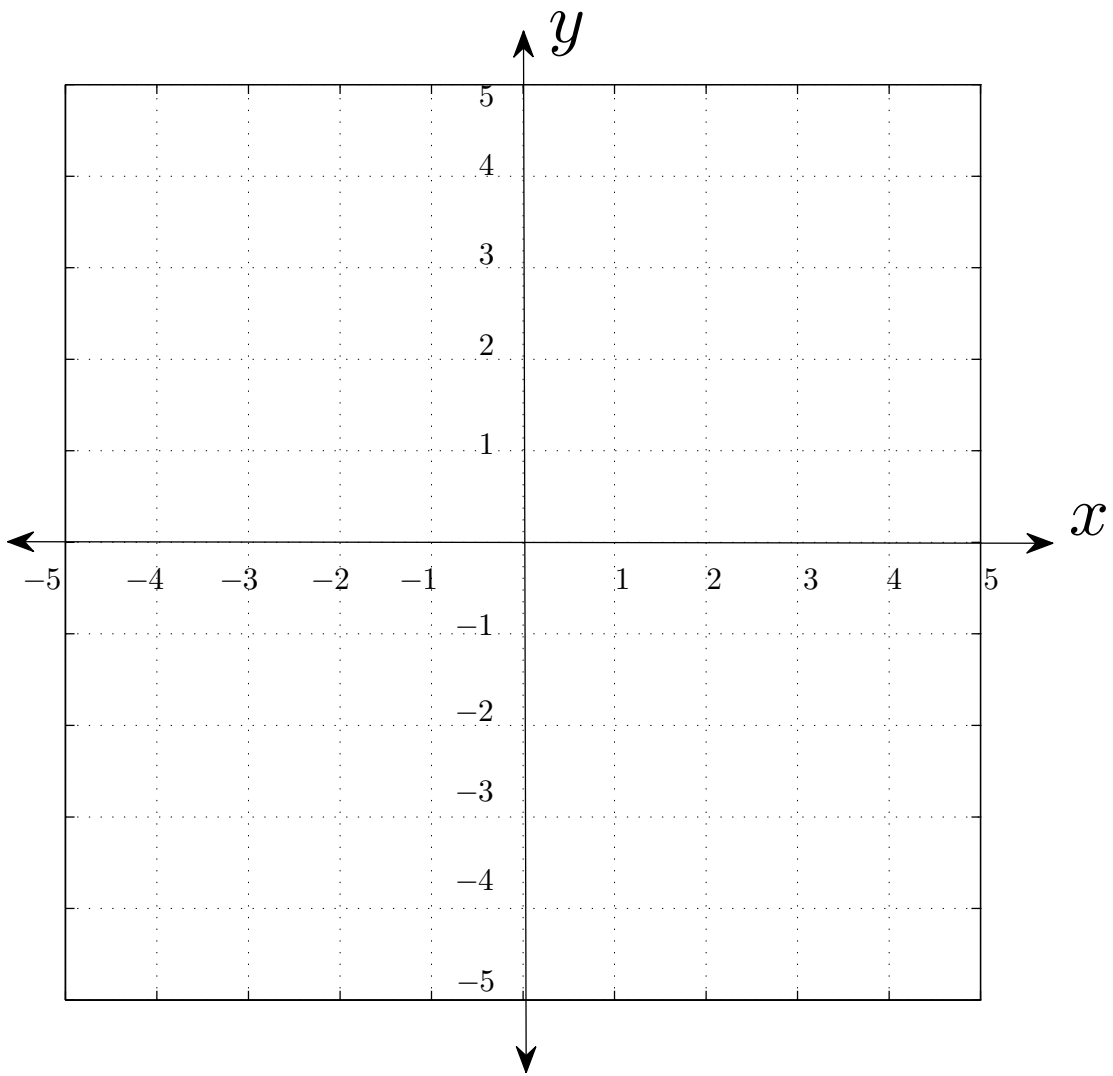
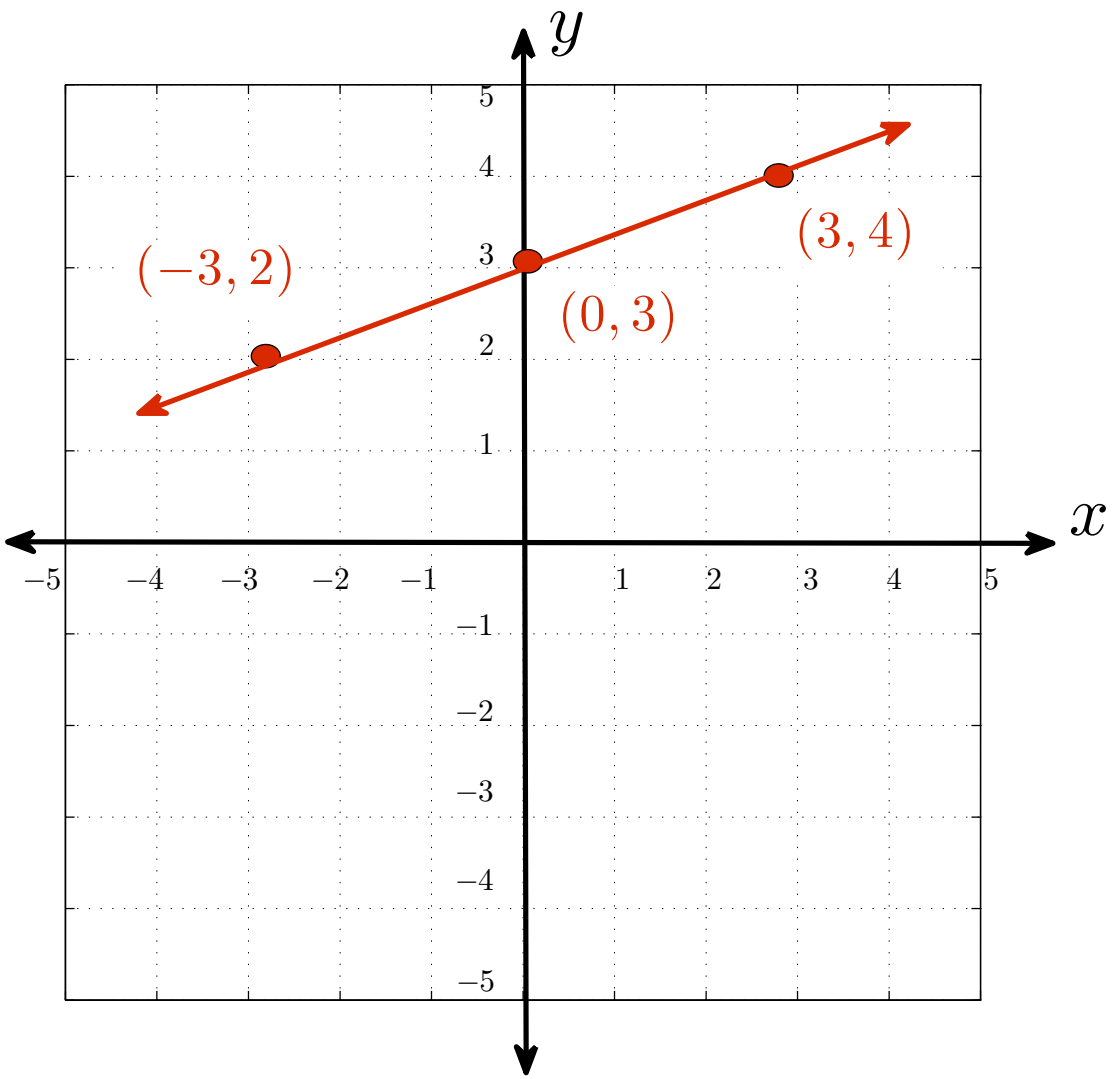
Concept Check: Graph $y = \frac{1}{3}x + 3$



