Welcome to Spring 2020 Math Jam!

Prerequisite Refresher for Statistics

with *Professor Tim Busken*

Review Topics

1. Study skills for math
2. Rounding and Estimation (basics)
3. Exponents and Scientific Notation
4. Reading and Interpreting Graphs, Charts and Tables
5. Percents, Decimals, and Fractions
6. Order of Operations
7. Solving Linear Equations
8. Proportions
9. Translating Words into Math and Vice Versa
10. Slope, Intercepts and Equations of Lines
11. Applications of Linear Graphs and Equations

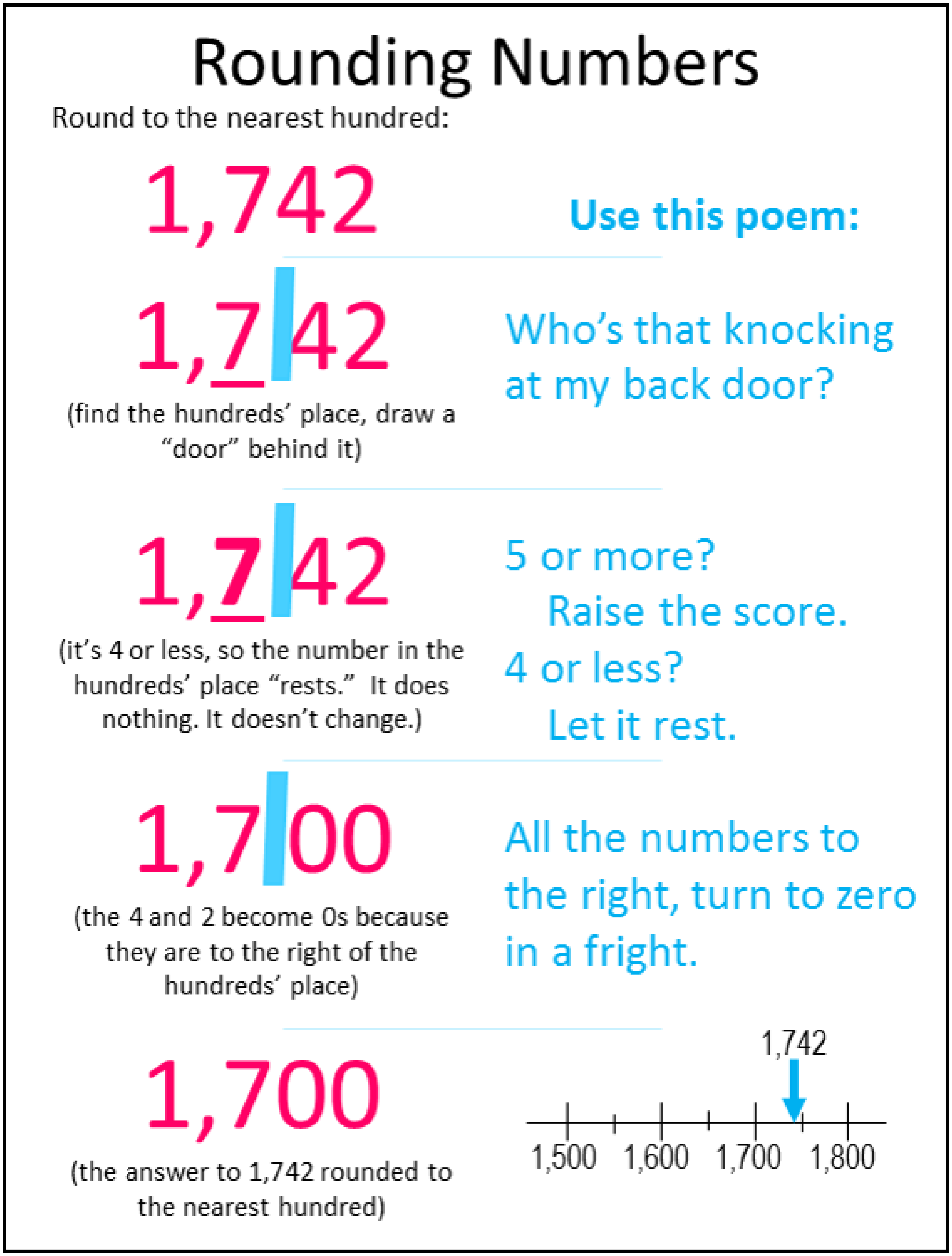


Figure 1: http://mycleverendeavors.blogspot.com/2015/07/favorites-for-friday-rounding-numbers.html

