

# The Slope of a Line

Professor Tim Busken

Department of Mathematics

September 16, 2014

# The Slope of a Line

## Learning Objectives:

- Find the slope of a line from its graph.

# The Slope of a Line

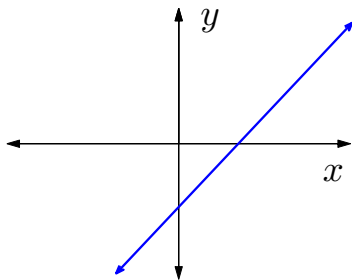
## Learning Objectives:

- Find the slope of a line from its graph.
- Find the slope of a line given two points on the line.

## Definition

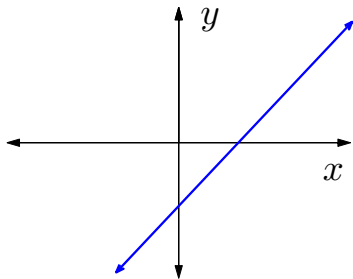
The slope of a line is a measure of the steepness of the line.

A line that rises from left to right  
has positive slope.

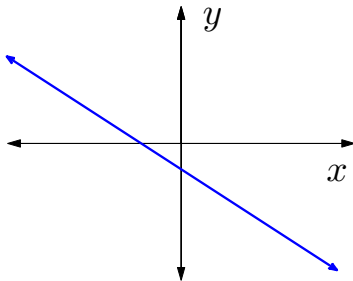


Positive Slope

A line that falls from left to right has negative slope.

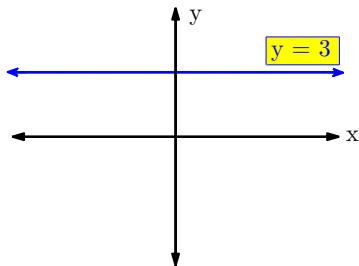


Positive Slope



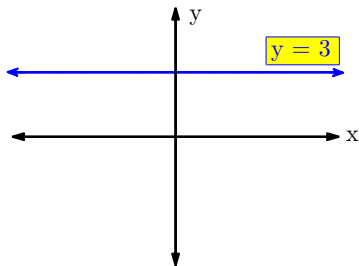
Negative Slope

# Horizontal lines have zero slope

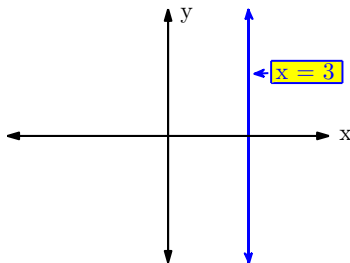


graph of  $x = 3$

Horizontal lines have zero slope,  
and vertical lines have no slope.



graph of  $x = 3$

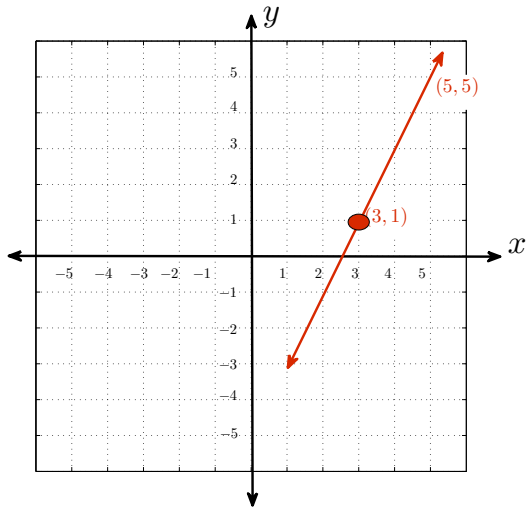


graph of  $y = 3$



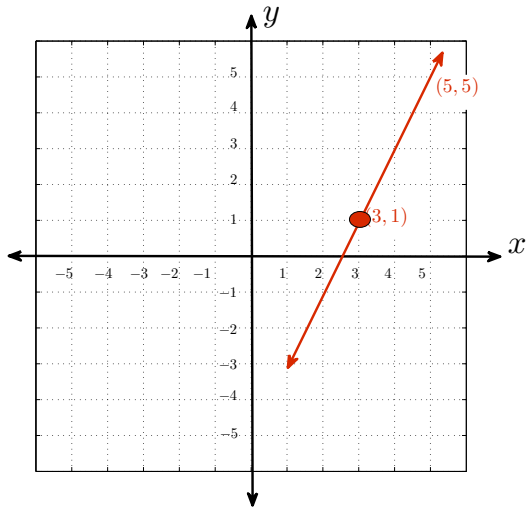
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



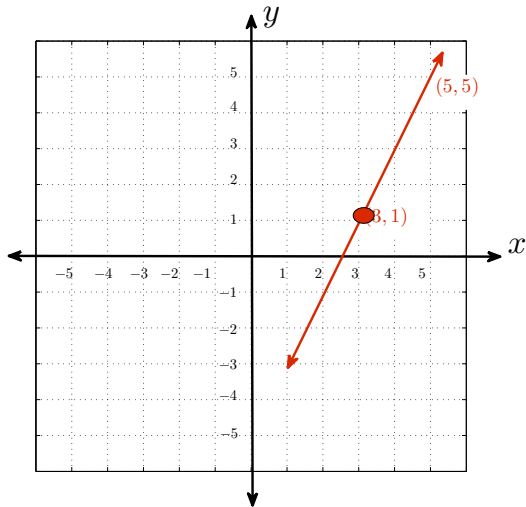
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



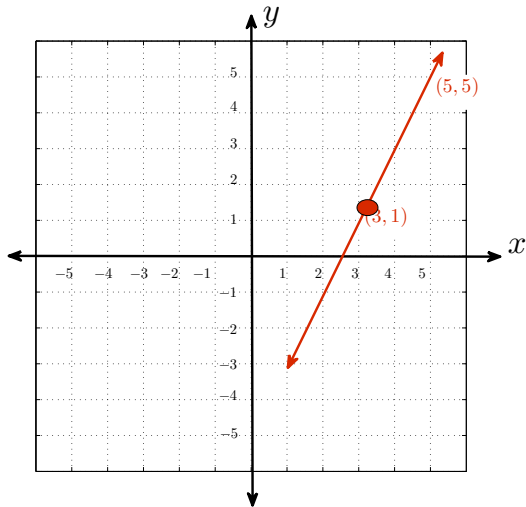
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



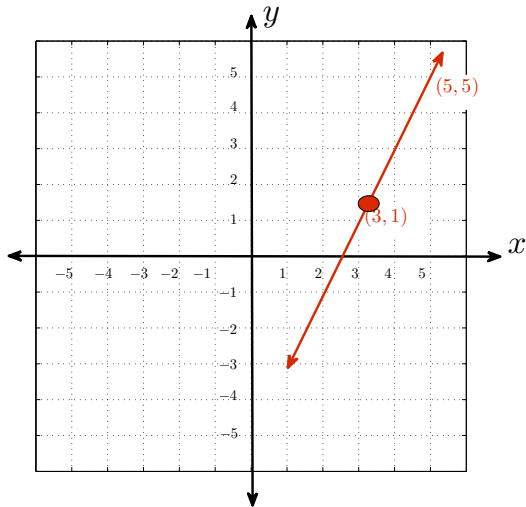
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