x	y	(x, y)

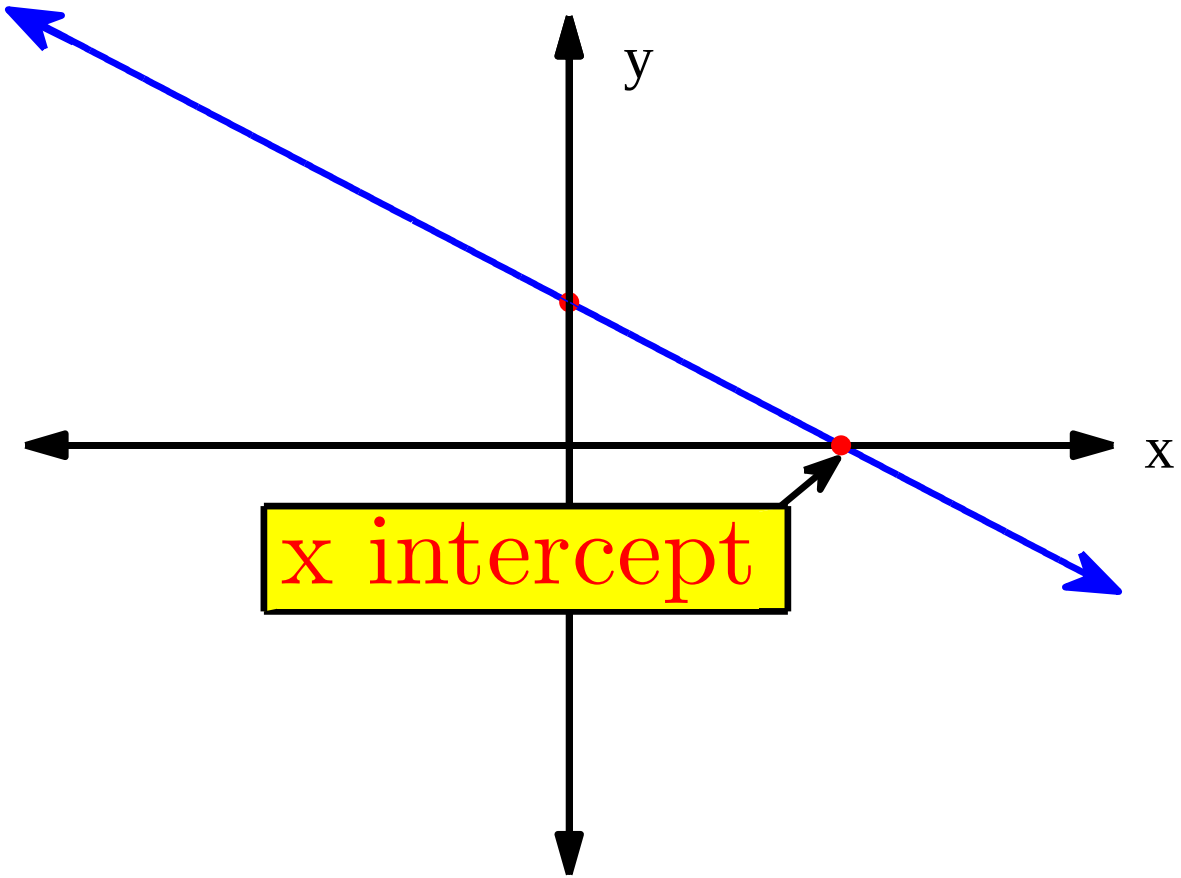
Concept Check: Graph $y = \frac{1}{3}x + 3$



x	y	(x, y)
-3	2	(-3,2)
0	3	(0,3)
3	4	(3,4)