Place Value and Decimals													
Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	and	Tenths	Hundredths	Thousandths	Ten-Thousandths	Hundred-Thousandths	Millionths
5,	1	7	4,	8	5	1	.	3	5	6	8	0	0

Exercises

1. Round 0.760 to the nearest one	2. Round 2.64 to the nearest one
3. Round 3.785 to the nearest one	4. Round 23.44 to the nearest one
5. Round 35,973.21 to the nearest hundred	6. Round 35,973.21 to the nearest hundredth
7. Round 3.785 to the nearest one	8. Round 23.44 to the nearest one
9. Round 1.099 to the nearest hundredth	10. Round 5.76892 to the nearest hundredth

11. Round 0.0000056 to the nearest ten-thousandth	12. Round 0.0000012656 to the nearest millionth
13. Round 1.6599 to the nearest hundredth	14. Round 7.23599 to the nearest thousandth
15. Round 0.00353535 to the nearest ten-thousandth	16. Round 0.02768002 to the nearest tenth
17. Round 0.45678 to the nearest one	18. Round 0.007899 to the nearest hundred-thousandth
19. Round 0.05399 to the nearest ten-thousandth	20. Round 5.72997 to the nearest hundredth

Web Link to Khan Academy Rounding Decimals Exercises

https://www.khanacademy.org/math/arithmetic/arith-decimals/arith-review-rounding-decimals/e/rounding-decimals

Exponents and Scientific Notation

We know that repeated addition of the same number could be written as a product (multiplication). For example,

$$5+5+5+5+5+5+5$$

(repeated addition of 5) can be written as the product 7×5, which is equal to 35. Is there a simplified way to write repeated multiplication like $2 \times 2 \times 2 \times 2 \times 2$? The answer is yes. For that we would use exponents (or powers). Then $2 \times 2 \times 2 \times 2 \times 2$ would be written as 2^5.

Definition: For an integer n and any number a, we write a to the power of n as a^n, where

$$a^n = \underbrace{a \times a \times a \times \cdots \times a}_{\text{n times}}$$

We call a the **base** and n the **power** or **exponent**.

Exercises

1. Write $5 \times 5 \times 5 \times 5$ using exponents	2. Write $x \cdot x \cdot x \cdot x$ using using exponents
3. Bill says 2^3 equals 6. Is Bill right?	4. Simplify $x^3 \cdot x^5$

Another way to write the solution to Exercise 4 is using the **"Product Property of Exponents"**, the property that says

$$x^N \cdot x^M = x^{N+M}$$

The solution to Exercise 4 can then be written: $x^3 \cdot x^5 = x^{3+5} = x^8$.

Exercises

5. Simplify $x^4 \cdot x^5 \cdot x^5$	6. Simplify $x^7 \cdot x^{-11}$
7. Simplify $x^6 \cdot y^5 \cdot x^3 \cdot y^3$	8. Simplify $(x^4)^3$

Another way to write the solution to Exercise 8 is using the **"Power of a Power Property of Exponents"**, the property that says

$$(x^N)^M = x^{N \times M}$$

The solution to Exercise 8 can then be written: $(x^4)^3 = x^{4\times 3} = x^{12}$

Exercises

8. Simplify $(x^5)^4$	9. Simplify $\left(x^2\right)^3 \cdot \left(x^4\right)^{-1}$
10. $\left(y^3\right)^{-1} \cdot \left(x^4\right)^3$	11. Simplify $(5x)^2$