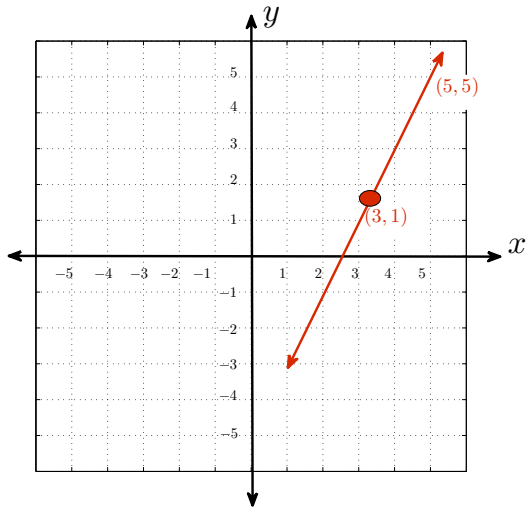
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



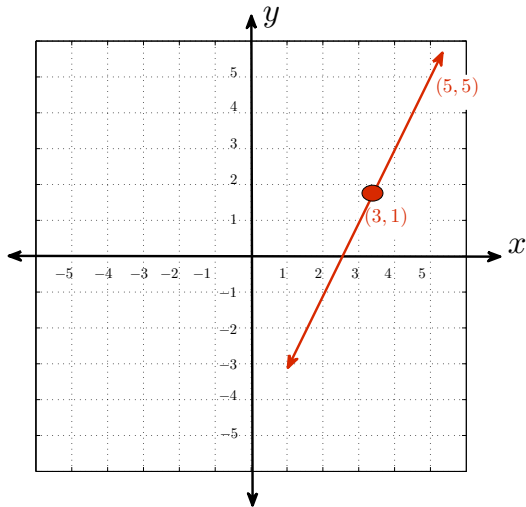
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



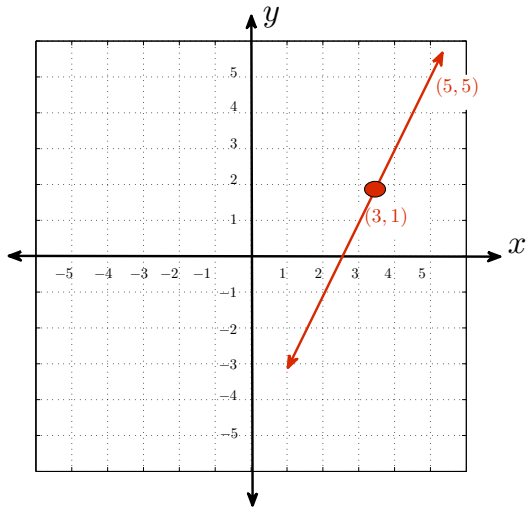
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



## The Slope of a Line

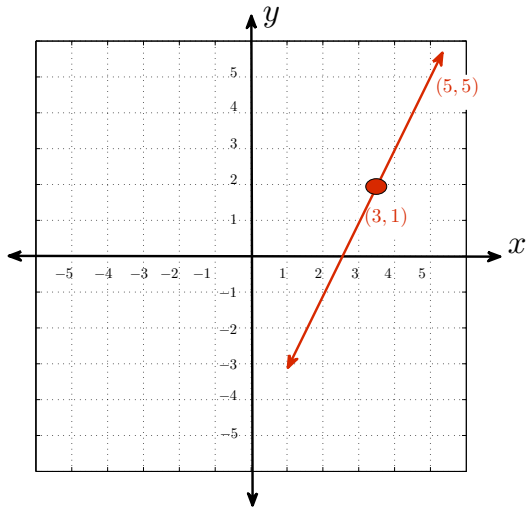
We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.





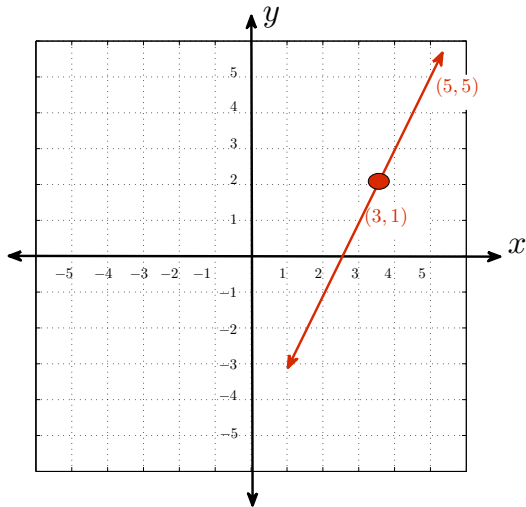
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



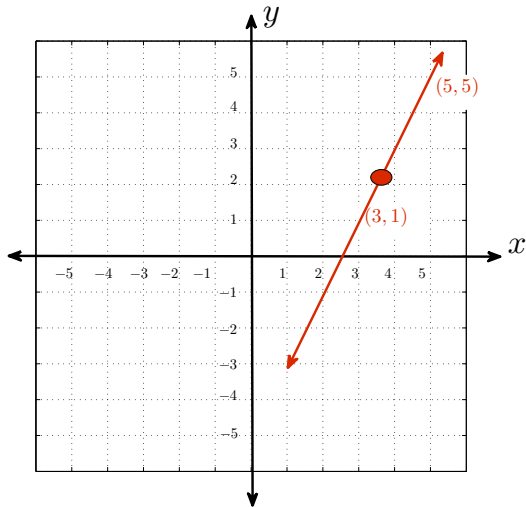
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



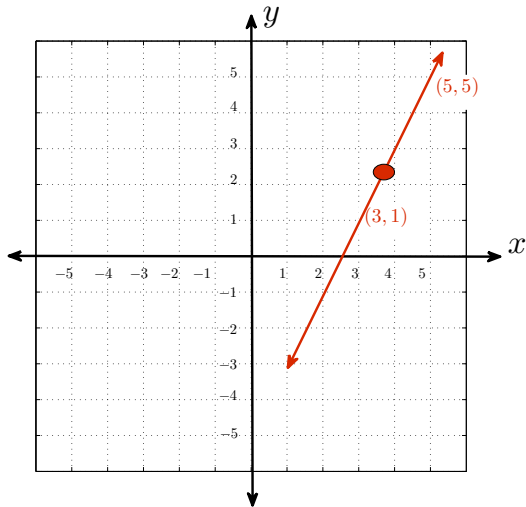
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