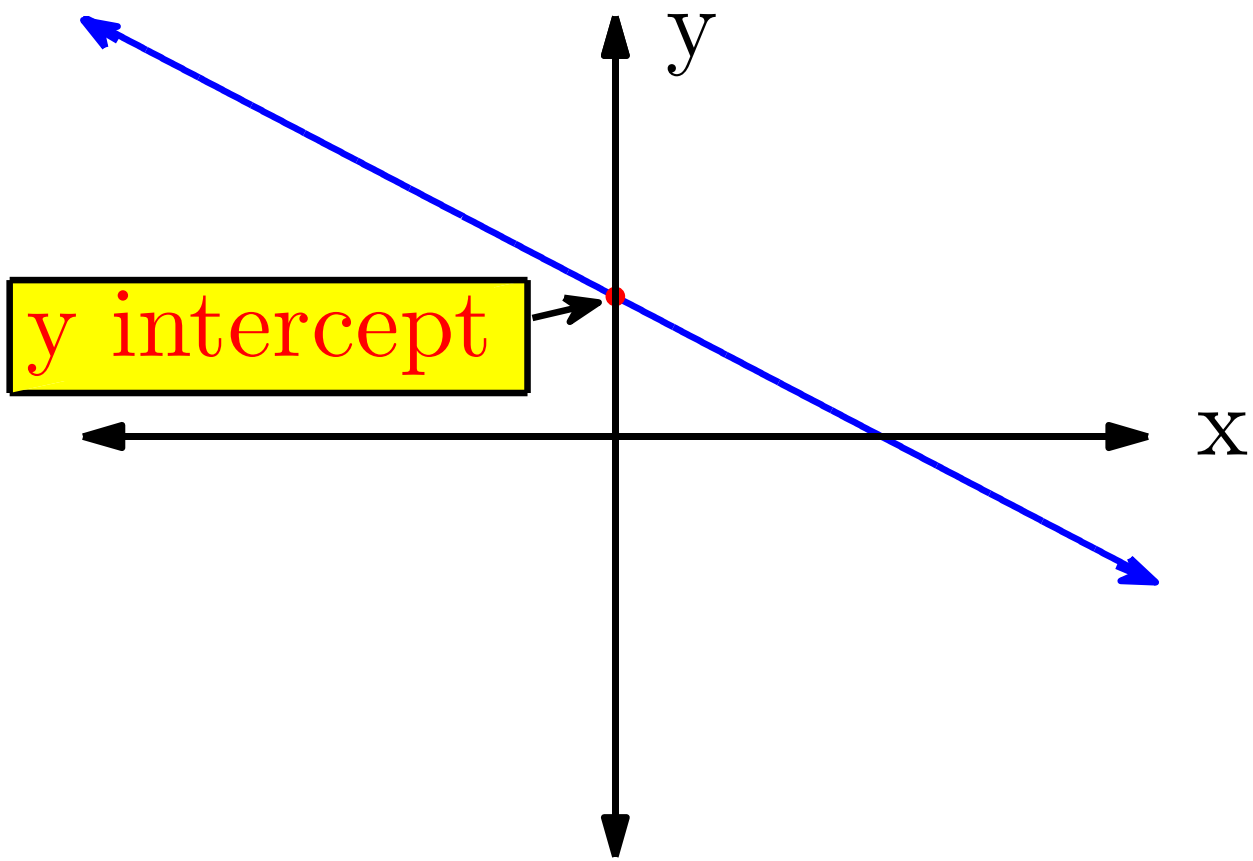
Definition

The graph of an equation has an **x-intercept** whenever the graph of the equation crosses the x axis. The x intercept always occurs when the value of y is equal to zero.



Definition

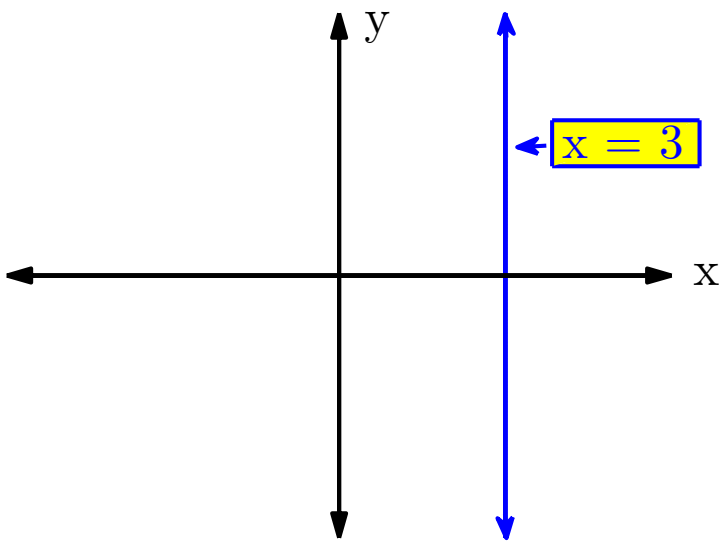
The graph of an equation has an **y-intercept** whenever the graph of the equation crosses the y axis. The y intercept always occurs when the value of x is equal to zero.



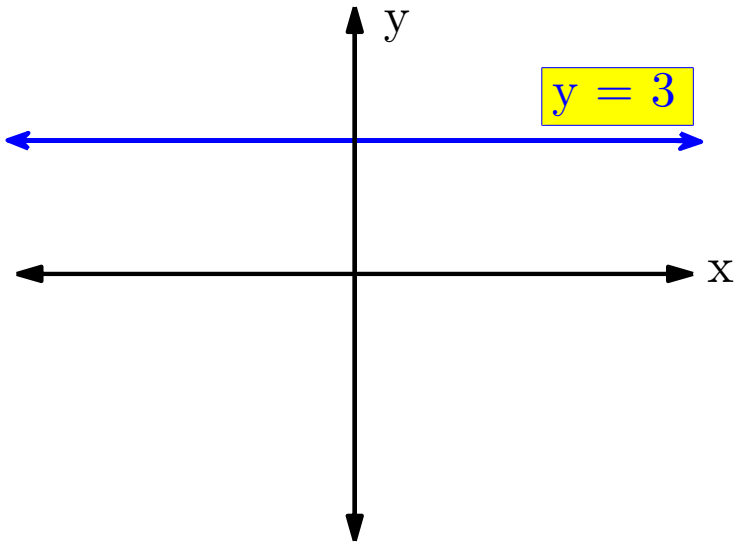
Example 4 Find the x- and y-intercepts for $5x - 7y = -35$, then graph the solution set.

Theorem

Suppose a and b are real numbers. Graphs of linear equations of the form $x = a$ are vertical lines and graphs of linear equations of the form $y = b$ are horizontal lines.



graph of $x = 3$



graph of $y = 3$

More Classroom Examples

Work Together! Stop at 8:35 a.m. for Quiz 1 Review
Graph each of the following lines:

- $y = \frac{1}{2}x$
- $x = -2$
- $y = -4$