Another way to write the solution to Exercise 11 is using the **"Power of a Product Property of Exponents"**, the property that says

$$(xy)^N = x^N \cdot y^N$$

The solution to Exercise 11 can then be written: $(5x)^2 = 5^2 \cdot x^2 = 25x^2$

Exercises Let's put all three properties together and try a couple more.

$$x^N \cdot x^M = x^{N+M} \qquad (x^N)^M = x^{N \times M} \qquad (xy)^N = x^N \cdot y^N$$

12. Simplify $(x^5y^3)^4 \cdot x^3$	13. Simplify $\left(x^{-5}y^{-3}\right)^{-2} \cdot y^9$
14. Simplify $10^4 \cdot 10^2$	15. Simplify $\left(10^{-5}\right)^{-3} \cdot 10^{-9}$

Now, we know that one way simplify, or write the fraction $\dfrac{15}{35}$ in lowest terms is to write it

$$\frac{15}{35} = \frac{3 \cdot 5}{7 \cdot 5} = \frac{3 \cdot \cancel{5}}{7 \cdot \cancel{5}} = \frac{3}{7}$$

But can we use this idea to simplify $\dfrac{x^7}{x^5}$??

$$\frac{x^7}{x^5} = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{\cdot x \cdot x \cdot x \cdot x \cdot x} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = \frac{x \cdot x}{1} = x^2$$

Alternatively, we could have used the "**Quotient Property of Exponents**", which says that $\dfrac{x^N}{x^D} = x^{N-D}$ to write the answer as

$$\frac{x^7}{x^5} = x^{7-5} = x^2$$

Exercises

16. Simplify $\frac{4^9}{4^7}$

17. Simplify $\frac{2^{53}}{2^{54}}$

18. Simplify $\frac{18^{55}}{18^{55}}$

The last two examples bring up some interesting ideas: $\frac{18^{55}}{18^{55}} = 18^{55-55} = 18^0$ from our rule. ***But what does a number raised to the 0 power mean?*** Let's look deeper.

Example: Find the result of $\frac{7^2}{7^2}$ in two different ways.

Solution: $\frac{7^2}{7^2} = 7^0$ from our rule, but what if we expanded the powers out: $\frac{7^2}{7^2} = \frac{7 \cdot 7}{7 \cdot 7}$. Simplifying this, we can see that $\frac{7^2}{7^2} = \frac{49}{49} = 1$, but our rule shows $\frac{7^2}{7^2} = 7^0$. This means that the two results must be the same, which means $7^0 = 1$. Use your calculator to confirm this fact.

Now what does 2^{-1} represent?

To determine the value of 2^{-1}, let's multiply it by something and see what happens. For this example, we'll multiply by 2.

Determine the value of: $2^{-1} \cdot 2$

But from the multiplicative inverse (reciprocal) property, we know that there is already something to multiply by 2 and end with 1, *that is*, the reciprocal of 2.

Determine the value of: $\frac{1}{2} \cdot 2$

Since both of these equations are equal to 1, we can say that $2^{-1} = \dfrac{1}{2}$

Summary: Properties of Exponents

1. $x^N \cdot x^M = x^{N+M}$
2. $(x^N)^M = x^{N \times M}$
3. $(xy)^N = x^N \cdot y^M$
4. $x^0 = 1$ (as long as x is not 0)
5. $x^{-N} = \dfrac{1}{x^N}$

Scientific Application: Negative and positive exponents allow us to rewrite large and small numbers in a way that makes them easier to handle. There are many ways to rewrite a number like 14,000.

	Written as standard multiplication	**Written as multiplication with exponents on 10**
A)	$14{,}000 \times 1$	$14{,}000 \times 10^0$
B)	1400×10	1400×10^1
C)	140×100	140×10^2
D)	$14 \times 1{,}000$	14×10^3
E)	$1.4 \times 10{,}000$	1.4×10^4
F)	$0.14 \times 100{,}000$	0.14×10^5

Since every expression in the last table is equal to 14,000, scientists had to choose the one that was easiest to work with. Standard multiplication becomes tedious when there are a large number of 0's in a number, so the exponent on 10 method was chosen; scientists write numbers so they look like $a \times 10^n$.