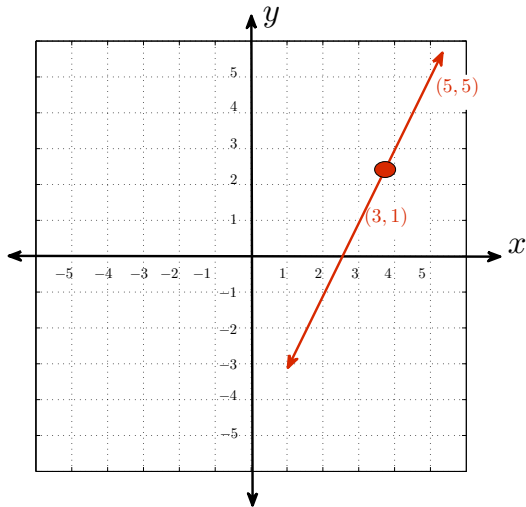
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



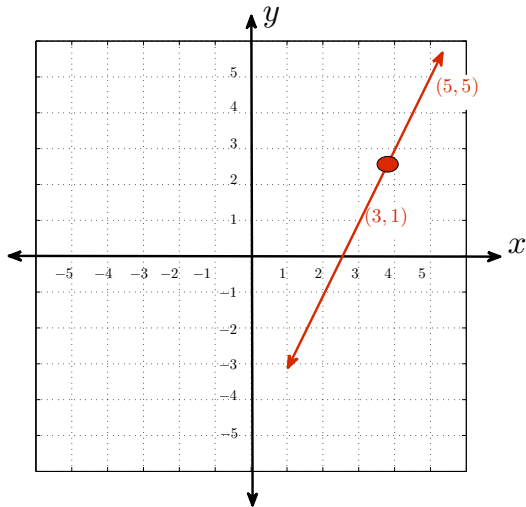
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



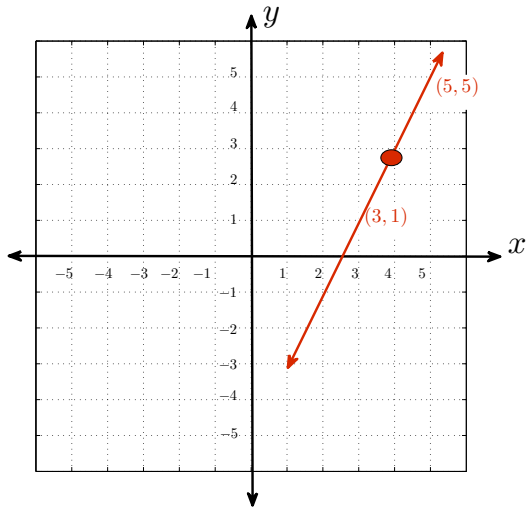
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



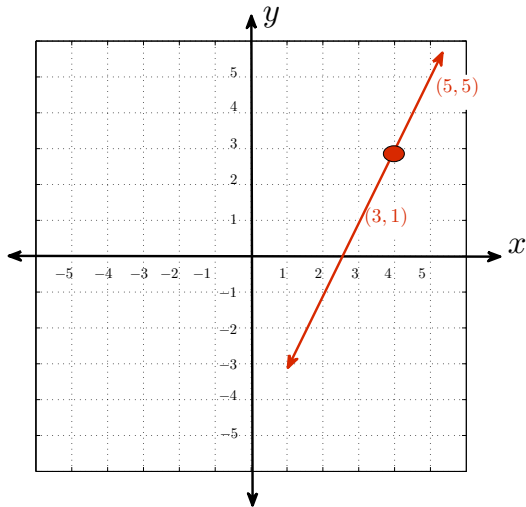
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



## The Slope of a Line

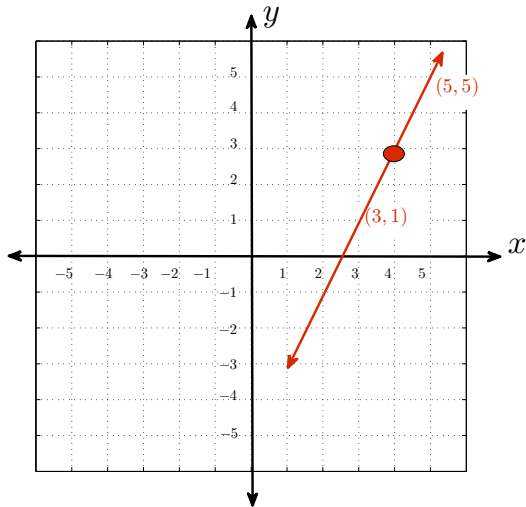
We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.





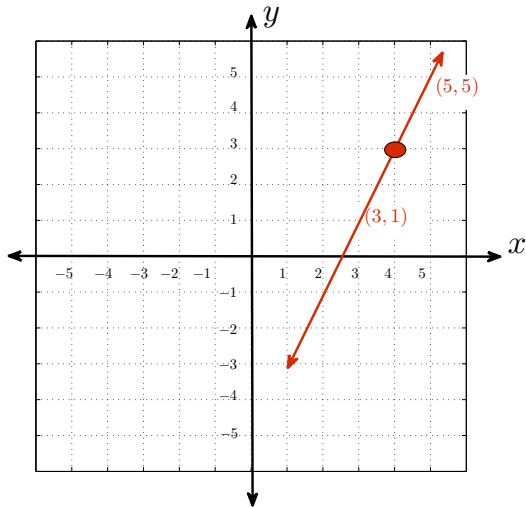
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



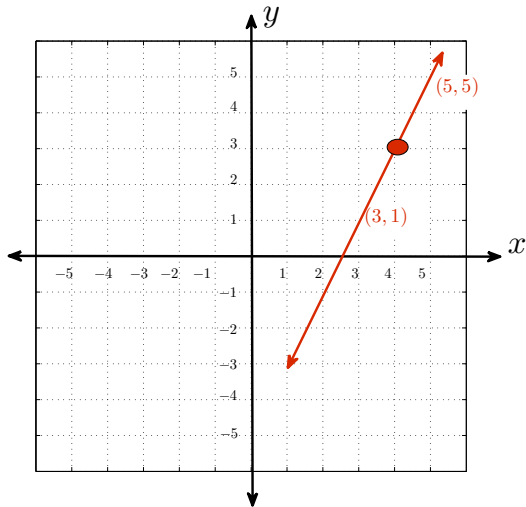
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



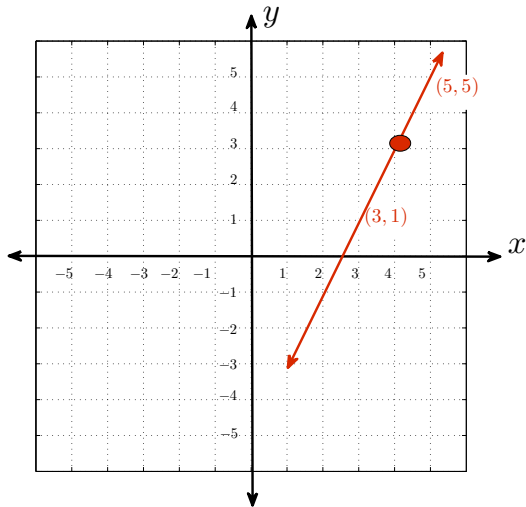
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



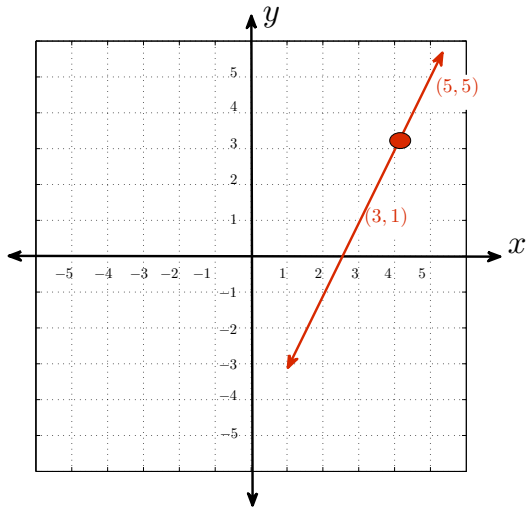
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



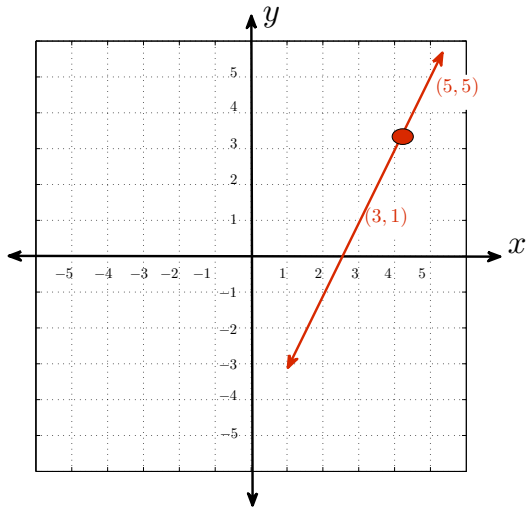
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



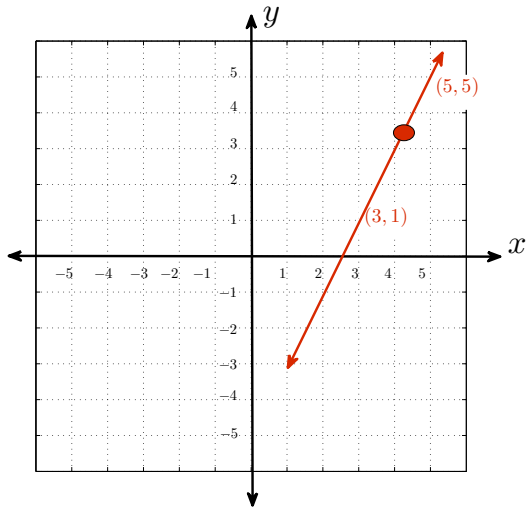
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



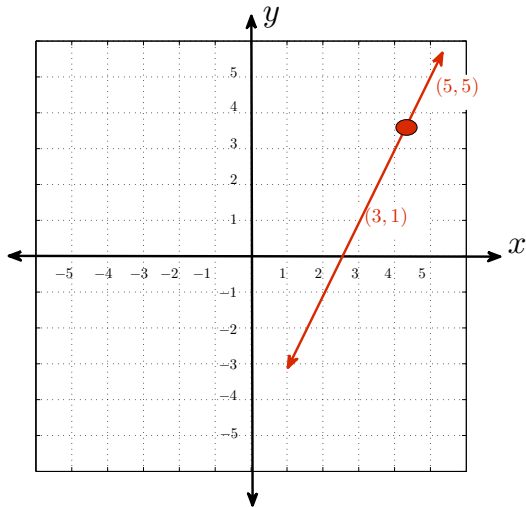
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



## The Slope of a Line

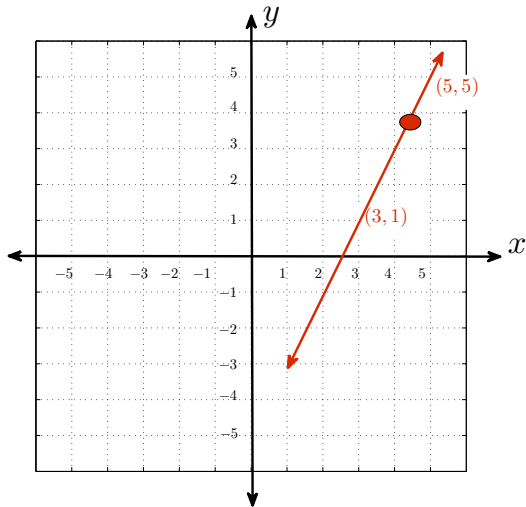
We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.





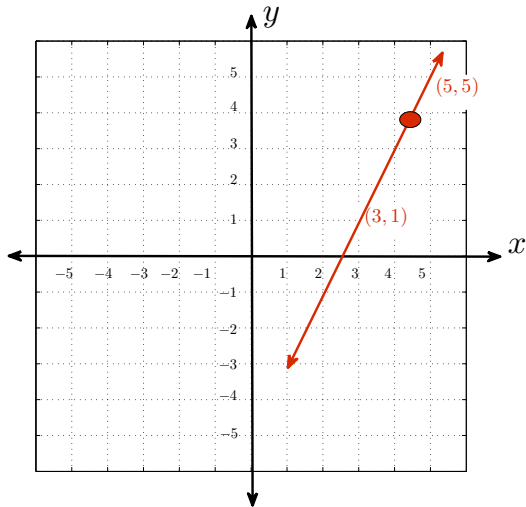
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



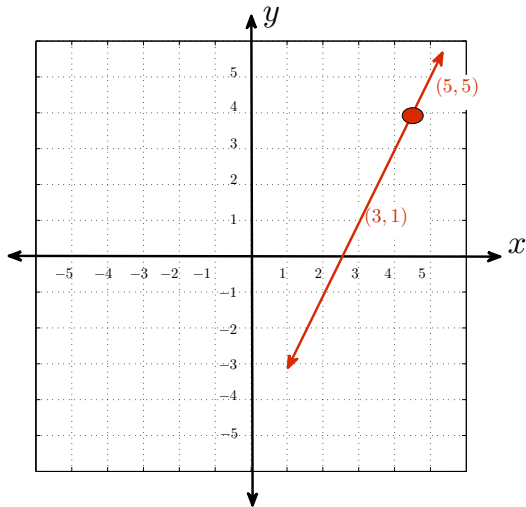
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