And the choice for the value of *a*? Scientists picked the version of the number so $1 \le |a| < 10$. From our previous table, only one number meets that condition: 1.4×10^4.

Writing a number in the form $a \times 10^n$ where $1 \le |a| < 10$ and *n* is an integer, is called **scientific notation**. Remember that other formats will have the same value, but only one is scientific notation.

Let's do one more table for the number 0.00078. Remember that $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$.

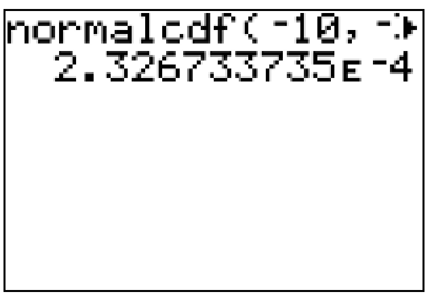
	Written as standard multiplication	**Written as multiplication with exponents on 10**
A)	0.00078×1	0.00078×10^0
B)	$0.0078 \times \frac{1}{10}$	0.0078×10^{-1}
C)	$0.078 \times \frac{1}{100}$	0.078×10^{-2}
D)	$0.78 \times \frac{1}{1{,}000}$	0.78×10^{-3}
E)	$7.8 \times \frac{1}{10{,}000}$	7.8×10^{-4}
F)	$78 \times \frac{1}{100{,}000}$	78×10^{-5}

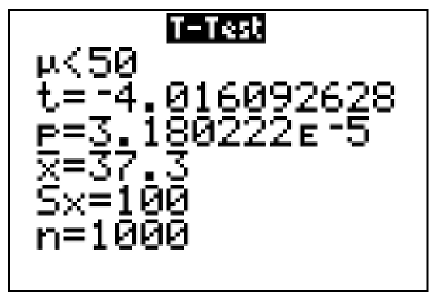
Interactive Example 4: Which of the formats in the table above is in scientific notation?

Exercises Write each number in scientific notation

1. 0.003	2. 0.00003215
3. 0.00000689	4. 0.0003215
5. 15.235	6. 147
9. 256,450	8. 91,258,694

10. Both calculator screens below show a number in scientific notation. Round both numbers to the ten-thousandths. Are both numbers less than or greater than 5%?





Percents

Definition "Percent" means "per 100" or "for every 100" or "number of parts out of 100."

Recall that the word "per" means "divided by." Therefore, 3% is equal to $\frac{3}{100}$, or 0.03 since 3 divided by 100 is 0.03. Additionally, notice that $100\% = \frac{100}{100} = 1$.

Exercises

1. Write 23% as a decimal
2. Write 57% as a fraction
3. Write 0.12 as a percent
1. Write 7% as a decimal
4. Write 0.4 as a percent
5. Write 0.06 as a percent
6. Write 2.46 as a percent
7. Write 0.0049 as a percent
8. Write $\frac{11}{7}$ as a percent
9. Write $3\frac{2}{5}$ as a percent

10. Jen thinks that $10^{-1} = 10\%$. Is she correct?
11. Write the ratio "7 out of 100" as a percent 12. Write the ratio "7 out of 40" as a percent 13. It is found that the residents of Hugetown City have been infected with a particular virus. The CDC estimates that 3.23508% of it's 3.4 million residents have been infected.

    a.) About how many of the 3.4 million residents have been infected?
    b.) About how many residents out of every 100 are infected?
    c.) About how many residents out of every 1000 are infected?
    d.) About how many residents out of every 10,000 are infected?
    e.) About how many residents out of every 100,000 are infected?

14. The rate of homelessness in Bigquake County is 1.23456%. About how many residents out of every 10,000 are homeless?

Learning Objectives:

- ☞ **Directly translate word phrases into equations**
- ☞ **Solve Percent Problems**

Recognizing key words in a percent problem is helpful in writing the problem as an equation. Three key words in the statement of a percent problem and their meanings are:

__of__ means __multiplication__ (·)
__is__ means __equal__ (=)
__what__ (or some equivalent) means __the unknown number__

Example 1: Translate this phrase to an equation: 3 is what percent of 12?