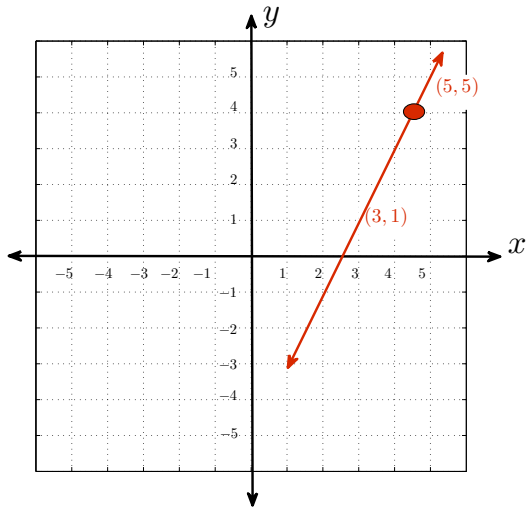
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



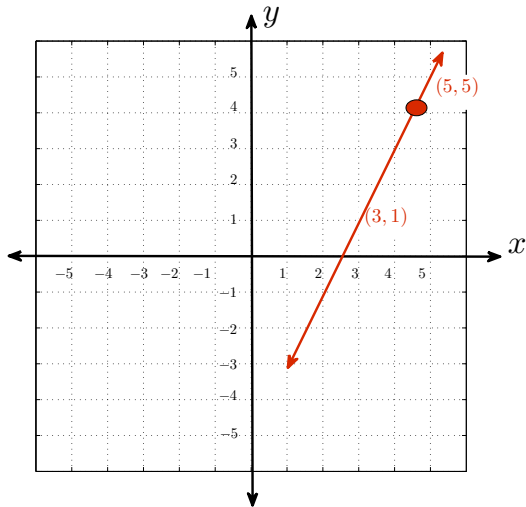
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



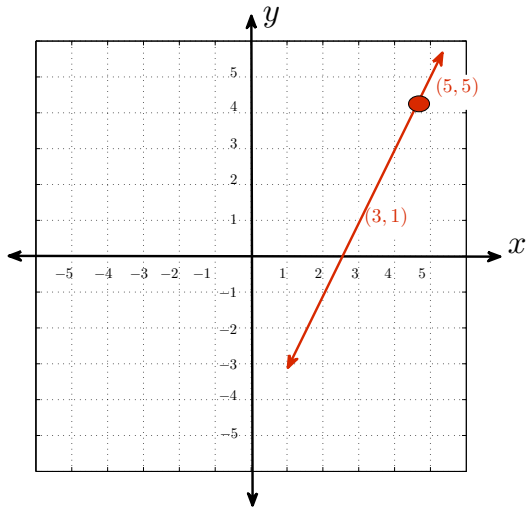
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



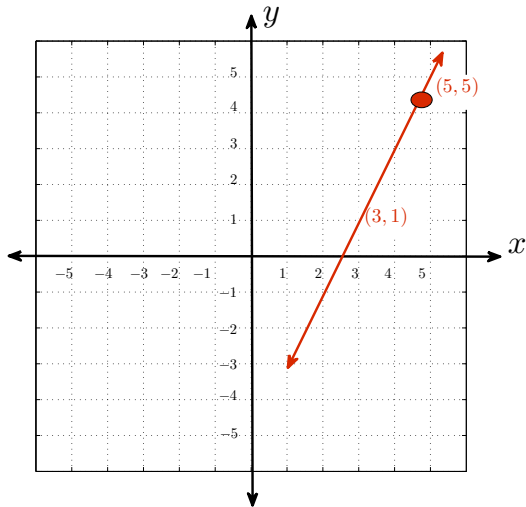
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



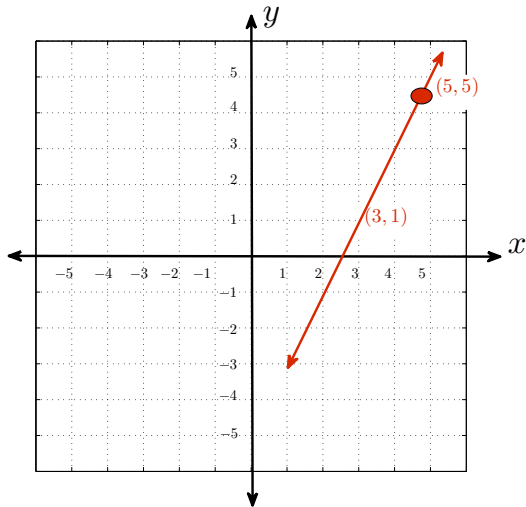
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



## The Slope of a Line

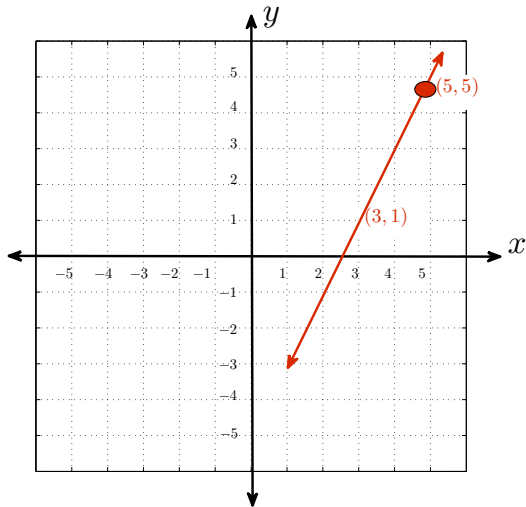
We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.





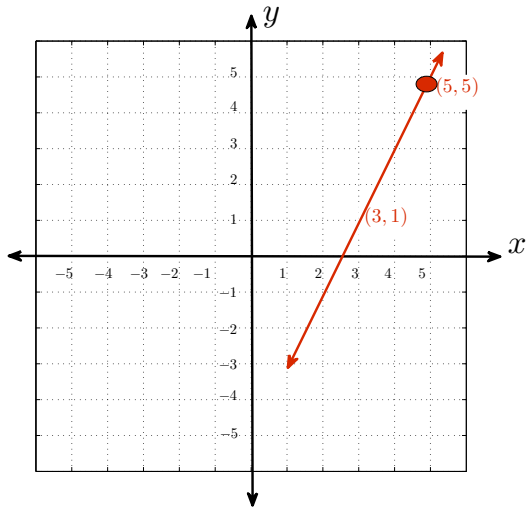
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



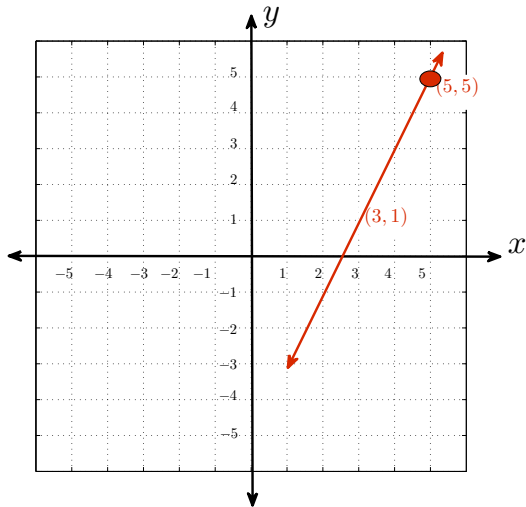
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



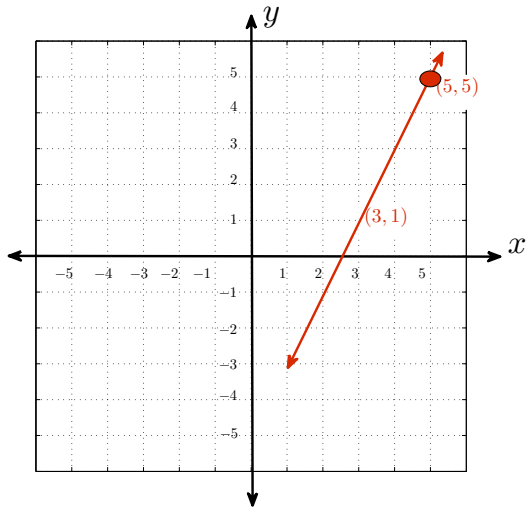
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



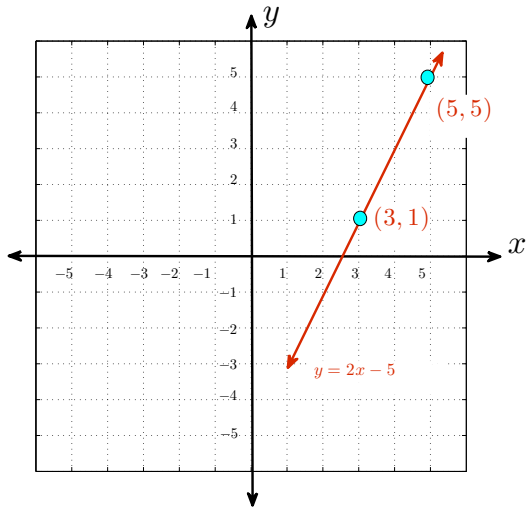
## The Slope of a Line

We can define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



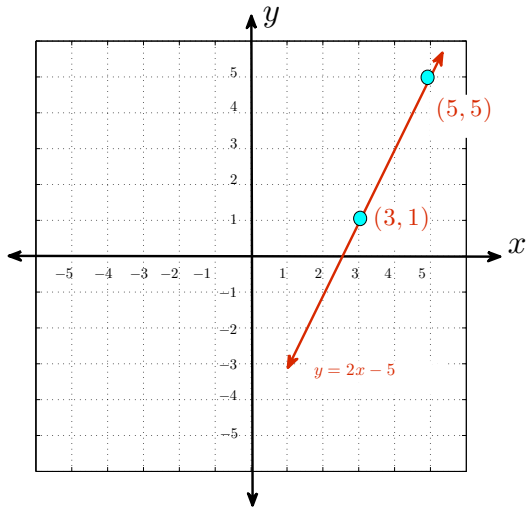
## The Slope of a Line

Geometrically, we define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



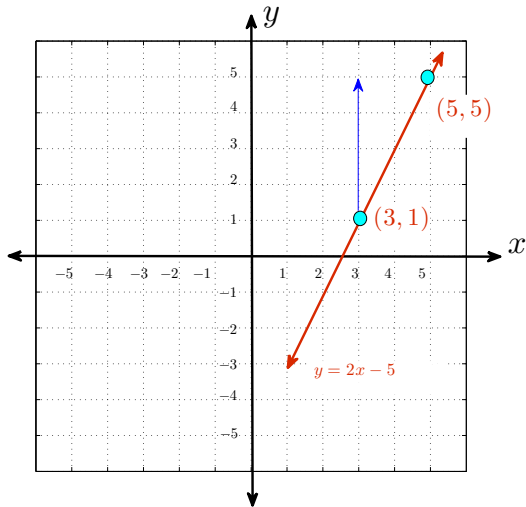
## The Slope of a Line

Geometrically, we define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



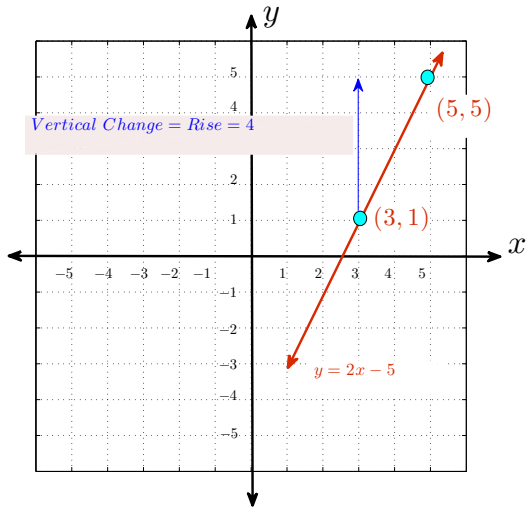
## The Slope of a Line

Geometrically, we define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



## The Slope of a Line

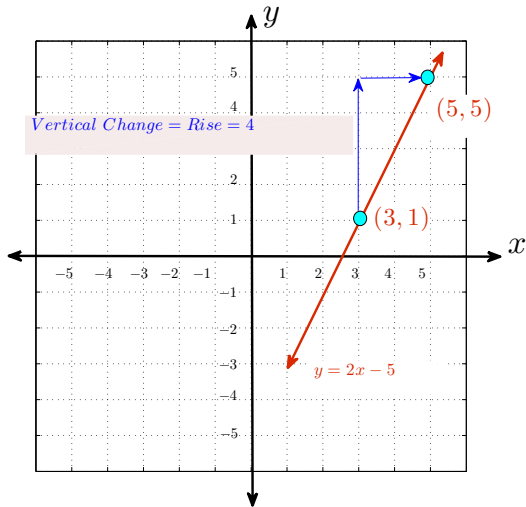
Geometrically, we define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.





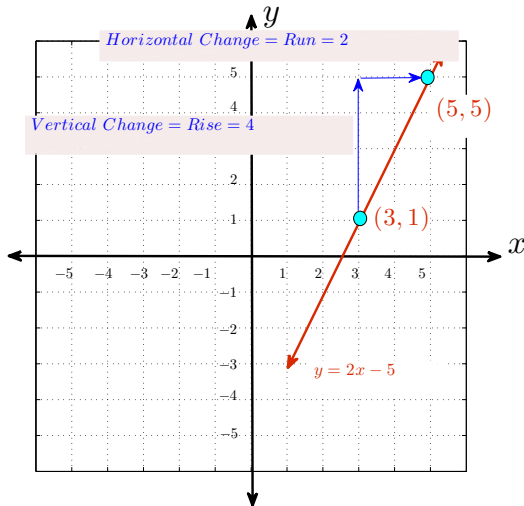
## The Slope of a Line

Geometrically, we define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



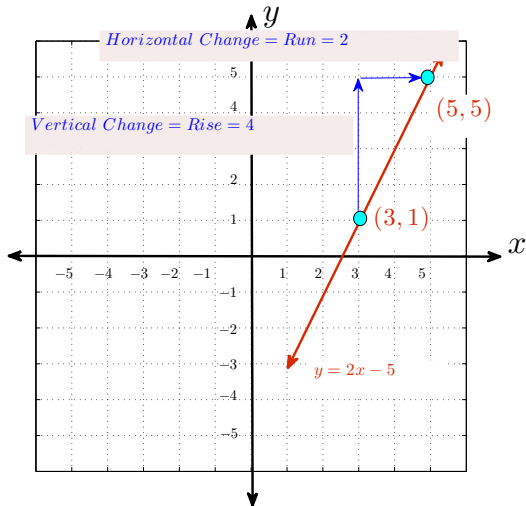
## The Slope of a Line

Geometrically, we define the **slope of a line** as the ratio of the vertical change to the horizontal change when moving from one point to another on the line.