Solution: 3 is what percent of 12?

↓ ↓ ↓ ↓ ↓

$3 = x \cdot 12$

Example 2: Translate this phrase to an equation: 3.2 is 60% of what number?

Solution: 3.2 is 60% of what number?

↓ ↓ ↓ ↓ ↓

$3.2 = 60\% \cdot x$

Example 3: Translate this phrase to an equation: What number is 20% of 0.07

Solution: What number is 20% of 0.07?

↓ ↓ ↓ ↓ ↓

$x = 20\% \cdot 0.07$

✎ ***Exercises***

1. What is 73% of 900? 1. ____________
2. 45% of 360 is what number? 2. ____________
3. 65% of what number is 75? 3. ____________
4. What percent of 17 is 58? 4. ____________

Our percent problems have 3 numbers, two are known and one (x) is unknown. Each of these three numbers has a name.

$$\begin{array}{c} 7 \text{ is } 50\% \text{ of } 14 \\ \downarrow\ \downarrow\ \ \downarrow\ \ \ \downarrow\ \downarrow \\ 7 = 50\% \cdot 14 \\ \swarrow \quad \downarrow \quad \searrow \\ \text{amount} = \text{percent} \cdot \text{base} \end{array} \longrightarrow \text{This is called the Percent Equation.}$$

✎ ***Solving Percent Equations***

5. What number is 25% of 90? 5. ____________

6. 95% of 400 is what number? 6. ____________

7. 63 is what percent of 45? 7. ____________

Learning Objectives:

- ☞ **Write Percent Problems as Proportions**
- ☞ **Solve Percent Problems using Proportions**

In this section, we set up a proportion instead of a percent equation when solving for the unknown.

Percent Proportion

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100} \longleftarrow \text{always } 100$$

When translating percent problems to proportions, we use the letters p, b and a. We use p to represent the percent, b to represent the base, *or whole*, and a to represent the amount, *or part compared to the whole*. **The base, b, usually follows the word *of*.**

$$\begin{array}{r} \text{amount} \longrightarrow \\ \text{base} \longrightarrow \end{array} \frac{a}{b} = \frac{p}{100} \longleftarrow \text{percent}$$

Example 1: Translate to a proportion: 35% of what number is 84?
Solution:

$$\begin{array}{r} \text{amount} \longrightarrow \\ \text{base} \longrightarrow \end{array} \frac{84}{b} = \frac{35}{100} \longleftarrow \text{percent}$$

Example 2: Translate to a proportion: 1.2 is 47% of what number?
Solution:

$$\begin{array}{r} \text{amount} \longrightarrow \\ \text{base} \longrightarrow \end{array} \frac{1.2}{b} = \frac{47}{100} \longleftarrow \text{percent}$$

Example 3: Translate to a proportion: What number is 23% of 574?
Solution:

$$\begin{array}{r} \text{amount} \longrightarrow \\ \text{base} \longrightarrow \end{array} \frac{a}{574} = \frac{23}{100} \longleftarrow \text{percent}$$

Example 4: Translate to a proportion: 157 is what percent of 119?
Solution:

$$\begin{array}{r} \text{amount} \longrightarrow \\ \text{base} \longrightarrow \end{array} \frac{157}{119} = \frac{p}{100} \longleftarrow \text{percent}$$

✎ ***Translate each question to an equation or proportion. Afterwards, solve for x. Round to the nearest tenth, if necessary.***

1. 2.5% of 90 is what number? 1. ________

2. 30 is 6% of what number? 2. ________

3. 550.4 is what percent of 172? 3. ________

4. What number is 53% of 130? 4. ________

5. What is 65% of 2400? 5. ________

Translating Phrases into Equations and Inequalities

at least $\geq$
not less than $\geq$
at most $\leq$
not more than $\leq$
no difference $=$
is different from $\neq$
a majority means more than 50%
over $>$
under $<$

Bibliography

Math Fundamentals for Statistics I, by Scott Fallstrom and Brent Pickett