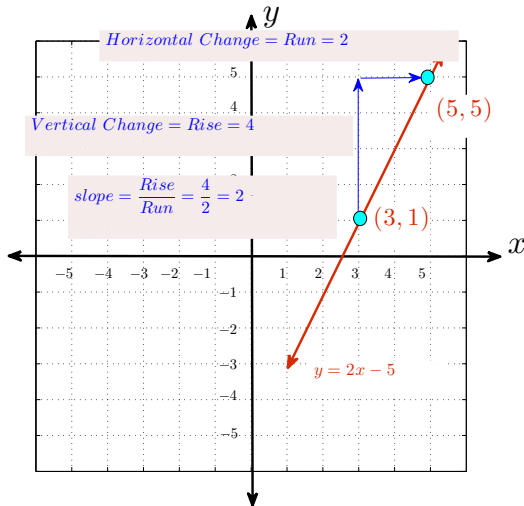
## The Slope of a Line

$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{4}{2} = 2$$



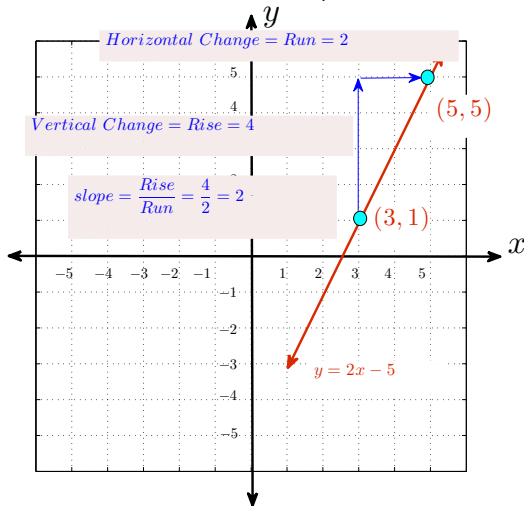
## The Slope of a Line

Notice that the vertical change is measured by subtracting the y-coordinates of the two points,  $5 - 1 = 4$ .

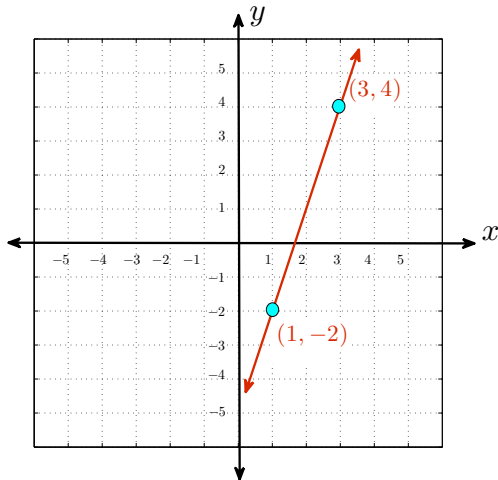


## The Slope of a Line

Notice that the vertical change is measured by subtracting the y-coordinates of the two points,  $5 - 1 = 4$ . The horizontal change is the difference between the x-coordinates,  $5 - 3 = 2$ .

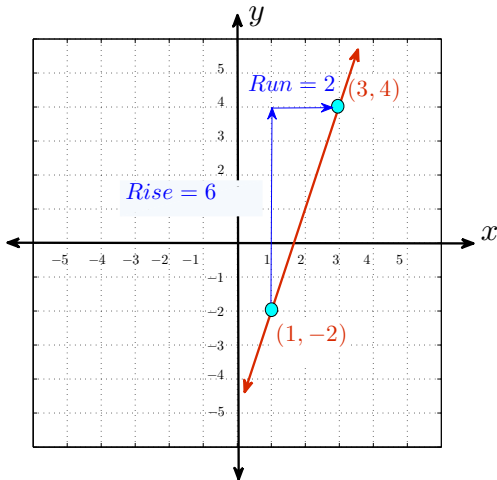


Ex. Find the slope of a line through  $(3, 4)$  and  $(1, -2)$ .



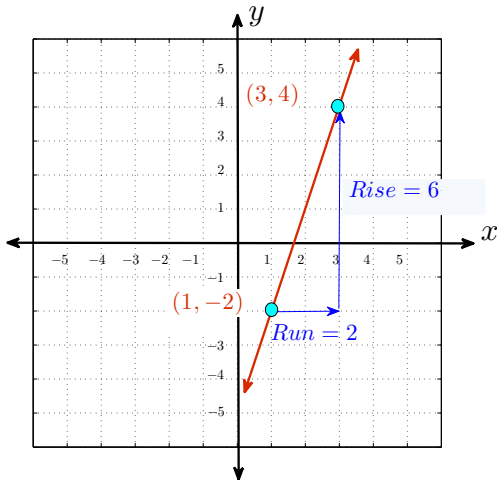
Ex. Find the slope of a line through (3, 4) and (1, -2).

$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{6}{2} = 3$$



Ex. Find the slope of a line through  $(3, 4)$  and  $(1, -2)$ .

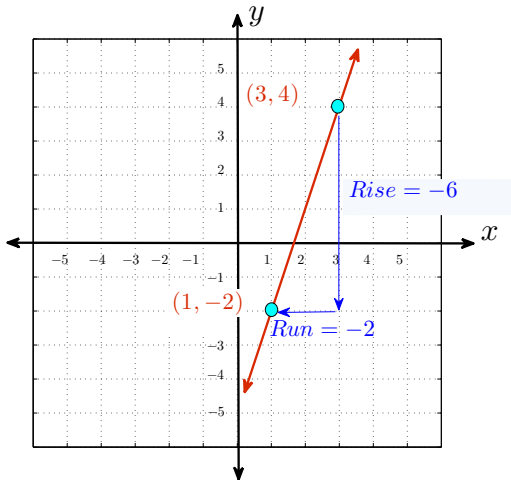
$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{6}{2} = 3$$





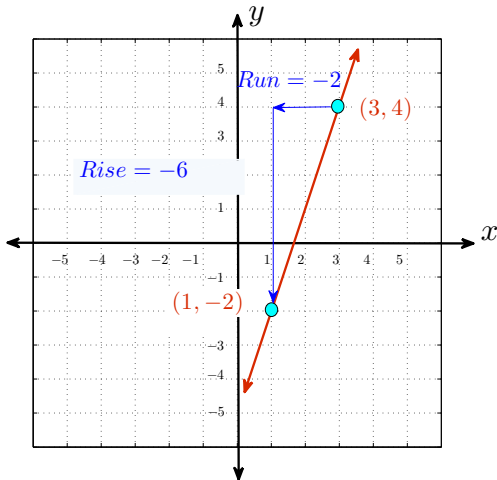
Ex. Find the slope of a line through (3, 4) and (1, -2).

$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{-6}{-2} = 3$$

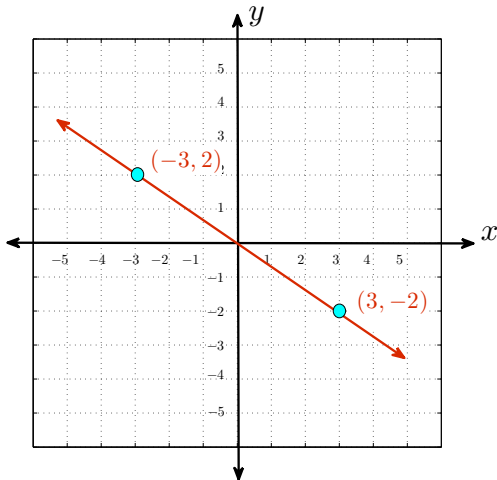


Ex. Find the slope of a line through (3, 4) and (1, -2).

$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{-6}{-2} = 3$$

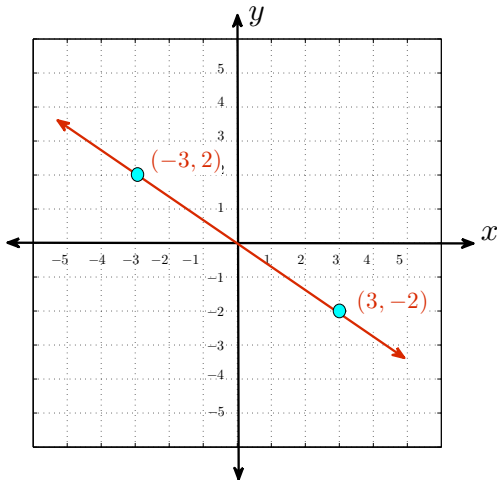


Ex. Find the slope of a line through  $(-3, 2)$  and  $(3, -2)$ .



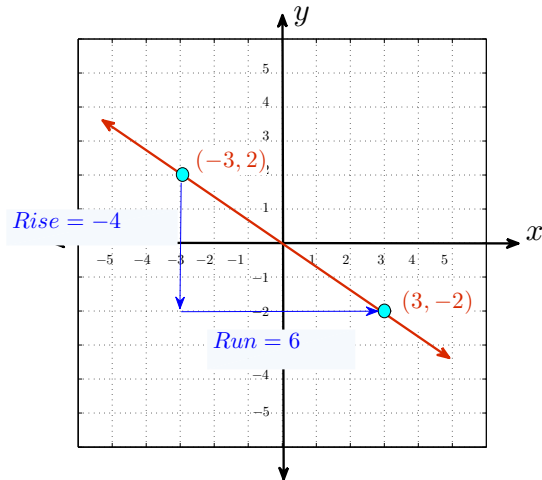
Ex. Find the slope of a line through  $(-3, 2)$  and  $(3, -2)$ .

We expect a negative slope for the solution because the graph of the line falls from left to right.



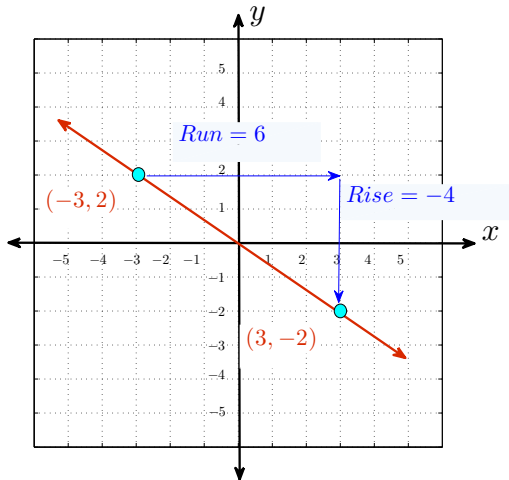
Ex. Find the slope of a line through  $(-3, 2)$  and  $(3, -2)$ .

$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{-4}{6} = \frac{2 \cdot (-2)}{2 \cdot 3} = \frac{\cancel{2} \cdot (-2)}{\cancel{2} \cdot 3} = -\frac{2}{3}$$



Ex. Find the slope of a line through  $(-3, 2)$  and  $(3, -2)$ .

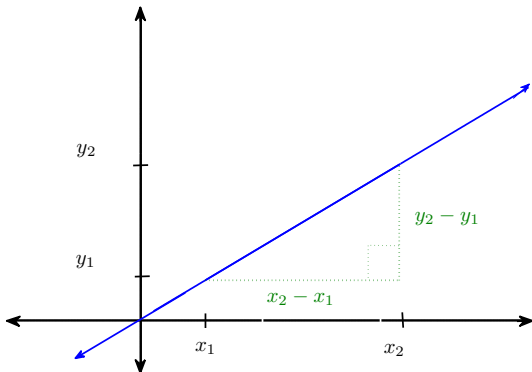
$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{-4}{6} = \frac{2 \cdot (-2)}{2 \cdot 3} = \frac{\cancel{2} \cdot (-2)}{\cancel{2} \cdot 3} = -\frac{2}{3}$$



## Definition

Let  $(x_1, y_1)$  and  $(x_2, y_2)$  be any two points on the rectangular coordinate plane. The **SLOPE** of a line which passes through the points  $(x_1, y_1)$  and  $(x_2, y_2)$  is  $m$ , where  $m$  is given by the formula:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$



## Classroom Examples: Take the next three minutes to work these 2 problems.

Use the slope formula,  $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ , to find the slope of a line containing the given points.

- $(7, -4)$  and  $(4, 2)$
- $(2, -3)$  and  $(-1, -3)$



## Theorem

*If line  $L_1$  has slope  $m_1$  and line  $L_2$  has slope  $m_2$ , then*

*$L_1$  is parallel to  $L_2$  if and only if  $m_1 = m_2$*

## Theorem

*If line  $L_1$  has slope  $m_1$  and line  $L_2$  has slope  $m_2$ , then*

*$L_1$  is perpendicular to  $L_2$  if and only if  $m_1 \cdot m_2 = -1$  (or  $m_1 = -\frac{1}{m_2}$ )*

## Theorem

If line  $L_1$  has slope  $m_1$  and line  $L_2$  has slope  $m_2$ , then

$L_1$  is perpendicular to  $L_2$  if and only if  $m_1 \cdot m_2 = -1$  (or  $m_1 = -\frac{1}{m_2}$ )

